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ON HOMOMORPHISMS (GOOD HOMOMORPHISMS) BETWEEN COMPLETELY $\mathcal{J}^{\circ}$-SIMPLE SEMIGROUPS

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ABSTRACT

It is known that every completely $\mathcal{J}^{\circ}$-simple semigroup is isomorphic to a normalized Rees matrix semigroup over a $\circ$- monoid. Utilizing this result, we show that the homomorphism of a completely $\mathcal{J}^{\circ}$-simple semigroup is a good homomorphism. Consequently, we give a construction theorem of homomorphisms between completely $\mathcal{J}^{\circ}$-simple semigroups. This result strengthens the one given by Ren *et al*. (2018) on homomorphisms of completely $\mathcal{J}^{*}$-simple semigroups.

Keywords: Completely $\mathcal{J}^{*}$-simple semigroups, Normalizes Rees matrix semigroup, $\circ$- monoid, Good homomorphism.

Mathematics Subject Classification: 20M10

INTRODUCTION

In generalizing regular semigroups to abundant semigroups, Fountain (1982) adopted the following relations defined as follows: let a and b be elements of the semigroup S,

$$\mathcal{L}^{*} = \{(a,b) \in S \times S : (\forall\, x, y \in S^{1})\ ax = ay \Leftrightarrow bx = by\},$$

$$\mathcal{R}^{*} = \{(a,b) \in S \times S : (\forall\, x, y \in S^{1})\ xa = ya \Leftrightarrow xb = yb\},$$

$$\mathcal{H}^{*} = \mathcal{L}^{*} \cap \mathcal{R}^{*},\ \mathcal{D}^{*} = \mathcal{L}^{*} \vee \mathcal{R}^{*}.$$

It is well known that the above relations is called the Green's *-relations. The following relations given below generalizes the Green's *-relations and are called Green's $\circ$- relations.

Let a and b be elements of the semigroup S,

$$\mathcal{L}^{\circ} = \{(a,b) \in S \times S : (\forall\, x, y \in S^{1})\ ax = a \Leftrightarrow bx = b\},$$

$$\mathcal{R}^{\circ} = \{(a,b) \in S \times S : (\forall\, x, y \in S^{1})\ xa = a \Leftrightarrow xb = b\},$$

$$\mathcal{H}^{\circ} = \mathcal{L}^{\circ} \cap \mathcal{R}^{\circ},\ \mathcal{D}^{\circ} = \mathcal{L}^{\circ} \vee \mathcal{R}^{\circ}.$$

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We denote the relations $\mathcal{L}^{\circ}, \mathcal{R}^{\circ}, \mathcal{H}^{\circ}$ and $\mathcal{D}^{\circ}$ in S by $\mathcal{L}^{\circ}(S), \mathcal{R}^{\circ}(S), \mathcal{H}^{\circ}(S)$ and $\mathcal{D}^{\circ}(S)$ respectively. The $\mathcal{L}^{\circ}$ -class containing an element a of the semigroup S is denoted by L_a° or $L_a^{\circ}(S)$ in case of ambiguity. The corresponding notation will be used for classes of the other $\circ$- relations.

Following Fountain (1982), a semigroup S is called an abundant semigroup if each $\mathcal{L}^{*}$-class and each $\mathcal{R}^{*}$ -class contains an idempotent. An abundant semigroup is said to be super abundant if each $\mathcal{H}^{*}$-class contains an idempotent, see Ren and Shum (2004). We call a semigroup S $\circ$- abundant if every $\mathcal{L}^{\circ}$-class and$\mathcal{R}^{\circ}$-class of S contains an idempotent of S,moreover, if every $\mathcal{H}^{\circ}$-class of an $\circ$- abundant S contains an idempotent, then we call such a semigroup $\circ$- superabundant. Clearly, all regular semigroups are $\circ$- abundant and all completely regular semigroups are $\circ$- superabundant. Obviously, both $\circ$- abundant semigroups and $\circ$- superabundant semigroups are natural generalization of regular semigroups and also completely regular semigroups are within the class of $\circ$- abundant semigroups.

