DESIGN SLIDING MODE CONTROLLER FOR ROBOT MANIPULATOR WITH ARTIFICIAL TUNEABLE GAIN

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ABSTRACT
One of the most active research areas in the field of robotics is robot manipulators control, because these systems are multi-input multi-output (MIMO), nonlinear, and uncertainty. At present, robot manipulators are used in unknown and unstructured situation and caused to provide complicated systems, consequently strong mathematical tools are used in new control methodologies to design nonlinear robust controller with satisfactory performance (e.g., minimum error, good trajectory, disturbance rejection). Robotic systems controlling is vital due to the wide range of application. Obviously stability and robustness are the most minimum requirements in control systems; even though the proof of stability and robustness is more important especially in the case of nonlinear systems. The strategies of robotic manipulators control are classified into two main groups: classical and non-classical methods, where the conventional control theory uses the classical method and the artificial intelligence theory (e.g., fuzzy logic, neural network, and neuro fuzzy) uses the non-classical methods. However both of classical and non-classical theories have applied successfully in many applications, but they also have some limitations. One of the best nonlinear robust controllers which can be used in uncertainty nonlinear systems is sliding mode controller (SMC). Sliding mode controller has two most important challenges: chattering phenomenon and nonlinear dynamic equivalent part. This paper is focused on the applied non-classical method (e.g., Fuzzy Logic) in robust classical method (e.g., Sliding Mode Controller) in the presence of uncertainties and external disturbance to reduce the limitations. Applying the Mamdani’s error based fuzzy logic controller with 7 rules is the main goal that causes the elimination chattering phenomenon with regard to the variety of uncertainty and external disturbance; as a result this paper focuses on the sliding mode controller with artificial tuneable gain (SMCAT) to adjusting the sliding surface slope coefficient depends on applying fuzzy method.

Keywords: Uncertain nonlinear systems, classical control, non-classical control, fuzzy logic, robot manipulator, sliding mode controller with artificial tuneable gain and chattering phenomenon.

INTRODUCTION
Controller (control system) is a device that can sense data from plant (e.g., robot manipulator) to improve the plants behavior through actuation and computation (Ogata, 2009). SMC is one of the influential nonlinear controllers in certain and uncertain systems which are used to present a methodical solution for two main important controllers’ challenges, which named: stability and robustness. Conversely, this controller is used in different applications; sliding mode controller has subsequent drawback i.e. chattering phenomenon. To reduce or remove this challenge, one of the best techniques is applying non-classical method in robust classical such as sliding mode controller method (Kurfess 2005; Siciliano and Khatib, 2008).

A robot is a machine which can be programmed as a reality of tasks which it has divided into three main categories i.e. robot manipulators, mobile robots and hybrid robots. PUMA-560 robot manipulator is an articulated 6 DOF serial robot manipulator. This robot is widely used in industrial and academic area and also dynamic parameters have been identified and documented in the literature (Armstrong et al., 1986). From the control point of view, robot manipulator divides into two main sections i.e. kinematics and dynamic parts. Estimate dynamic parameters are considerably important to control, mechanical design and simulation (Siciliano and Khatib, 2008).

In order to solve the uncertain and complicated systems with a set of IF-THEN rules, fuzzy logic teach should be applied so beginning able to recommended and approximate model in the main motivation (Reznik, 1997). Conversely fuzzy logic method is constructive to control complicated mathematical models; the design quality may not always be so high. Besides using fuzzy logic in the main controller of a control loop, it can be used to design adaptive control, tuning parameters, working in a parallel with classical controllers (e.g., sliding mode controller) and design sliding mode fuzzy or

As mentioned above to reduce a fuzzy logic and sliding mode limitations one of the significant method is design fuzzy logic in a parallel with sliding mode controller (Shahnazi et al. 2008; Hsueh et al., 2009). AFGSMC is sliding mode controller where adjusted by fuzzy logic technique to simple implement, most excellent stability and robustness. AFGSMC has the following advantages; reducing the number of fuzzy rule base and increasing robustness and stability (Hsu and Fu, 2002; Hsu and Malki, 2002; Hsueh et al., 2009).

