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UNDERSTANDING THE PLANCK CONSTANT AND THE BEHAVIOUR OF PHOTON PARTICLES FROM A MECHANICAL PERSPECTIVE

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ABSTRACT

The Planck constant is derived from the analysing of the energy and frequency relationship of a simple harmonic oscillator model for photon particles from a mechanical perspective. The correlation between the Planck constant, the inertial mass, the angular frequency, and the corresponding radius of the harmonic oscillation of the photon particle is derived. The photon particles have equal mechanical angular momentum. This insight is applied to the propagation of photon particles in the free space and other transparent media based on the model of the cycloid motion of harmonic oscillators. The interdependence of the properties of photon particles and their entanglements with their surrounding spaces are elicited. Kepler's second and third laws of motion are deduced for the specified photon particle in the free space and other transparent media. For any particle in periodic motion with the rotational symmetry, its inertial mass, its time period, and its space displacement are entangled together by its conserved mechanical angular momentum. The centre of the periodic motion with the rotational symmetry is proposed as a universally applicable reference frame to simplify and unify the laws of physics. The generalized Planck constant, generalized Planck-Einstein relation, and generalized de Broglie relation are proposed for applications in both microcosms and macrocosms.

Keywords: Planck constant, mechanical angular momentum, cycloid motion, generalized Planck constant, generalized Planck-Einstein relation, generalized de Broglie relation.

INTRODUCTION

The Planck constant is the pillar of modern quantum physics. The Planck constant has profound ramifications in three important areas: our technology, our understanding of reality, and our understanding of life (Stein, 2011. Planck's constant: The number that rules technology, reality, and life. http://www.pbs.org/wgbh/nova/blogs/physics/2011/10/plancks-constant). The Planck constant h entered physics in the beginning of the 20th century as the result of Max Planck's attempts to provide a theoretical explanation for the empirically discovered laws of thermal blackbody radiation (Planck, 1901; Planck, 1900). He found that the experimental observations of thermal blackbody radiation spectrum can be speculated in perfect agreement, if one adopted the concept that matter was a collection of discrete harmonic oscillators (Oldershaw, 2013) emitting electromagnetic radiation in packages (energy quanta) that obeyed an energy (E) / frequency (f) law of the following form:

$$E = hf \quad (1)$$

One of the earliest applications of the Planck constant was by Einstein to explain some aspects of Wien's law for blackbody radiation and to account for the photoelectric effect (Einstein, 1905). Einstein introduced the idea of a photon particle with energy related to its frequency as $E = hf$ called the Planck-Einstein relation. The establishment of the Planck-Einstein relation is generally regarded as the starting point of quantum physics. The Planck constanthas become one of the most essential universal constants. However, the physical origin and nature of the Planck constant have not been fully understood. Investigating the physical origin and nature of the Planck constant is not only important for advancing our understanding on the foundation of modern quantum physics for microcosms, it is also relevant to the current study of cosmology (Chang, 2017). Stellar Planck constant has been proposed to describe the characteristic quantities of angular momentum and action inherent in the objects of the stellar level of matter (Fedosin, 2015; Fedosin, 2014). Basically, every aspect of nature is associated with quantum phenomena involving the Planck constant. Understanding the physical origin and nature of the Planck constant may open the door to unify the physical laws of the microcosm and the macrocosm, the classical and the quantum.

The Planck constant is linked with periodic motion with rotational symmetry. In order to get a better insight into the physical origin and the nature of the Planck constant, an approach is discovered to derive the Planck constant through the analysing of the energy and frequency

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relationship of a simple harmonic oscillator model for a photon particle from a mechanical perspective. The gained insight into the physical origin and the nature of the Planck constant is both applicable and powerful in elucidating the property of photon particles in the free space (vacuum) and other transparent media. All photon particles have an equal mechanical angular momentum $\hbar$. The interdependence of the properties of photon particles and their entanglements with their surrounding spaces are elicited. The inertial mass of any particle in periodic motion with rotational symmetry should not be equal to zero. Any particle in periodic motion with rotational symmetry, its inertial mass, its time period, and its space displacement are entangled together with its conserved mechanical angular momentum.

