SOME STATIC SPHERICAL CLASSICAL SOLUTIONS INCLUDING THE COSMOLOGICAL TERM

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ABSTRACT

The general static spherical potential in the form of

\[ \phi = \frac{A}{r} + \frac{B}{r^2} + \frac{1}{2} k r^2 \]

is proposed. The first term is the usual Newton’s law. The second term refers to the negative field energy of the source which is rather small when comparing with the first term. The last term relates to a spring constant \( k \) of the source which acts as a repulsive force against the gravitational one. We point out that the spring effect has a limit distance depending on individual sources. Furthermore, the spring force acting against the gravitational one can be regarded as the fifth force. The spring theory is also applied in short range interaction.

Keywords: Classical electrostatics, general relativity and gravitational fifth force, short range interaction.

INTRODUCTION

When a source is placed in space, its external field is occupied by 2 constituents, namely, the negative energy of the source and tentacles (or springs) attaching to the source. The latter have a range limit, as they will break when being extended to a certain distance. Later in this paper we will investigate the properties of these two constituents. We start with the Yang’s pure space equations of (Yang, 1974)

\[ R_{\mu\nu} \; ; \; \Lambda = R_{\mu\lambda} \; ; \; \nu . \]

We try not to call (1) the vacuum equations. In spite of many unphysical conditions, Pavelle (1974, 1975) pointed out that there exists a possible solution if the cosmological constant \( \Lambda \neq 0 \) in Einstein’s equations, or

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 . \]

(2)

In case of spherical static symmetry, the line element is written as

\[ ds^2 = e^{\nu(r)} dt^2 - e^{\Lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 , \]

where

\[ e^\nu = e^{-\Lambda} = 1 + \frac{2A}{rc^2} + \frac{2B}{r^2 e^2} + \frac{\Lambda r^2}{3} , \]

(3)

(4)

In case of a non-vacuum exterior solution as we mentioned previously, (2) become

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = T_{\mu\nu} \]

(5)

and

\[ e^\nu = e^{-\Lambda} = 1 + \frac{2A}{rc^2} + \frac{2B}{r^2 e^2} + \frac{\Lambda r^2}{3} , \]

(6)

which has the same form as Reisner-Nördstrom (Adler, 1975) where \( A, B, \Lambda \) are constants to be determined. In fact, the non-zero cosmological term is nothing new to the physicists (Kottler metric, 1918). However, it cannot be regarded as a universal constant but a spring term relating to different sources.

The potential is

\[ \phi = \frac{A}{r} + \frac{B}{r^2} + \frac{1}{2} k r^2 \]

(7)

where the second term involves the exterior field of the source.

The following sections will deduce the values of \( A \) and \( B \) of (7) classically.

ELECTRIC FIELD

The electric field energy density surrounding a charge \( q \) is proportional to the square of the field intensity \( E \), or

\[ W \propto E^2 . \]

(8)

Since a charge is always accompanied by its electromagnetic mass \( \delta m \), its mass becomes total rest mass \( M = \text{mechanical mass} + \text{electromagnetic mass} \).
The above two masses on the right side of (9) are non-separable from each other. The surrounding field of this charge possesses a total mass of $-\delta m$ as shown in (13).

$E\mathbf{d}q$ (8) can be modified as

$$\nabla \cdot E = 4\pi \rho_e = \alpha E^2,$$

in which $\alpha$ is a constant to be determined.

Upon integration,

$$E = \frac{A}{r^2} (1 + \frac{B}{r})^{-1},$$

where $A, B$ are the constant of integration.

By setting $A = $ charge $q$, (8) becomes

$$W = \frac{E^2}{8\pi} = \frac{q^2}{8\pi r^2} (1 + \frac{B}{r})^{-2}.$$

The total electric field energy over the whole space is

$$W_{total} = \frac{q^2}{2} \int_0^1 \frac{1}{r} (1 + \frac{B}{r})^{-2} dr = -\delta m c^2.$$

We consider the field energy to be negative. Thus,

$$B = \frac{-q^2}{2\delta mc^2}.$$

The field intensity becomes

$$E = \frac{q}{r^2} (1 - \frac{q^2}{2\delta mc^2 r^2})^{-1}.$$

Obviously, for $\delta m \to 0, E \to 0$, indicating that always accompanies with the charge. The potential can be written in the form

$$\phi = \frac{q}{r} + \frac{q^3}{4\delta mc^2 r^2}.$$

Bohr’s theory of an orbiting electron seems to allow no rooms for the spring term $k$ as well as for the electromagnetic mass $\delta m$ term. If (13) fails, this section will be totally meaningless. To investigate the property of $\delta m$, we compare Newton’s law with that of Coulomb’s to obtain

$$\delta m = \frac{ie}{(4\pi\varepsilon_0 G)^\frac{3}{2}}.$$

Thus, the $2^{nd}$ term of (16) is complex and so a charged particle as described by (9) refers to a composite particle of real and complex parts. The energy-momentum tensor of (5) can be written as

$$T^{\mu \nu} = -\frac{\rho + i\rho_e}{(4\pi\varepsilon_0)^\frac{3}{2}} U^\mu U^\nu,$$

where $\rho$ is the mass density and $\rho_e$ is the charge density same as (10). The above (19) can be a good attempt to combine electromagnetic field into general relativity. Moreover, we suspect that the second term of (19) may be related to the dark matter.