A semigroup S is a unipotent monoid Wang *et al.* (2004), if S has a unique idempotent which is its identity. A unipotent monoid S with the identity e is called a $\circ$- monoid if S satisfies the condition that for any $x, y \in S$, $xy = x$ or $yx = x$ implies that $y = e$. Clearly, a cancellative monoid is a $\circ$-monoid. For a more detailed knowledge, see Fountain (1982), Howie (1995), Ren and Shum (2007),Wang *et al.* (2004).

It is well known that completely simple semigroups is a very important semigroup in the class of completely regular semigroups. Chualin and Yonghua (2011) gave the structure of completely $\mathcal{J}^{\circ}$-simple semigroups which can be regarded as a natural generalization of a completely simple semigroups given in Petrich and Reilly (1999)and completely $\mathcal{J}^{*}$-simple semigroups given in Ma *et al.* (2011)in the class of $\circ$- abundant semigroups. Moreover, in the theory of $\circ$- abundant semigroups, a homomorphic image of an $\circ$- abundant semigroup need not be $\circ$- abundant and so the notion of good homomorphism for $\circ$- abundant is given as follows: a semigroup homomorphism $\theta : S \to T$ is said to be good if for any $a, b \in S$, $a\, \mathcal{L}^{\circ}\, b$ implies $a\theta\, \mathcal{L}^{\circ}\, b\theta$ and $a\theta\, \mathcal{R}^{\circ}\, b\theta$.

Our aim is to consider the homomorphisms and good homomorphisms between completely $\mathcal{J}^{*}$-simple semigroups and describe the method of construction of the homomorphisms (good homomorphisms) between completely $\mathcal{J}^{*}$-simple semigroups.In this paper, for the undefined notion and notations the reader is referred to Chualin and Yonghua (2011).

2. Preliminaries

If a semigroup S has an idempotent$e$, the following characterization is known.

Lemma 2.1 (Chualin and Yonghua, 2011)**.**Let S be a semigroup and e be an idempotent in S.Then the following conditions are equivalent:

i) $a\, \mathcal{L}^{\circ}\, e$.
ii) $ae = a$ and for all $x \in S$, $ax = a$ implies $ex = e$.

It is easy to show that if a, b are regular elements of a semigroup S, then $a \mathcal{L}\, b\, (a\, \mathcal{R}\, b)$ if and only if $a\, \mathcal{L}^{*}\, b$ ($a\, \mathcal{R}^{*}\, b$) if and only if $a\, \mathcal{L}^{\circ}\, b$ ($a\, \mathcal{R}^{\circ}\, b$). Futhermore, it is easy to check that $\mathcal{L}^{\circ}$and $\mathcal{R}^{\circ}$ are a right and a left congruence respectively in a semigroup S. It is important to note that $\mathcal{L}^{\circ}$ is not always a right congruence as illustrated in the example below.

Example 2.2 (Chualin and Yonghua, 2011). Suppose $S = \{p, q, r, s, t, u, v\}$ with the following Cayley Table:

$\cdot$	p	q	r	s	t	u	v
p	p	r	r	t	t	u	v
q	s	q	u	s	v	u	v
r	t	r	v	t	v	v	v
s	s	u	u	v	v	v	v
t	t	v	v	v	v	v	v
u	v	u	v	v	v	v	v
v	v	v	v	v	v	v	v

The semigroup S is generated by two idempotents$p$and q. It can be easily checked that S is associative. Then $(p, s) \in \mathcal{L}^{\circ}$, but $(ps, ss) = (t, v) \notin \mathcal{L}^{\circ}$. Therefore $\mathcal{L}^{\circ}$ is not a right congruence.
A semigroup S is said to satisfy $(CR)((CL))$condition if $\mathcal{L}^{\circ}$ ($\mathcal{R}^{\circ}$) is a right (left) congruence and S satisfy (C) condition if $\mathcal{L}^{\circ}$and $\mathcal{R}^{\circ}$ are right and left congruence, respectively.

Proposition 2.3 (Chualin and Yonghua, 2011). Suppose S be a semigroup satisfying (C) and $e \in E(S)$.ThenH_e° is a $\circ$- monoid.
If S does not satisfy (C), then the H°-class which contains an idempotent may not be a $\circ$- monoid. It is well-known inFountain(1982) that $\mathcal{H}^{*}$-class which contains an idempotent is a cancellativemonoid. But this does not always hold on $\mathcal{H}^{\circ}$, see Chualin and Yonghua (2011).
The connection between the Green's *-relations and the Green's $\circ$- relations lies in the following Lemma.