This paper is organized as, in section 1, main subject of sliding mode controller and formulation are presented. This section covered the following details, classical sliding mode controller for robotic manipulator, equivalent control and chatter free sliding control. In section 2, modelling of robotic manipulators is presented. Detail of fuzzy logic controllers and fuzzy rule base is presented in section 3. In section 4, design Adaptive Fuzzy Gain scheduling sliding mode controller (AFGSMC); this method is used to reduce the uncertainty and variation in dynamic parameter. In section 5, the simulation results and discussion are presented.

1. Classical sliding mode control for robot manipulator

The control law for six degrees of freedom PUMA-560 robot manipulator is written as (Kurfess, 2005; Siciliano and Khatib, 2008):

\[ \tau = \tau_{eq} + \tau_{sat} \]  
\[ \text{(1)} \]

Where, the model-based component \( \tau_{eq} \) is compensated the nominal dynamics of systems. Therefore \( \tau_{eq} \) can calculate as follows:

\[ \tau_{eq} = [M^{-1}(B + C + G) + \dot{S}]M \]  
\[ \text{(2)} \]

Where

\[ M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} \\ 0 & 0 & 0 & M_{44} & M_{45} & M_{46} \\ 0 & 0 & 0 & 0 & M_{55} & M_{56} \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix} \]

Suppose that \( \tau_{sat} \) is computed as;

\[ \tau_{sat} = K_{sat} \left( \frac{S}{\theta} \right) \]  
\[ \text{(3)} \]

where

\[ \tau_{sat} = \begin{bmatrix} \tau_{sat1} \\ \tau_{sat2} \\ \tau_{sat3} \\ \tau_{sat4} \\ \tau_{sat5} \\ \tau_{sat6} \end{bmatrix}, \quad K = \begin{bmatrix} K_{1} \\ K_{2} \\ K_{3} \\ K_{4} \end{bmatrix}, \quad S = \lambda e + \tau \]

and \( \text{Sat} \left( \frac{S}{\theta} \right) \) can be defined as

\[ \text{Sat} \left( \frac{S}{\theta} \right) = \begin{cases} 1 & (\frac{S}{\theta} > 1) \\ -1 & (\frac{S}{\theta} < -1) \\ sgn(S) & (-1 < \frac{S}{\theta} < 1) \end{cases} \]  
\[ \text{(4)} \]

Moreover by replace the formulation (3) in (1) the control output is written as ;

\[ \tau = \tau_{eq} + K_{sat} \left( \frac{S}{\theta} \right) \]  
\[ \text{(5)} \]

Figure 1 shows the position classical sliding mode control for PUMA-560 robot manipulator. By (5) and (2) the sliding mode control of PUMA-560 robot manipulator is calculated as;

\[ \tau = [M^{-1}(B + C + G) + \dot{S}]M + K_{sat} \left( \frac{S}{\theta} \right) \]  
\[ \text{(6)} \]
2. Modeling of robotic manipulator

It is well known that the equation of a multi degrees of freedom (DOF) robot manipulator governed by the following equation (Siciliano and Khatib, 2008):

\[ M(q) \ddot{q} + N(q, \dot{q}) = \tau \]  
(7)

Where \( \tau \) is \( n \times 1 \) vector of actuation torque, \( M(q) \) is \( n \times n \) symmetric and positive definite inertia matrix, \( N(q, \dot{q}) \) is the vector of nonlinearity term, and \( q \) is \( n \times 1 \) position vector. In equation 2.8 if vector of nonlinearity derive as Centrifugal and Coriolis and Gravity terms, consequently robot manipulator dynamic equation can also be written in a following form:

\[ N(q, \dot{q}) = V(q, \dot{q}) + G(q) \]  
(8)

\[ V(q, \dot{q}) = B(q)[\dot{q}] + C(q)[\dot{q}]^2 \]  
(9)

\[ \tau = M(q)\ddot{q} + B(q)[\dot{q}] + C(q)[\dot{q}]^2 + G(q) \]  
(10)

