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EFFECT OF CONFINEMENT OF GLUONS ON GROUND STATE HEAVY MESON SPECTRUM IN THE RELATIVISTIC HARMONIC MODEL

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ABSTRACT

In the frame work of relativistic harmonic model (RHM), we have investigated the mass spectrum of the S-wave heavy mesons. The full Hamiltonian used in the investigation has Lorentz scalar plus vector harmonic potential and confined one gluon exchange potential (COGEP). The mass of the mesons was obtained by diagonalsing a 5x5 matrix. A good agreement is obtained with the experimental masses of heavy mesons. The role of COGEP is discussed. The limitation of the perturbative treatment of estimating η_c-J/ψ is pointed out.

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INTRODUCTION

The meson spectroscopy is a broad subject covering from few hundred MeV masses of the light mesons to the 10 GeV scale of the $b\bar{b}$ system (Namgung and Lichtenberg, 1984). Such a wide energy region allows us to address perturbative and nonperturbative phenomena of the underlying Quantum Chromodynamics (QCD) theory. Though QCD is accepted as the fundamental theory of strong interactions, there exist no exact solutions to the theory in the non-perturbative low energy regime. The QCD is not exactly solvable in the non-perturbative regime which is required to obtain the physical properties of the hadrons. Hence various approximation methods have been employed to solve QCD in the non-perturbative regime. The most promising of these is through lattice gauge theories (Hooft, 1976). The lattice gauge theories involve gigantic computation, hence the progress has been slow and detailed predictions of the hadron properties have not been made. As a consequence, our understanding of hadrons continues to rely on insights obtained from the experiments and QCD motivated models in addition to lattice QCD results. The phenomenological models developed to explain observed properties of hadrons are either non-relativistic quark models (NRQM) (Bhaduri *et al*., 1981; Godfrey and Isgur, 1985; Blask *et al*., 1990; Brau *et al*., 2000; Vijande *et al*., 2005) with suitably chosen potential or relativistic quark models (RQM) (Jena 1983; Khadkikar and Vijaya, 1991; Vinodkumar *et al*., 1992) where the interaction is treated perturbatively. There are successful NRQM and RQM to explain the meson spectra (Vijaya *et al*., 1998; Bhavyshri *et al*., 2005 and 2008). The NRQM usually contain three main ingredients: the kinetic energy, confinement potential and a hyperfine interaction term which has often been taken as an effective one-gluon-exchange potential (OGEP) (De Rujula *et al*., 1975). On the other hand, the relativistic models have a confinement potential which is usually taken to be Lorentz scalar plus vector potential (Khadkikar and Gupta, 1983). There are models both non-relativistic and relativistic employed to explain meson spectra with OGEP (Semay and Silvestre-Brac, 1997, 1999; Takayuki *et al*., 2007).

In the present work, an attempt has been made to obtain the ground state mass of the heavy mesons in the frame work of relativistic harmonic model (RHM) (Khadkikar and Gupta, 1983; Khadkikar and Vijaya, 1991; Vijaya *et al*., 2004). The objective of present study was to obtain the ground state mass of the heavy mesons with minimum number of parameters and to investigate the relativistic effects on the mass spectrum. Infact, the non-relativistic models should work better for heavy quark mesons, since a particle of mass m, localised in a volume of radius R, has a momentum ~ 1/R through the uncertainty relation its kinetic energy $(< T >) << m$ only if $m R >> 1$. In the constituent quark model this is satisfied for the c,b and t quarks. Also, in NRQM the spurious excitation of the centre-of-mass (CM) motion can be eliminated easily. But as has been shown in reference (Rosner, 2007), the non-relativistic description for charmonium is quite crude, whereas it is substantially better for $b\bar{b}$ systems. Hence, to study the S wave spectra of heavy mesons we have made use of the successful RHM in which the confinement potential is a Lorentz scalar plus vector potential. Both scalar and vector potential are harmonic oscillator potentials. The total Hamiltonian has Lorentz scalar plus vector potential along with COGEP. In our model, the effect of confinement of gluons also has been taken into account. In the existing models though the