Lemma 2.4 (Chualin and Yonghua, 2011).Let S be a left $*$- abundant semigroup, then $\mathcal{L}^{\circ} = \mathcal{L}^{*}$.

According to Chuanlin and Yonghua (2011) an $\circ$-abundant semigroup S without zero is called a completely $\mathcal{J}^{\circ}$-simple semigroup if S itself is primitive and the idempotents of S generate a regular subsemigroup of S.

Completely $\mathcal{J}^\circ$ -simple semigroups have been extensively studied in Chualin and Yonghua (2011). We now state the following useful Lemmas.

Lemma 2.5 (Chualin and Yonghua, 2011). Let S be a semigroup. Then the following statements are equivalent:

i) S is a completely $\mathcal{J}^\circ$-simple semigroup

ii) S is $\circ$ - superabundant and $\mathcal{J}^\circ$-simple

iii) S is isomorphic to a mormalized Rees matrix semigroup $M[T; I, \Lambda: P]$ over a $\circ$ - monoid T, in which I, Λ are non-empty sets, each entry in P is a unit of T.

Lemma 2.6 (Chualin and Yonghua, 2011). Let S be a completely $\mathcal{J}^\circ$ -simple semigroup. Then the following statements are true:

i)each $\mathcal{H}^\circ$-class contain a regular element

ii) each $\mathcal{H}^\circ$-class of S is isomorphic to a $\circ$ - monoid.

The following Lemma is evident.

Lemma 2.7(Chualin and Yonghua, 2011).Let S be a completely $\mathcal{J}^\circ$-simple semigroup and $a, b \in S$.Then $|H_a^\circ| = |H_b^\circ|$.

Lemma 2.8 (Chualin and Yonghua, 2011). Let S be a normalized Rees matrix semigroup and $M[T; I, \Lambda: P]$ is a matrix semigroup over a $\circ$ - monoid T in which each entry in$P$ is a unit of T, and let $(i, x, \lambda), (j, y, v) \in S = M[T; I, \Lambda: P]$. Then the following statements hold:

i)$(i, x, \lambda)\mathcal{L}^\circ(j, y, v)$if and only if $\lambda = v$.

ii) $(i, x, \lambda)\mathcal{R}^\circ(j, y, v)$if and only if $i = j$.

The following Lemma due to Fountain, 1982 for abundant semigroups can be easily adopted for $\circ$ - abundant semigroups.

Lemma 2.9. Let S be an $\circ$ - abundant semigroup and $\theta : S \to T$be a homomorphism of semigroups. Then the following statements are equivalent:

i) θ is a good homomorphism

ii) For each elements $a \in S$, there exists idempotents e, f with $e \in L_a^\circ$ and $f \in R_a^\circ$ such that $a\theta\, \mathcal{L}^\circ(T)\, e\theta, a\theta\, \mathcal{R}^\circ(T)\, f\theta$.

3. Homomorphisms

We begin with an important property of completely $\mathcal{J}^\circ$-simple semigroups.

Theorem 3.1. Let $S = M[T; I, \Lambda: P]$ and $S' = M[T'; I', \Lambda': Q']$ be two completely $\mathcal{J}^\circ$ -simple semigroups and let $\theta : S \to S'$ be a homomorphism. Then θ is a good homomorphism.

Proof. Let $S = M[T; I, \Lambda: P]$ be a completely $\mathcal{J}^\circ$-simple semigroup and let $a = (i, x, \lambda)$, $b = (j, y, v) \in S$. It follows from Lemma 2.4 and Lemma 2.5 that there exist an idempotent $e = (i, p_{\lambda i}^{-1}, \lambda) \in S$ such that $ae = a$. Suppose that $\theta : S \to S'$ is a homomorphism, then we have that

$$a\theta = (i, x, \lambda)\theta = (i', x', \lambda') \in S',$$
$$b\theta = (j, y, v)\theta = (j', y', v') \in S',$$
$$e\theta = (i, p_{\lambda i}^{-1}, \lambda)\theta = (i^*, (p_{\lambda i}^{-1})^*, \lambda^*) \in S'.$$