The exploration of the physical origin and the nature of the Planck constant from a mechanical perspective

The exploration begins from analysing the energy and frequency relationship of a simple harmonic oscillator with rotational symmetry as shown schematically in Figure 1. For harmonic oscillation to occur, the system must possess the following two quantities: elasticity and inertia. The simplest example of a harmonic oscillating system is a mass connected to a rigid foundation by way of a spring. At this stage, we assume that the spring is undamped and massless under ideal approximation. However, a nonzero spring mass or damped spring can be easily accommodated under some simplifying assumptions and can be developed further for the more accurate understanding of reality (Garret, 2017; Zhang, 2021a; Zhang, 2021b). The elasticity of the spring k (spring constant) provides the elastic restoring force that enables the system to return to its equilibrium when the system is displaced from its equilibrium position. The inertia of the mass m (inertial mass) provides the overshoot that warrants the system to pass the equilibrium position. The natural angular frequency of the oscillation ω is related to the elastic and inertia properties of the system.

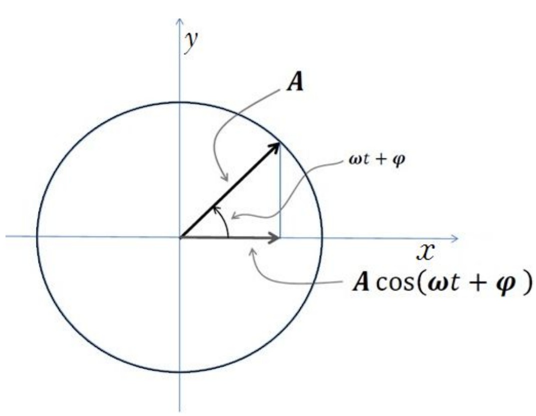

Fig. 1. The corresponding harmonic circular motion of the simple harmonic oscillator.

By applying Newton's second law $F = ma$ and Hooke's law $F = -kx$, the equation of motion for the simple mass-spring harmonic oscillating system is obtained as follows:

$$ma = -kx \rightarrow m\frac{d^2x}{dt^2} + kx = 0 \rightarrow$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$$

where ω is the natural angular frequency of the oscillation, i.e.

$$\omega = \sqrt{elasticity/inertia} = \sqrt{k/m} \quad (2)$$

The solution to the equation of motion takes the following form:

$$x(t) = A\cos(\omega t + \varphi)$$

where A is the amplitude of the oscillationor the corresponding radius of the harmonic circular motion and φ is the phase constant. Both the constants A and φ are determined by the initial condition at a chosen time of $t = 0$ under the ideal approximation that the spring is undamped and massless. Figure 1 shows schematically the corresponding harmonic circular motion of the simple harmonic oscillator. The period T (the time to complete one corresponding circle), the frequency f, and the angular frequency ω of the oscillating are defined as follows:

$$T = 1/f \text{ and } \omega = 2\pi f = \frac{2\pi}{T} \quad (3)$$

As the system oscillates, the total energy E of the system remains constant and time-independent, and depends only on the elasticity parameter k and the maximum displacement A (or the mass m and the maximum magnitude of velocity $V_m = \omega A$), i.e.

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mV^2 = \quad (4)$$

$$\frac{1}{2}kA^2cos^2(\omega t + \varphi) + \frac{1}{2}m\omega^2A^2sin^2(\omega t + \varphi)$$

$$= \frac{1}{2}kA^2 = \frac{1}{2}mV_m^2 = \frac{1}{2}m\omega^2A^2$$

where V is the magnitude of velocity (speed) of the point of the mass m. For the corresponding harmonic circular motion as shown schematically in Figure 1, one can write down that

$$V = A\omega \quad (5)$$

Differentiation of the total energy $E = \frac{1}{2}m\omega^2A^2$ in equation (4) with respect to the angular frequency ω generates the following expression:

$$\frac{dE}{d\omega} = mA^2\omega = A^2\sqrt{km} \quad (6)$$

Hence,

$$\Delta E = A^2\sqrt{km}\,\Delta\omega = 2\pi A^2\sqrt{km}\,\Delta f \quad (7)$$

Comparing equation (7) with equation (1), the Planck constant h for the photon particle at the reference frame of constant linear light velocity can be derived as follows:

$$h = 2\pi A^2\sqrt{km} (\text{or } \hbar = A^2\sqrt{km}) \quad (8)$$

where A is the radius of the harmonic circular motion, k is the elasticity parameter of the space, m is the equivalent inertial mass of the photon particle and $\hbar$ is the reduced Planck constant or Dirac's constant, $\hbar = h/2\pi$. Incorporating equation (2), (5), and (8), especially for the harmonic circular motion of the photon particle at the reference frame of constant linear light velocity, it is derived that

$$\hbar = A^2\sqrt{km} = m\omega A^2 = mVA \quad (9)$$

where V is the average of the magnitude of velocity (speed) of the photon that represents the constant speed c in the free space (vacuum) and the constant V in other transparent homogeneous media. While mVA is the mechanical angular momentum of the harmonic circular motion of photon particles. Hence, the reduced Planck constant $\hbar$ is the mechanical angular momentum of the harmonic circular motion of photon particles.