Where,

- \( B(q) \) is matrix of coriolis torques,
- \( C(q) \) is matrix of centrifugal torque,
- \( [\dot{q}, \dot{q}] \) is vector of joint velocity that it can give by:

\[ [\dot{q}_1, \dot{q}_2, \dot{q}_3, \ldots, \dot{q}_1, \dot{q}_2, \dot{q}_3, \ldots]^T \text{, and } [\dot{q}]^2 \text{ is vector, that it can given by: } [\dot{q}_1, \dot{q}_2, \dot{q}_3, \ldots] \]

To derive the dynamic modeling of the robot manipulators, some researchers introduced the kinetic energy matrix and gravity vector symbolic elements by performing the summation of either Lagrange’s or the Gibbs-Alembert formulation (Kurfess, 2005; Siciliano and Khatib, 2008).

\[ \ddot{q} = M^{-1}(q)\{\tau - N(q, \dot{q})\} \]  
(11)

From a control point of view this technique is very attractive since the nonlinear and coupled robot manipulator dynamics is replaced by a linear and decoupled second order system. The first step to determine the dynamic equation of robot manipulator by the formulation of (11) is finding the kinetic energy matrix (M) parameters by used of Lagrange’s formulation. The second step is finding the Coriolis and Centrifugal matrix which they can calculate by partial derivatives of kinetic energy. The last step to determine the dynamic equation of robot manipulator is to find the gravity vector by performing the summation of Lagrange’s formulation.

Therefore the kinetic energy matrix in \( n \) DOF is a \( n \times n \) matrix that can be calculated by the following matrix

\[ M(q) = \begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1n} \\ M_{21} & M_{22} & \cdots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & \cdots & M_{nn} \end{bmatrix} \]  
(12)

The Coriolis matrix (B) is a \( n \times \frac{n(n-3)}{2} \) matrix that can be calculated by the following matrix;

\[ B(q) = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \]  
(13)

The Centrifugal matrix (C) is a \( n \times n \) matrix that can be calculated by the following matrix;

\[ C(q) = \begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \cdots & C_{nn} \end{bmatrix} \]  
(14)

The Gravity vector (G) is a \( n \times 1 \) vector that can be calculated by the following vector;

\[ G(q) = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_n \end{bmatrix} \]  
(15)

3. Design Fuzzy logic controller

After the invention of fuzzy logic theory in 1965 by Zadeh (1997), this theory was used in wide range area. Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory. This controller can be used to control of nonlinear, uncertain, and noisy systems. Fuzzy logic control systems, do not use complex mathematically models of plant for analysis. This method is free of some model-based techniques that used in classical controllers. It must be noted that application of fuzzy logic is not limited only to modelling of nonlinear systems (Reznik, 1997) but also this method can help engineers to design easier controller.

The fuzzy inference mechanism provides a mechanism for referring the rule base in fuzzy set. There are two most commonly method that can be used in fuzzy logic controllers, namely, Mamdani method and Sugeno method, which Mamdani built one of the first fuzzy controller to control of system engine and Michio Sugeno suggested to use a singleton as a membership function of the rule consequent. The Mamdani fuzzy inference method has four steps, namely, fuzzification, rule evaluation, aggregation of the rule outputs and defuzzification. Sugeno method is very similar to Mamdani method but Sugeno changed the consequent rule base that he used the mathematical function of the
input rule base instead of fuzzy set. The following define can be shown the Mamdani and Sugeno fuzzy rule base;

- **Mamdani**
  
  \[ \text{IF} \ x \ \text{is} \ A \ \text{and} \ y \ \text{is} \ B \ \text{then} \ z \ \text{is} \ C \]

- **Sugeno**
  
  \[ f(x,y) = 1 \]