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effect of confinement of quarks has been taken into account the effect of confinement of gluons has not been taken into account. Hence, a consistent scheme has been employed for the confinement of gluons. For the confinement of gluons, we have made use of the current confinement model (CCM) (Vinodkumar, 1992). The confined gluon propagators (CGP) are derived in CCM has been used to obtain the confined one gluon exchange potential (COGEP). Our present model along with instanton induced interaction has been successful in obtaining the mass spectra of S and P wave light mesons (Vijaya *et al*., 2004, 2009; Bhavyshri *et al*., 2005, 2008).

The Relativistic harmonic model

In RHM (Khadkikar and Gupta 1983; Vijaya *et al*., 2004) quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic-oscillator potential

$$V_{conf}(r) = \frac{1}{2}\left(1+\gamma_0\right)A^2 r^2 + M \tag{5}$$

where γ_0 is the Dirac matrix:

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{6}$$

M is the quark mass and A^2 is the confinement strength. They have a different value for each quark flavour. In RHM, the confined single quark wave function (ψ) is given by:

$$\psi = N\begin{pmatrix} \phi \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M}\phi \end{pmatrix} \tag{7}$$

with the normalization

$$N = \left(\frac{2(E+M)}{3E+M}\right)^{1/2} \tag{8}$$

where E is an eigenvalue of the single particle Dirac equation with the interaction potential given in (1). The lower component is eliminated by performing the similarity transformation,

$$U\psi = \phi \tag{9}$$

Where U is given by,

$$\frac{1}{N\left[1+\dfrac{\mathbf{P}^2}{(E+M)^2}\right]}\begin{pmatrix} \mathbf{1} & \dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M} \\ -\dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M} & \mathbf{1} \end{pmatrix} \tag{10}$$

Here, U is a momentum and state (E) dependent transformation operator. With this transformation, the upper component ϕ satisfies the harmonic oscillator wave equation.

$$\left[\frac{\mathbf{P}^2}{E+M} + A^2 r^2\right]\phi = (E-M)\phi, \tag{11}$$

which is like the three dimensional harmonic oscillator equation with an energy-dependent parameter Ω_n^2 :

$$\Omega_n = A\left(E_n + M\right)^{1/2} \tag{12}$$

The eigenvalue of (11) is given by,

$$E_n^2 = M^2 + (2n+1)\Omega_n^2. \tag{13}$$

Note that eqn. (11) can also be derived by eliminating the lower component of the wave function using the Foldy-Wouthuysen transformation.

Adding the individual contributions of the quarks we obtain the total mass of the hadron. The spurious centre of mass (CM) is corrected by using intrinsic operators for the $\sum_i r_i^2$ and $\sum_i \nabla_i^2$ terms appearing in the Hamiltonian. This amounts to just subtracting the CM motion zero point contribution from the E^2 expression. It should be noted that this method is exact for the 0S-state quarks as the CM motion is also in the 0S state.
In addition, the Hamiltonian has COGEP

The COGEP is obtained from the scattering amplitude (Khadkikar and Vijaya, 1991;Vijaya and Khadkikar, 1993),

$$M_{fi} = \frac{g_s^2}{4\pi}\bar{\psi}_i\gamma^\mu\frac{\lambda_i^a}{2}\psi_i D_{\mu\nu}^{ab}(q)\bar{\psi}_j\gamma^\nu\frac{\lambda_j^b}{2}\psi_j, \tag{14}$$

where, $\bar{\psi} = \psi^+\gamma_0$, $\psi_{i/j}$ are the wave functions of the quarks in the RHM, $D_{\mu\upsilon}^{ab} = \partial_{ab}D_{\mu\upsilon}$ are the CCM gluon propagators in momentum representation, $g_s^2/4\pi$ (= α_s) is the quark-gluon coupling constant and λ_i is the color $SU(3)_c$ generator of the i^{th} quark. Below we give the expressions for the central part of the COGEP.
The central part of COGEP is (Khadkikar and Vijaya Kumar, 1991),

$$V_{COGEP}^{cent}(\vec{r}_{ij}) = \frac{\alpha_s N^4}{4}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\left[D_0(\vec{r}_{ij}) + \frac{1}{(E+M)^2}\left[4\pi\delta^3(\vec{r}_{ij}) - c^4 r^2 D_1(\vec{r}_{ij})\right]\left[1-2/3\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right]\right] \tag{15}$$