It is worth to point out that the mechanical angular momentum of photon particles from a mechanical perspective is different to the optical angular momentum of the corresponding photon waves from an electromagnetic perspective (Bliokh and Nori, 2015), although they have interdependence. The conservation of mechanical angular momentum is fundamentally associated with periodic motions of the rotational symmetry and can be calculated using Noether's theorem (Bliokh and Nori, 2015). For the photon particles, equations (1), (8), and (9) disclose the correlation and interdependence of E, f, m, k, V, ω, A, and h. Physical science is all about the correlation of physical quantities. If we take h ($\hbar$) and the velocity of photon $V = c$ in the free space (vacuum) as some universal constants, then the m (equivalent inertial mass), the ω (time period) and the A (space displacement) of the photon particles are entangled together with the constant mechanical angular momentum $\hbar$.

From equation (9), it also can be seen that the reduced Planck constant $\hbar$ is a constant determined by the interacting of the photon particle and its surrounding space. If a simple dissipative element as a viscous damper was added to the simple harmonic oscillator model to take into account the weak friction force of the free space, it is derived theoretically that the energy and the inertial mass of the photon particle can decrease slowly, its corresponding radius of harmonic circular motion increases slowly (red-shift), its mechanical angular momentum is kept as a constant according to equation (9) (Zhang, 2021a; Zhang, 2021b). The harmonic oscillator model of photon particles also paves the way for a possible explanation that the energies and the inertial mass of photon oscillators can be increased under driving forces and works, meanwhile, their corresponding radii are decreased (blue shift) according to equation (9). A driving and dampening harmonic oscillator model may be further developed to explain the production of high energy photons, electron-positron pairs, and other elementary particles.

Conservation of mechanical angular momentum in macrocosms is well-known as Kepler's second law or Kepler's equal area law. In microcosms, it is well-known as the quantization of angular momentum, which was initially proposed as one of Bohr's key hypotheses in the Bohr model of hydrogen. The mechanical angular momentum is a conservative quantity in both microcosms and macrocosms for periodic motions with rotational symmetry, although the real trajectories of particles in microcosms are quite different with the trajectories of particles in macrocosms. The trajectories of particles in microcosms are blurred to clouds in approximately round shapes by transient energy fluctuations. It is unveiled that the quantization of mechanical angular momentum in quantum mechanics and Kepler's second law in astronomy arise from the same fundamental law of the conservation of mechanical angular momentum for periodic motions with rotational symmetry. For a particle in periodic motion with rotational symmetry, its mechanical angular momentum is a constant and can never be equal to zero. Therefore, its energy forms into quantum (discrete energy) in the frequency domain according to equation (7). The Planck-Einstein relation can be generalized for any particle in the periodic motion with rotational symmetry as follows:

$$E = H_A\,\omega \quad (10)$$

$$H_A = A^2\sqrt{km} = m\omega A^2 = mVA \quad (11)$$

where E is the energy of the particle in periodic motion with the rotational symmetry, ω is the angular frequency of the particle, H_A is the conserved mechanical angular momentum. H_A may be named as the generalized Planck constant. The particle can be a photon particle, an elementary particle, an atom, a planet, a star, or a galaxy. It shall be emphasized that from equation (7) to equations (1) and (10), a zero-energy point is assigned while the frequency of the periodic motion is equal to zero. If the frequency is equal to zero, the motion shall be a motion along an absolute straight line without spinning or oscillation. This kind of motion may not exist in the Universe. The typical motions at elementary particle scale, atomic scale, stellar scale, and galactic scale are periodic motions with the rotational symmetry. Hence, the conservation of mechanical angular momentum is fundamental in both microcosms and macrocosms. All particles in periodic motion with the rotational symmetry in both microcosms and macrocosms, their inertial masses, their time periods, and their space displacements are entangled together through their conserved mechanical angular momentums. Einstein's general relativity is a kind of beautiful and abstractive mathematical description of the physical world famously depicted as: Matter tells space-time how to curve; space-time tells matter how to move (Wheeler and Ford, 1998, 2000) that indicates the entanglement of matter, space, and time. However, it does not reveal the fundamental nature of the physical world, why and how? Now it is uncovered that the fundamental nature of the physical world is the entanglement of the inertial mass, the time period, and the space displacement of the particle in periodic motion with the rotational symmetry, which arises from the fundamental law of the conservation of mechanical angular momentum for motions with rotational symmetry. Within the three parameters of inertial mass, time period, and space displacement of a particle in periodic motion with rotational symmetry, only two parameters are independent.

Although the Planck constant is derived from analysing the energy and frequency relationship of the simple harmonic oscillator, it is applicable and powerful in elucidating the property and propagation of photon particles in both the free space (vacuum) and other transparent media, which will be illustrated in the following sections. It will be shown that Kepler's third law (the Period law) can be derived for a specified photon particle based on the new insight into the physical nature and the physical origin of the Planck constant.

The property and propagation of photon particles in the free space (vacuum)

It is proved experimentally that the velocity of photons in the free space is the constant speed *c* over a wide range of frequencies, the relation between energy E and momentum p of a propagating single frequency photon particle is $E = pc$ (Nelson and Kinder, 2017). The propagation of the single frequency photon particle in the free space can be treated as the cycloid motion of a harmonic oscillator from a mechanical perspective shown schematically in Figure 2. This model of the cycloid motion of the harmonic oscillator for the single frequency photon particle illustrates vividly the Wave Particle Durability of a photon and it can greatly assist in the derivation of correlations between the properties of photons. In the period T (one cycle, $\theta = 2\pi$), the single frequency photon particle rolls the wavelength λ, which equals to one circumference $2\pi R = 2\pi A$ as shown in Figures 1 and 2, where

$$\lambda = 2\pi A \tag{12}$$

In the free space, it is experimentally approved that

$$\lambda / T = c \tag{13}$$

From equations (2), (3), (8), (12), and (13), it can be derived that

$$m = h/c\lambda = \hbar\omega/c^2 \tag{14}$$

$$k = 4\pi^2 hc/\lambda^3 \tag{15}$$

Incorporating equations (1), (2), (3), and (14), it is obtained that

$$E = hf = \hbar\omega = hc/\lambda = mc^2 \tag{16}$$

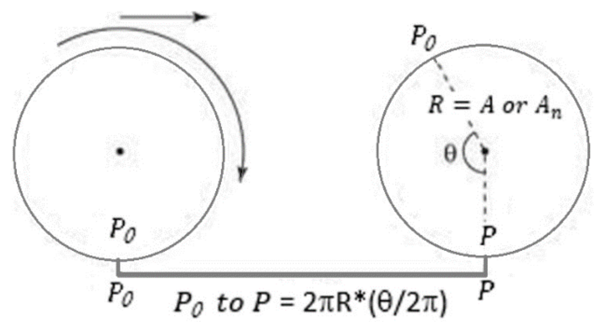

Fig. 2. The schematic diagram showing the cycloid motion model for a photon particle.

The derived equality $E = mc^2$ is the Einstein mass-energy equation. However, the mass of the single frequency photon particle in equation (16) shall be the inertial mass of the single frequency photon particle from a mechanical perspective. In this article, for the mass of the single frequency photon particle, it is called either equivalent mass or inertial mass to distinguish it with the rest mass in Einstein's special relativity. The single frequency photon particle will be simply called the photon particle below.