Suppose that $a\, \mathcal{L}^\circ\, b$. Thus from Lemma 2.1, we have $ae = a$and $be = b$. But θ is a homomorphism, so it follows that

$$a\theta = (ae)\theta = a\theta e\theta,$$
$$b\theta = (be)\theta = b\theta e\theta.$$

Consequently, we have

$$(i', x', \lambda') = (i', x', \lambda')(i^*, (p_{\lambda i}^{-1})^*, \lambda^*)$$
$$= (i', x' q_{\lambda' i^*}(p_{\lambda i}^{-1})^*, \lambda^*),$$
$$(j', y', v') = (j', y', v')(i^*, (p_{\lambda i}^{-1})^*, \lambda^*)$$
$$= (j', y' q_{v' i^*}(p_{\lambda i}^{-1})^*, \lambda^*).$$

Hence, $\lambda' = \lambda^* = v'$. From Lemma 2.8, we have that $(i, x, \lambda)\theta\, \mathcal{L}^\circ(j, y, v)\theta$.

Similarly, it follows that$(i, x, \lambda)\theta\, \mathcal{R}^\circ(j, y, v)\theta$. Thus, θ is a good homomorphism.

The following corollary is an immediate consequence of Theorem 3.1.

Corollary 3.2. Let $S = M[T; I, \Lambda: P]$ and $S' = M[T'; I', \Lambda': Q']$ be two completely $\mathcal{J}^\circ$ -simple semigroups. Suppose $\theta : S \to S'$, then θ maps each $\mathcal{L}^\circ$-class of S into $\mathcal{L}^\circ$-class of S' and maps each $\mathcal{R}^\circ$-class of S into $\mathcal{R}^\circ$-class of S'.

The theorem below which is analogous to the one given in Ren*et al.* (2018) presents a structure homomorphism theorem of completely $\mathcal{J}^\circ$-simple semigroups.

Theorem 3.3. Let $S = M[T; I, \Lambda: P]$ and $S' = M[T'; I', \Lambda': Q']$ be two completely $\mathcal{J}^\circ$ -simple semigroups and let $r : i \mapsto r_i$ be a mapping of I intoT' and $l : \lambda \mapsto l_\lambda$ be a mapping of ΛintoT' respectively. Let $\alpha : I \to I'$, $\beta : \Lambda \to \Lambda'$ and define a homomorphism $\sigma : \mathrm{T} \to \mathrm{T}'$ by the rule

$$p_{\lambda i}\sigma = l_\lambda q_{\lambda\beta, i\alpha} r_i$$

for all $i \in I$ and $\lambda \in \Lambda$. Define a map $\theta : S \to S'$ by the rule that

$$(i,x,\lambda)\theta = (i\alpha, r_i(x\sigma)l_\lambda, \lambda\beta)$$
where $(i,x,\lambda) \epsilon S$. Then θ is a homomorphism. Conversely, every homomorphism is of this type.

Proof. For the direct part of the theorem, we are to show that for $a = (i,x,\lambda)$, $b = (j,y,v) \epsilon S$, we have $a\theta b\theta = (ab)\theta$.

So we have that
$(i,x,\lambda)\theta(j,b,v)\theta$
$= (i\alpha, r_i(x\sigma)l_\lambda, \lambda\beta)\ (j\alpha, r_j(y\sigma)l_v, v\beta)$
$= (i\alpha, r_i(x\sigma)l_\lambda q_{\lambda\beta,j\alpha} r_j(y\sigma)l_v, v\beta)$
$= (i\alpha, r_i(x\sigma)(p_{\lambda j}\sigma)(y\sigma)l_v, v\beta)$
(since $p_{\lambda j}\sigma = l_\lambda q_{\lambda\beta,i\alpha} r_i$)
$= (i\alpha, r_i(xp_{\lambda j}y)\sigma l_v, v\beta)$
(since σ is a homomorphism)
$= \left(i, (xp_{\lambda j}y, v\right)\theta$
$= \left((i,x,\lambda)(j,b,v)\right)\theta.$

Thus θ is a homomorphism.