To calculate the matrix elements (ME) of COGEP, we have fitted the exact expressions of $D_0(\vec{r})$ and $D_1(\vec{r})$ by Gaussian functions. It is to be noted that the $D_0(\vec{r})$ and

$D_1(\vec{r})$ are different from the usual Coulombic propagators. However, in the asymptotic limit ($\vec{r} \to 0$) they are similar to Columbic propagators and in the infra-red limit ($\vec{r} \to \infty$) they fall like Gaussian. In the above expression the c (fm^{-1}) gives the range of propagation of gluons. The $D_0(r)$ and $D_1(r)$ are given by,

$$D_0(\vec{r}) = \left(\frac{\alpha_1}{r} + \alpha_2\right) \exp\left[\frac{-r^2 c_0^{\,2}}{2}\right]; \qquad D_1(\vec{r}) = \frac{\gamma}{r} \exp\left[\frac{-r^2 c_2^{\,2}}{2}\right]$$

Where α_1 = 1.035994, α_2 = 2.016150, c_0 = $(3.001453)^{1/2}$ fm^{-1} γ = 0.8639336

And $c_2 = (4.367436)^{1/2}$ fm^{-1}. It should be noted that in the limit c → 0, the central part of the COGEP goes over to the corresponding potential OGEP of the NRQM (Vinodkumar *et al*., 1992).

Fitting Procedure

The parameters in our model are the masses of up (M_u), down (M_d), strange (M_s), charm (M_c) and bottom (M_b) which are taken as parameters. The α_s is fixed by the J/Ψ - η_c mass splitting. The mass difference arises from the colour magnetic term of COGEP. The, A^2 is the confinement strength parameter and Ω (=1/b) is the oscillation size parameter. The b is fixed by minimizing the expectation value of the Hamiltonian for the pseudo scalar mesons. The confinement strength A^2 is fixed by the stability condition for the variation of the mass of the mesons against the size parameter b. The additional parameter c, termed CCM parameter was fitted to iota (1440 MeV), $J^{pc} = 0^{-+}$ (the oldest glue ball candidate) as a digluon glue ball (Vijaya *et al*., 1998). The values of the parameters used in our calculation are listed in table 1.

Table 1.Values of the parameters used in our model.

$M_{u/d}$ (MeV)	160.6
M_s (MeV)	402
M_c (MeV)	847
M_b(MeV)	2156
Ω(fm)	0.77
α_s	0.6
C (fm^{-1})	1.74
A^2 (MeV fm^{-2})	3693

Results of S wave Meson Spectra

In the present study, the product of quark-antiquark oscillator wave functions is expressed in terms of oscillator wave functions corresponding to the relative and CM coordinates. The oscillator quantum number for the CM wave functions are restricted to N_{CM} = 0. The Hilbert space of relative wave functions is truncated at radial quantum number n_{max} = 5. The Hamiltonian matrix is constructed for each meson separately in the basis states of $\left|N_{CM} = 0, L_{CM} = 0;\ {}^{2S+1}L_J\right\rangle$. The masses of the pseudo scalar mesons (PSM) and vector mesons (VM) after diagonalisation for successive values of n_{max} are listed in table 2 and table 3 respectively. We get a very good agreement with the experimental masses (Amsler, 2008) both for PSM and VM. The COGEP is attractive for PSM hence the diagonalisation in the space of radially excited states brings down the value of PSM to their physical mass. For example, with n_{max} = 1, the naive masses of the η_c turned out to be 5039.18 MeV. The colour-electric term of COGEP are attractive both for PSM and VM and contribute significantly to the masses. It is satisfying to note that all the masses converge reasonably well to respective experimental values for the same value of α_s (= 0.6). It is to be noted here that we need to enhance the value of α_s when diagonalisation is carried out in smaller configuration space. For the VM, the range of values over which α_s is tuned is considerably smaller than that for the PSM as the OGEP is repulsive. It is clear from tables 2 and 3 that the difference between the masses of the successive values of n_{max} decreases.