Based on equation (16), the momentum of the photon particle can be deduced as follows:

$$p = E/c = mc = h/\lambda \tag{17}$$

where $p = h/\lambda$ is the de Broglie relation. From equations (3), (16), and (17), for the photon particle within one period of T or one wavelength of λ, it can be derived that

$$ET = h \tag{18}$$

$$p\lambda = mc\lambda = h \tag{19}$$

Fascinatingly, for the specified photon particle with the fixed frequency or wavelength, Kepler's third law (the Period law) can be derived from equations (2), (3), and (15) as follows:

$$T^2 = 4\pi^2 m A^3/\hbar c = K_{kep} A^3 \tag{20}$$

where K_{kep} is Kepler's constant for the specified photon particle in periodic motion in the free space. It is defined by

$$K_{kep} = 4\pi^2 m/\hbar c \tag{21}$$

Equations (20) and (21) disclose the correlation of $T, A, c, m, \hbar$, and K_{kep} for the photon particles in periodic motions in the free space. Kepler's laws of motion are applicable in both microcosms and macrocosms. If we substitute equation (14) into equation (20), it can be derived that

$$T^2 = 4\pi^2 A^2/c^2 \tag{22}$$

How could equations (20) and (22) be both true? The answer lies in the conserved mechanical angular momentum of the cycloid motion of photon particles in the free space, $m/\hbar = 1/(cA)$ for the specified photon particle. It may be easier to be understood from another point of view, equation (22) is true for any photon particle in any inertial reference frame at some constant speed in the free space. Equation (20) is only true for an imagined observer at the centre of the harmonic motion of the photon particle. This is an imagined inertial reference frame at constant linear light velocity moving together with the rotation centre of the photon particle. Because of the cycloid motion of photon particles, if we accept $\lambda = 2\pi A$, the measured parameters T, c, λ and the calculated parameter A in any inertial reference frame agree with each other. Similar to K_{kep}, the inertial mass m of the photon particle shall be the inertial mass in the imagined reference frame at constant linear light velocity moving together with the rotation centre of the specified photon particle. According to equation (14), the photon particle has the mass equivalent to the inertial mass of the photon particle in the imaged reference frame at constant linear light velocity from a mechanical perspective, which can be calculated as $m = h/(c\lambda) = 2.21021906 \times 10^{-42}\ (1/\lambda)$. From the imagined reference frame at constant linear light velocity moving together with a stream of photon particles to a laboratory reference frame on earth or in a satellite, the inertial masses of photon particles cannot be transferred to a zero inertial mass because the zero inertial mass cannot be transferred back to a range of inertial masses. Hence, the inertial masses of photon particles in a laboratory reference frame on the Earth or inside a satellite cannot be equal to zero. From a theoretical perspective, a finite inertial mass for the photon particle will lead to Proca's Lagrangian and Yukawa potential (Proca, 1936; Tu *et al*., 2005; Caccavano and Leung, 2013; Nyambuya, 2014), which is perfectly compatible with the general principles of elementary particle physics, which may lead to unified physics laws. Maxwell's Equations and Coulomb's Law with great successes in science and engineering are based on the ideal assumptions of massless photon and the frictionless free space, i.e. a vacuum. They are ideal approximations of physical realities, which are brilliant enough for the scale of size of the Earth-Moon system.

However, on a cosmic scale such as the Milky Way Galaxy and above, modifications are needed (Zhang, 2021a; Zhang, 2021b). Einstein's theories of relativity sprang out of Maxwell's equations. Hence, the ideal approximations of the massless photon and the frictionless free space were inherited. Modern cosmology theories originated from Einstein's relativities have met difficulties and challenges (Zhang, 2021a; Zhang, 2021b; Potter, 2009; Traunmüller, 2017). It is well-known that it is extremely difficult to reconcile Einstein's relativities with quantum mechanics. The quantitative understanding of the inertial masses and the entanglement of inertial masses, time periods, and space displacements of particles in periodic motion with rotational symmetry from a universally applicable reference frame may help to resolve the difficulties and challenges. The reference frame of the centre of the periodic motion with rotational symmetry can be a universally applicable reference frame, which may simplify and unify the laws of physics.

Let us now focus back on the photon particles travelling in the transparent media with refractive indices. Interestingly, the inertial mass of the photon particle in a transparent medium can have a sizable increase in comparison with the one in the free space (vacuum) that will be elucidated in the following section.

The property and propagation of photon particles in transparent media with refractive indices

In the early 20th century, two possible candidates for the momentum of a photon propagating through a transparent medium emerged. One was proposed by Hermann Minkowski (1908) that the photon's momentum was given by $p_n = n\hbar\omega/c$. A year later, Max Abraham (1909) came up with a different expression for the photon's momentum: $p_n = \hbar\omega/nc$. This controversy is known as the Abraham-Minkowski dilemma(Partanen *et al*., 2017).