Conversely, let $S' = M[T'; I', \Lambda': Q']$ be a completely $\mathcal{J}^\circ$-simple semigroup. Denote the set of $\mathcal{R}^\circ$-classes of S and S' by I and I' and then the set of $\mathcal{L}^\circ$-classes of S and S' by Λ and Λ'. Infact, we shall treat I and Λ as index sets and write $\mathcal{R}^\circ$-classes as $R_i^\circ (i \epsilon I)$ and the $\mathcal{L}^\circ$-classes as $L_\lambda^\circ (\lambda \epsilon \Lambda)$. Without loss of generality we may suppose that there exists an element $1 \epsilon I \cap \Lambda$ such that $H_{11}^\circ = T$. It is obvious from Corollary 3.2 that there exists a map $\alpha : I \to I'$ and $\beta : \Lambda \to \Lambda'$ respectively. Also, by Lemma 2.5, each $\mathcal{H}^\circ$-class of S is isomorphic to a $\circ$ - monoid. Put $H_{11}^\circ = T$ and $H_{1\alpha,1\beta}^\circ = T'$ respectively.

Now let $p_{11}^{-1} \epsilon T$ be an element of maximal subgroup in the $\circ$ - monoid and let $p_{11}^{-1}\theta = {q_{1\beta,1\alpha}}^{-1}$. Thus every element $x \epsilon T$ can be expressed as
$$(1, p_{11}^{-1}x, 1)\theta = \left(1\alpha, {q_{1\beta,1\alpha}}^{-1}(x\sigma), 1\beta\right).$$

For $\alpha : T \to T'$, suppose e is the identity element in the $\circ$ - monoid T, then $e\sigma$ is certainly the identity element in the $\circ$ - monoid T'. Define a mapping $r : i \longmapsto r_i$ of I into T' by the rule
$$(i,e,1)\theta = (i\alpha, r_i, 1\beta).$$

In a similar manner, define a mapping $l : \lambda \longmapsto l_\lambda$ of Λ into T' by the rule
$$(1, p_{11}^{-1}, \lambda)\theta = \left(i\alpha, {q_{1\beta,1\alpha}}^{-1} l_\lambda, \lambda\beta\right).$$
Consequently, we have
$$[(i,e,\lambda)(i,e,\lambda)]\theta = (i, p_{\lambda i}, \lambda)\theta$$
$$= [(i,e,1)(1, p_{11}^{-1}p_{\lambda i}, 1)(1, p_{11}^{-1}, \lambda)]\theta$$
$$= (i,e,1)\theta(1, p_{11}^{-1}p_{\lambda i}, \lambda)\theta(1, p_{11}^{-1}, \lambda)\theta$$
$$= (1\alpha, r_i, 1\beta)(1\alpha, {q_{1\beta,1\alpha}}^{-1}(p_{\lambda i}\sigma), 1\beta)(1\alpha, {q_{1\beta,1\alpha}}^{-1} l_\lambda, \lambda\beta)$$
$$= (i\varphi, r_i q_{1\beta,1\alpha}{q_{1\beta,1\alpha}}^{-1}(p_{\lambda i}\sigma), 1\beta)(1\alpha, {q_{1\beta,1\alpha}}^{-1} l_\lambda, \lambda\beta)$$
$$= (i\alpha, r_i(p_{\lambda i}\sigma)l_\lambda, \lambda\beta)$$
for $i \epsilon I, \lambda \epsilon \Lambda$.

Also, we have that
$$[(i,e,\lambda)(i,e,\lambda)]\theta = [(i,e,\lambda)\theta][(i,e,\lambda)\theta]$$
$$= (i\alpha, r_i(e\sigma)l_\lambda, \lambda\beta)\ (i\alpha, r_i(e\sigma)l_\lambda, \lambda\beta)$$
$$= \left(i\alpha, r_i(e\sigma)l_\lambda q_{\lambda\beta,i\alpha} r_i(e\sigma)l_\lambda, \lambda\beta\right).$$
Now,
since $[(i,e,\lambda)(i,e,\lambda)]\theta = (i\alpha, r_i(p_{\lambda i}\sigma)l_\lambda, \lambda\beta)$ and
$[(i,e,\lambda)(i,e,\lambda)]\theta$
$$= \left(i\alpha, r_i(e\sigma)l_\lambda q_{\lambda\beta,i\alpha} r_i(e\sigma)l_\lambda, \lambda\beta\right),$$

comparing the middle coordinates gives
$$r_i(p_{\lambda i}\sigma)l_\lambda = r_i(e\sigma)l_\lambda q_{\lambda\beta,i\alpha} r_i(e\sigma)l_\lambda$$
$$= r_i l_\lambda q_{\lambda\beta,i\alpha} r_i l_\lambda$$
(since $e\sigma$ is an identity element of T').