Fuzzification is used to determine the membership degrees for antecedent part when \( x \) and \( y \) have crisp values. Rule evaluation focuses on operation in the antecedent of the fuzzy rules. This part can used AND/OR fuzzy operation in antecedent part after that the output fuzzy set can be calculated by using individual rule-base inference. There are several methodologies in aggregation of the rule outputs that can be used in fuzzy logic controllers, namely, Max-Min aggregation, Sum-Min aggregation, Max-bounded product, Max-drastic product, Max-bounded sum, Max-algebraic sum and Min-max. In this paper we used Max-min aggregation. Max-min aggregation defined as below:

\[
\mu_{F}^{\mu}(x_{k},y_{k}) = \max_{i=1}^{r} \left[ \mu_{A_{i}}(x_{k}) \cdot \mu_{B_{i}}(y_{k}) \right] = \min_{i=1}^{r} \left[ \mu_{A_{i}}(x_{k}) + \mu_{B_{i}}(y_{k}) \right]
\]

where \( r \) is the number of fuzzy rules activated by \( x_{k} \) and \( y_{k} \) and also \( \mu_{F}^{\mu}(x_{k},y_{k}) \) is a fuzzy interpretation of \( l = \text{thr} \) rule. The last step in the fuzzy inference in any fuzzy set is defuzzification. This part is used to transform fuzzy set to crisp set, therefore the input for defuzzification is the aggregate output and the output is a crisp number. There are several methodologies in defuzzification of the rule outputs that can be used in fuzzy logic controllers but this paper focuses on Center of gravity method \( \text{COG} \), which \( \text{COG} \) method used the following equation to calculate the defuzzification:

\[
\text{COG}(x_{k},y_{k}) = \frac{\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{A_{i}}(x_{k}) \cdot \mu_{B_{i}}(x_{k},y_{k})}{\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{A_{i}}(x_{k},y_{k})}
\]

where \( \text{COG}(x_{k},y_{k}) \) illustrates the crisp value of defuzzification output. \( U_{i} \in U \) is discrete element of an output of the fuzzy set, \( \mu_{A_{i}}(x_{k}) \) is the fuzzy set membership function, and \( r \) is the number of fuzzy rules.

4. Design Adaptive Fuzzy Gain scheduling sliding mode controller (AFGSMC)

Adaptive control used in systems whose dynamic parameters are varying and need to be training online. In general states adaptive control classified in two main groups: traditional adaptive method and fuzzy adaptive method, that traditional adaptive method need to have some information about dynamic plant and some dynamic parameters must be known but fuzzy adaptive method can training the variation of parameters by expert knowledge. Combined adaptive method to sliding mode controllers can help to controllers to have better performance by online tuning the nonlinear and time variant parameters.

For any plants (e.g., robot manipulators) whose have variation in parameter, adaptive control can learn the dynamic parameter to design an acceptable controller. All pure classical and fuzzy controllers have common difficulty, which they need to find several scale factors. Therefore, adaptive method can adjust and tune parameters (Hwang and Chao, 2005; Mohan and Bhanot, 2006; Hsueh et al., 2009).

The addition of adaptive methodology to a sliding mode controller caused to improve the tracking performance by online tuning the parameters. The adaptive sliding mode controller is used to estimate the unknown dynamic parameters and external disturbances.

Design supervisory FIS for classical SMC has five steps:

1. **Determine inputs and outputs:** This controller has one input \( S \) and one output \( \alpha \). The input is sliding surface \( S \) and the output is tuning coefficient \( \alpha \).

2. **Find membership function and linguistic variable:** The linguistic variables for sliding surface \( S \) are; Negative Big(N.B), Negative Medium(N.M), Negative Small(N.S), Zero(Z), Positive Small(P.S), Positive Medium(P.M), Positive Big(P.B), and it is quantized in to thirteen levels represented by: -1, -0.83, -0.66, -0.5, -0.33, -0.16, 0, 0.16, 0.33, 0.5, 0.66, 0.83, 1, and the linguistic variables to find the tuning coefficient \( \alpha \) are; Negative Big(N.B), Negative Medium(N.M), Negative Small(N.S), Zero(Z), Positive Small(P.S), Positive Medium(P.M), Positive Big(P.B), and it is quantized in to thirteen levels represented by: -1, -0.83, -0.66, -0.5, -0.33, -0.16, 0, 0.16, 0.33, 0.5, 0.66, 0.83, 1.