These rivalling momenta differ by n^2, which is a sizeable factor in most media: in water, n is approximately 1.33 and in glass n is ~ 1.46 (Leonhardt, 2006). Over the years, physicists have found supporting evidences, from both first-principles arguments and experiments, for each expression in different contexts, some of them have even proposed ways that the apparent paradox might be resolved (Partanen *et al*., 2017; Leonhardt, 2006; Cho, 2010; Barnett, 2010; Sheppard and Kemp, 2016; Kemp, 2011; Brevik, 1979; Brevik, 2017; Testa, 2013; Crenshaw, 2013). Through computer simulations, Partanen *et al.* (2017) claimed that they proved the transfer of mass with the light pulse representing the photon mass drag effect. The photon mass drag effect gives an essential contribution to the total momentum of the light pulse, which becomes equal to Minkowski's momentum. It is claimed that the Minkowski momentum is the total momentum of the system, the Abraham momentum is the portion of the momentum carried by the light field, and the difference between the two is carried by the atomic mass density wave. The claimed experiments have only been taken in computer simulations so far. Due to the coupling of the field and matter, Abraham's momentum is unable to be measured directly, only the total momentum of the light pulse (Minkowski's momentum) can be directly measured.

The behaviour of photon particles in the transparent media with refractive indices can be elucidated in a simple and clear way based on the new insight into the physical origin and nature of the Planck constant. Subtly, some materials are transparent at least to some kinds of light. For example, water and glass are transparent to visible light, air is transparent to a wide range of frequencies of light. When a stream of photon particles travels from the free space (vacuum) to a transparent medium and travels through it with negligible energy absorption, their energies and frequenciescan be viewed as no change. However, the elastic interactions with electrons inside the medium do have the net effect of slowing the photons down (Nelson and Kinder, 2017). The velocity of the photon in the transparent medium is c/n, where c is the velocity of photons in the free space, n is the medium's refractive index, a dimensionless number larger than 1. More precisely, the index depends on the frequency f, so the relation can be better written as c/n_f (Nelson and Kinder, 2017). As now we focus theoretically on monochromatic light with single frequency photon particles, n instead of n_f will be used for simplicity.

Let's recall the model of cycloid motion for photon particles, as schematically shown in Figure 2. In the period T, the photon particle rolls the wavelength λ_n in the medium with the refractive index n. The wavelength λ_n equals to one circumference $2\pi A_n$, where A_n is the amplitude of the harmonic motion in the transparent medium with the refractive index n, the radius of the corresponding harmonic circular motion $R = A_n$. Hence,

$$\lambda_n = 2\pi A_n \tag{23}$$

$$\lambda_n/T = \lambda_n f = V = c/n \tag{24}$$

$$\omega = 2\pi f = \sqrt{elasticity/inertia} = \sqrt{k_n/m_n} \tag{25}$$

where k_n is the elasticity of the medium, m_n is the inertial mass of the photon particle in the transparent medium. The Planck constant is still a constant from the free space to the transparent medium as the mechanical angular momentum of the photon is conservative based on Noether's theorem, i.e.

$$h = 2\pi A^2\sqrt{km} = 2\pi A_n^2\sqrt{k_n m_n} \tag{26}$$

$$\hbar = m\omega A^2 = mcA = m_n\omega A_n^2 = m_n V A_n \tag{27}$$

From equations (23)-(26), it can be derived that

$$m_n = h/V\,\lambda_n \tag{28}$$

$$k_n = 4\pi^2 hV/\lambda_n^3 \tag{29}$$

Incorporating equations (1), (24) and (28), it is obtained that

$$E = \hbar\omega = hf = hV/\lambda_n = m_n V^2 \tag{30}$$

Based on equation (30), the momentum of the photon particle can be deduced as follows:

$$p_n = m_n V = h/\lambda_n = nE/c = n\hbar\omega/c \tag{31}$$

where $p_n = h/\lambda_n$ is the de Broglie relation. From equations (30) and (31) for the photon particle within one period of T or one wavelength of λ_n of its periodic motion, it is derived as follows:

$$ET = h \tag{32}$$

$$p_n\lambda_n = m_n V\,\lambda_n = h \tag{33}$$

Equations (32) and (33) may be generalized for all particles in periodic motion with the rotational symmetry, i.e.

$$ET = 2\pi H_A \tag{34}$$

$$p\lambda = mV\lambda = 2\pi H_A \tag{35}$$

where H_A is the mechanical angular momentum of the particle in periodic motion with rotational symmetry. $E, p, \lambda, m,$ and V are subsequently the energy, momentum, wavelength, inertial mass, and the velocity of the particle in periodic motion with rotational symmetry. Equation (35) may be named generalized de Broglie relation. Equation (34) is essentially another form of equation (10).