The masses of the PSM and VM after diagonalisation for successive values of n_{max} are listed in table 2 and 3 respectively. With n_{max} = 5, the masses of η_c and J/ψ mesons are found to be 2980.3 MeV and 3097.9 MeV respectively. Further increase of the oscillator basis does not lead to any significant change in the masses. The calculation clearly indicates that masses for both PSM and VM converge to the experimental values when the diagonalisation is carried out in a larger basis, but the convergence is achieved in a smaller basis for VM. The CMP of COGEP has an additional term $c^4 r^2 D_1(r)$ whose overall sign is opposite to the δ(r) term and hence has a repulsive contribution to the masses of PSM. Hence, to obtain the physical mass of the PSM it was sufficient to carry out the diagonalisation in a smaller configuration space truncated at n_{max} = 5.

Summary and conclusions

In our present work, we have investigated the masses of ground state heavy mesons in the frame work of RHM with COGEP. The calculation shows that the computation of mesonic masses and mass splittings using COGEP is adequate for both PSM and VM. Hence, it is justified to use a combination of the COGEP along with a Lorentz scalar plus vector potential to reproduce the masses of

Table 2. The PSM masses (in MeV) for successive values of n_{max} .

n_{max}	$\eta_c\,(c\bar{c})$	$\eta_b\,(b\bar{b})$	$D(c\bar{d})$	$D_S^{\pm}\,(c\bar{s})$	$B(u\bar{b})$	$B_S^0(s\bar{b})$	$B_C^{\pm}(c\bar{b})$
1	5039.18	15988.55	3159.12	3331.74	8889.84	9174.22	10822.42
2	3963.4	12583.13	2486.17	2618.11	7021.07	7152.3	8512.01
3	3337.6	10596.32	2056.23	2204.72	5912.48	6098.16	7168.01
4	3045.56	9669.14	1906.69	2014.81	5395.14	5494.23	6540.81
5	2980.3	9467.84	1873.19	1969.43	5279.08	5378.89	6405.77
Expt.	2980±1.2	9461	1869.3±0.5	1968.5±0.6	5279.0±0.5	5369.6±2.4	6400±0.4

Table 3. The VM masses (in MeV) for successive values of $n_{max.}$

n_{max}	$J/\psi\,(c\bar{c})$	$\gamma\,(b\bar{b})$	$D^*(c\bar{d})$	$D_S^{*\pm}\,(c\bar{s})$	$B^*(u\bar{b})$	$B_S^*(s\bar{b})$
1	3243.91	9872.45	2136.57	2182.11	5500.72	5591.42
2	3134.08	9573.82	2034.12	2137.75	5388.9	5477.75
3	3109.92	9500.03	2018.44	2127.27	5347.36	5435.53
4	3098.77	9465.98	2011.21	2122.03	5328.19	5416.05
5	3097.9	9462.04	2010.89	2117.28	5321.62	5413.88
Expt.	3096.916±0.011	9460.3±0.26	2010.0±0.5	2112.4±0.7	5325.0±0.6	5412.8±1.3

both PSM and VM. For the attractive COGEP for PSM, the contribution from the off-diagonal elements is found to be significant. For the VM, there is no substantial change in the masses by increasing n_{max} as COGEP is repulsive and hence perturbative techniques are adequate and are justified. There is a significant contribution from the colour electric term of COGEP in all the models. The masses for both PSM and VM converge to the experimental values when the diagonalisation is carried out in a larger basis, but the convergence is achieved in a smaller basis for VM for reasons stated earlier. The results show that RHM with COGEP provide a quite a good description of the S wave mesons.

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