3. **Choice of shape of membership function:** In this part the researcher select the triangular membership function that it is shown in figure 1.

![Fig. 1. Membership function: a) sliding surface b) Tuning coefficient.](image-url)
4. Design fuzzy rule table: design the rule base of fuzzy logic controller can play important role to design best performance AFGSMC, suppose that two fuzzy rules in this controller are:
   F.R\(^1\): IF \(S\) is \(Z\), THEN \(\alpha\) is \(Z\).
   F.R\(^2\): IF \(S\) is \((P.B)\) THEN \(\alpha\) is \((L.R)\).

The complete rule base for this controller is shown in table 1.

Table 1. Fuzzy rule table.

<table>
<thead>
<tr>
<th>(S)</th>
<th>N.B</th>
<th>N.M</th>
<th>N.S</th>
<th>Z</th>
<th>P.S</th>
<th>P.M</th>
<th>P.B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>N.B</td>
<td>N.M</td>
<td>N.S</td>
<td>Z</td>
<td>P.S</td>
<td>P.M</td>
<td>P.B</td>
</tr>
</tbody>
</table>

The control strategy that deduced by table 1 are
- If sliding surface (\(S\)) is N.B, the control applied is N.B for moving \(S\) to \(S=0\).
- If sliding surface (\(S\)) is Z, the control applied is Z for moving \(S\) to \(S=0\).

5. Defuzzification: The final step to design fuzzy logic controller is defuzzification, in this controller the COG method will be used.

The block diagram of AFGSMC controller is shown in figure 2.

RESULTS AND DISCUSSION

Adaptive Fuzzy Gain scheduling sliding mode controller (AFGSMC), Adaptive Fuzzy Inference System (AFIS), and Sliding Mode Controller (SMC) were tested for step response trajectories. In this simulation the first, second, and third joints move from home to final position without and with external disturbance. The simulation was implemented in Matlab/Simulink environment. Tracking performance, error, robustness (disturbance rejection), and chattering rejection are compared.

Tracking performances: From the simulation for first, second, and third trajectory without any disturbance, it was seen that AFGSMC and SMC have the same performance. This is primarily because the manipulator robot parameters do not change in simulation. The AFGSMC and SMC give significant trajectory good following when compared to FLC. Figure 3 shows tracking performance without any disturbance for AFGSMC, AFIS and SMC.

Disturbance rejection: An unknown output disturbance is applied in different time. Figure 4 shows disturbance rejection for AFGSMC, AFIS and SMC. However the AFGSMC gives the better performance than AFIS but AFIS also has an acceptable performance.

Errors in the model: However the AFIS gives significant
error reduction when compared to pure FLC, but it is not as good as AFGSMC. The error profile for AFGSMC is smoother compared to the other controllers. Figure 5 shows a comparison of error performance for all three controllers that study in this paper.

Chattering phenomenon: An unknown output disturbance is applied in different time. Figure 6 shows the chattering rejection for step AFGSMC and SMC.

CONCLUSION

This paper presents a new methodology for designing an adaptive fuzzy gain scheduling sliding mode controller for PUMA robotic manipulator. From the simulation, it was seen that AFGSMC has 7 rule base because it has one input for supervisory controller but AFIS has 49 rules for supervisory and 49 rules for main controller therefore implementing of AFIS most of time has many problems and expensive and also the AFGSMC performance is better than SMC and AFIS in most of time, Because this controller can auto tune as SMC with change the robot arm parameters, but pure SMC cannot do it.

The pure sliding mode controller has some problems in parameter variations. In the worst case, the adaptive controller has the potential to perform as well as a sliding mode controller. In AFGSMC, the fuzzy supervisory controller can changed the \( \lambda \) to achieve the best performance and in AFIS the supervisory controller can changed the gain updating factor of main FIS to have the best performance.

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