It is interesting to notice that the inertial mass of the single frequency photon particle in the medium (m_n) increased in comparison with the one in the free space (m):

$$m_n = h/V\ \lambda_n = n^2(h/c\lambda) = n^2\frac{\hbar\omega}{C^2} = n^2 m \qquad (36)$$

where λ is the wavelength of the photon in the free space (vacuum). Partanen *et al.* (2017) showed that with the light pulse in a medium, the mass transfer equals to $(n^2-1)\frac{\hbar\omega}{c^2}$ with an assumption that the inertial mass of the photon particle in the free space equals to zero. Our calculations show that the inertial mass of the photon particle in the free space is $\frac{\hbar\omega}{C^2}$ and the inertial mass of the photon particle in the transparent medium equals to $n^2\frac{\hbar\omega}{c^2}$. The increase of inertial mass from the free space to the transparent medium is $(n^2-1)\frac{\hbar\omega}{C^2}$. The results from equations (23) to (36) imply that there are interdependences of the property of photon particles, and the property of photon particles is also entangled with their surrounding spaces in the transparent media. Their surrounding spaces can be different in the processes of emitting, propagation, and the measurement of the photon particles.

It is fascinating to notice from equations (31) that the de Broglie relation is kept the same form in the transparent medium as in the free space. According to equations (24) and (36), the inertial mass of the photon particle in the transparent medium is $m_n = n^2 m$, the photon particle gains the inertial mass while it slows down. The square of the velocity of the photon particle in the transparent medium is $V^2 = \frac{c^2}{n^2}$, hence, $E = m_n V^2 = mc^2$, the energy is conservative. It can be seen also that from the free space (vacuum) to the other transparent medium, the frequency of the photon is kept the same and the Planck constant is no change, thus the laws of conservation of both energy and angular momentum are obeyed. The conservation of energy arises from the negligible energy dissipation (under the ideal approximation) in both the free space and the transparent medium. The conservation of optical angular momentum is associated with the rotational symmetry and can be calculated using Noether's theorem. Amazingly, Kepler's third law can be elicited from equations (23), (24), (25), and (29) as follows:

$$T^2 = 4\pi^2 m_n A_n^3/\hbar V = K_{kep} A_n^3 \qquad (37)$$

where K_{kep} is Kepler's constant for the specified photon particle with the inertial mass m_n in the transparent medium, namely

$$K_{kep} = 4\pi^2 m_n/\hbar V = 4\pi^2 n\, m_n/\hbar c \qquad (38)$$

Equations (37) and (38) disclose the correlation or entanglement of $T, A_n, V, m_n, \hbar,$ and K_{kep} for the photon particles in the transparent medium. The entanglement of the photon particles with their surrounding spaces is elicited. According to the derived equation (31), the photon particle's momentum calculated from the cycloid motion model agrees with Minkowski's prediction $p_n = n\hbar\omega/c$. The Snell law of refraction may be re-analysed and clarified as a piece of supportive experimental evidence. The refraction of light in transparent media obeys Snell's law as shown schematically in Figure 3. The angles of incidence (θ) and refraction (ϕ) at an interface are inversely proportional to the respective indices of refraction n_1 and n_2, which are characteristics for each medium:

$$n_1\sin(\theta) = n_2\sin(\phi) \qquad (39)$$

The bending of light rays is regarded as the evidence for the existence of a force acting at the interface between the two media. The resultant force acting on the light is managed in the direction normal to the interface (Buenker and Muiño, 2004). Because of Newton's second law, the component of the momentum of the photons, which is parallel to the interface, shall be constant (Fig. 3). Thus, one can write that

$$p_1\sin(\theta) = p_2\sin(\phi) \qquad (40)$$

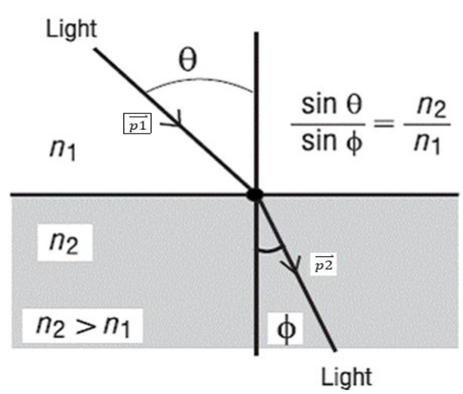

Fig. 3. The schematic diagram showing the refraction of light at an interface between two transparent media.