From $\theta : S \to S'$, it is known that the element $(i,e,1) \epsilon S$ is a completely regular element likewise $(i,e,1)\theta = (i\alpha, r_i, 1\beta) \epsilon S'$. Thus r_i belongs to the maximal subgroup of T'. Infact, there exists $r_i' \epsilon T'$ such that $r_i' r_i = e\sigma$. In a similar manner, there exists l_λ' such that $l_\lambda l_\lambda' = e\sigma$.

Now from $r_i(p_{\lambda i}\sigma)l_\lambda = r_i l_\lambda q_{\lambda\beta,i\alpha} r_i l_\lambda$, if we then multiply the LHS by r_i' and the RHS by l_λ', we have that
$$r_i' r_i(p_{\lambda i}\sigma)l_\lambda l_\lambda' = r_i' r_i l_\lambda q_{\lambda\beta,i\alpha} r_i l_\lambda l_\lambda'$$
$$\Longrightarrow e\sigma(p_{\lambda i}\sigma)e\sigma = e\sigma l_\lambda q_{\lambda\beta,i\alpha} r_i e\sigma$$
$$\Longrightarrow p_{\lambda i}\sigma = l_\lambda q_{\lambda\beta,i\alpha} r_i$$
(since $e\sigma$ is an identity element of T').

Thus, every element $(i,x,\lambda) \epsilon S$ can be expressed as $(i,x,\lambda) = (i,e,1)(1, p_{11}^{-1}x, 1)(1, p_{11}^{-1}, \lambda)$.

So we have that
$$(i,x,\lambda)\theta = [(i,e,1)(1, p_{11}^{-1}x, 1)(1, p_{11}^{-1}, \lambda]\theta$$
$$= (i,e,1)\theta(1, p_{11}^{-1}x, 1)\theta(1, p_{11}^{-1}, \lambda)\theta$$

$$= (i\alpha, r_i, 1\beta)(1\alpha, q_{1\beta,1\alpha}^{-1}(x\sigma), 1\beta)(1\alpha, q_{1\beta,1\alpha}^{-1} l_\lambda, \lambda\beta)$$
$$= (i\alpha, r_i q_{1\beta,1\alpha} q_{1\beta,1\alpha}^{-1}(x\sigma), 1\beta)(1\alpha, q_{1\beta,1\alpha}^{-1} l_\lambda, \lambda\beta)$$
$$= (i\alpha, r_i(x\sigma)l_\lambda, \lambda\beta)\ .$$

Finally, we show that θ is a homomorphism for $(i,x,\lambda), (j,y,v) \epsilon S$.

We now have that,

$$
\begin{aligned}
&(i,x,\lambda)\theta(j,y,v)\theta \\
&= (i\alpha, r_i(x\sigma)l_\lambda, \lambda\beta)(j\alpha, r_i(y\sigma)l_v, v\beta) \\
&= (i\alpha, r_i(x\sigma)l_\lambda q_{\lambda\beta,j\alpha} r_i(y\sigma)l_v, v\beta) \\
&= \big(i\alpha, r_i(x\sigma)\big(l_\lambda q_{\lambda\beta,j\alpha} r_i\big)(y\sigma)l_v\ , v\beta\big) \\
&\qquad = \big(i\alpha, r_i(x\sigma)\big(p_{\lambda j}\sigma\big)(y\sigma)l_v\ , v\beta\big) \\
&\qquad\quad = \big(i\alpha, r_i\big[\big(xp_{\lambda j}y\big)\sigma\big]l_v\ , v\beta\big) \\
&\qquad\qquad = \big(i, xp_{\lambda j}y, v\big)\theta \\
&= [(i,x,\lambda)(j,y,v)]\theta\ .
\end{aligned}
$$

Thus θ is a homomorphism.

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