Comparing equation (40) with equation (39), it can be deduced that the total momentum of the photon particle is always proportional to the refractive index n of the given medium. Assume that medium 1 represents the free space, i.e. a vacuum. Hence, we have $n_1 = 1$. Since the light is bent toward the normal in transparent medium 2

$(n_2 > n_1 = 1)$, we are led to the conclusion that the momentum of the photon particle is greater in transparent medium 2 in comparison with the one in the free space (medium 1), i.e.

$$p_2 = (n_2/n_1)\,p_1 = n_2 p_1 = n_2\,\hbar\omega/c \quad (41)$$

Hence, Snell's law of refraction supports the derived photon particle's momentum $p_n = n\hbar\omega/c$ that agrees with Minkowski's prediction. The Snell law of refraction can be viewed as a strong supportive experimental evidence of the cycloid motion model of photon particles (Padyala, 2019).

It is not by chance that the mechanical model of the cycloid motion of harmonic oscillator is able to illustrate vividly the wave-particle durability of photons and it is powerful in assisting the discovering of the interdependence and entanglement of the properties of photons in both the free space and other transparent media. The mechanical model must be linked with the physical reality of photon waves of the oscillating electric and magnetic fields from an electromagnetic perspective. Further research is needed to reveal the links in details.

CONCLUSION

The Planck constant is the pillar of modern physics. However, the physical origin and nature of the Planck constant have not been fully understood. In this article, the Planck constant was derived through analysing the energy and frequency relationship of the simple harmonic oscillator model for the photon particle from a mechanical perspective. It was derived that $h = 2\pi A^2\sqrt{km}$ and $\hbar = A^2\sqrt{km} = m\omega A^2 = mVA$. All single photon particles travelling in both the free space and other homogeneous transparent media have equal mechanical angular momentum $\hbar$ because they are in motion with identical rotational symmetry. These discoveries are applied to the propagation of photon particles in the free space by viewing the propagation of the photon particle as the cycloid motion of the harmonic oscillator. Amazingly, the Einstein mass-energy equation and the de Broglie relation are deduced subsequently. It is also elicited that the single frequency photon particle has an inertial mass in the free space such as $m = h/c\lambda = \hbar\omega/c^2$. It is further unveiled that the quantization of mechanical angular momentum in quantum mechanics and Kepler's second law (the equal area law in astronomy) arises from the same fundamental law of the conservation of mechanical angular momentum. Kepler's third law of motion (the Period law) is derived for the specified photon particle in both the free space and other homogeneous transparent media. From a widely applicable point of view, all particles in periodic motions with the rotational symmetry, their inertial masses, their time periods, and their space displacements are entangled together by their conserved mechanical angular momentums. These discoveries indicate the opening of the door to unify the physical laws of the microcosm and the macrocosm, the classical and the quantum. The reference frame of the centre of the periodic motion with the rotational symmetry is proposed as a universally applicable reference frame to simply and unify the laws of physics.

Applying the discoveries to the propagation of photon particles in the transparent medium with a refractive index, it is deduced that the de Broglie relation is kept the same form as in the free space. However, the equation for the energy of the single frequency photon particle transformed from $E = mc^2$ to $E = m_n V^2$, while its energy and mechanical angular momentum are both conservative. The Abraham-Minkowski dilemma was briefly reviewed. The derived momentum of the single frequency photon particle based on the new method agrees with Minkowski's prediction as most of experimental observations do. Snell's law of refraction was re-analysed and clarified as another piece of supportive experimental evidence to Minkowski's prediction. Snell's law of refraction is an important supportive evidence of the cycloid motion model of photon particles.

The new insight into the physical nature and origin of the generalized Planck constant, the generalized Planck-Einstein relation, and the generalized de Broglie relation can be equally applicable to atomic-scale systems, stellar scale systems, and galactic scale systems. Further research is needed to reveal the links between the cycloid motion of photon particles and the oscillating electric and magnetic fields of their corresponding electromagnetic waves.

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