**GRAVITATIONAL FIELD**

Since Newton’s law is analogous to that of the Coulombs, (13) in gravitation can be written as, by changing $q$ into $\sqrt{G} M$ for unit purpose (Treder, 1975):

$$\frac{GM^2}{2} \int_0^1 \frac{1}{r} (1 + \frac{B}{r})^{-2} dr = -M c^2,$$

where $-Mc^2$ is the negative energy of the field surrounding the source $M$. Hence, the result gives

$$B = -\frac{GM}{2c^2}.$$

It is interesting to know that the sum of the source $M$ and its surrounding field $-M$ is zero, showing that our universe is in fact “nothing” when sum up all the real and negative matters! Moreover, $M$ and $-M$ do not attract each other to avoid a true vacuum formed in the exterior of the source.

The gravitational force acting on a particle $m$ becomes

$$\frac{GM m}{r^2} + \frac{G^2 M^2 m}{2r^2 c^2} = ma.$$

A spring force $kr$ needs to be added into (22) eventhough we cannot derive it classically. That is:

field intensity $E$ + or - spring term $kr$ = acceleration

which is different from the traditional concept of field intensity equals acceleration. The plus or minus sign depends on whether the object is falling towards or darting away from the source. It needs to point out that $k$ is not a universal constant but depending on individual sources. To estimate the value of $k$ for the earth, we take the radius of the earth $R = 6.4 \times 10^6$ m, $a = 9.679$ m/s$^2$ (see appendix 1) and mass of the earth $M = 6 \times 10^{24}$ kg. Hence,
\[ a = \frac{GM}{r^2} + \frac{G^2M^2}{2r^2c^2} - kr = 9.679 \text{ m/s}^2 \]  

(23)

which yields the value of \( k \) of earth \( \sim 10^{-8}/\text{s}^2 \) 

(24)

Substitute (24) into (23) and set \( a = 0 \), we obtain \( r = 10^7 \) m.

THE FIFTH FORCE

The fifth force has a potential of the form (Fischbach, 1986, 1992)

\[ \phi = \frac{GM}{r}(1 + \alpha e^{-r/\lambda}) \],

(25)

in which

\[ \alpha = -(7.2 \pm 3.6) \times 10^{-3} \text{ , } \lambda = 200 \pm 50 \text{ m} \].

We find that the distance \( r \) is negative where there is no acceleration to any falling object, or \( a = 0 \). This is because (25) is only an empirical formula. Moreover, there are still experimental difficulties to detect the true nature of this additional exponential term. Some papers even criticized the existence, that included Thieberger (1987); Cowsik et al. (1988); Fitch et al. (1988); Bennet (1986); Nelson, Graham and Newman (1990); Stubbis et al. (1989); Mannheim (1991) and a more detail one by Franklin (1993). In spite of the difficulties in the experimental search for the fifth force, the theories do exist Kaluza-Klein (1921); Brans-Dicke (1961); Ramanand (1988) and Farrad-Rosen (2007). From (23), the so-called fifth force or the spring force \( kr \) in our theory is a repulsive force which is not only effective on earth but also affecting any heavenly object up to the critical distance.

The Newtonian inverse square term \( GM/r^2 \) of (23) of a planet must be greater than its gravity acceleration in order to obtain a positive \( k \). In the case of Jupiter, \( GM/r^2 = 24.79 \text{ m/s}^2 \), \( k \) for Jupiter \( = 3.2 \times 10^9/\text{s}^2 \) 

(26)

For other planets, \( GM/r^2 \text{ a (} M \text{ = mass of the planet) , which is difficult for us to calculate the value } k \text{ of these planets (Abell, 1987).} \)

The orbit of a planet

The usual Binet’s equation is of the form

\[ \frac{d^2 u}{d \varphi^2} + u = \frac{P}{h^2 u^2} \],

where \( P \) is the central force per unit mass. Since there are two additional terms in (23), this Binet’s equation should be re-written

\[ \frac{d^2 u}{d \varphi^2} + u = \frac{GM}{h^2} + \frac{G^2M^2}{2h^2c^2} - \frac{k}{h^2u^3} \]  

(27)

To solve for the above (27), we follow the same procedures as in Adler et al. (1975) [page 206-209]: rewrite (27) as

\[ u^* + bu = H - \frac{k}{h^2u_0} \]  

(28)

where

\[ H = \frac{GM}{h^2} \text{ b = } 1 - \frac{G^2M^2}{2h^2c^2} \]  

(28a)

and assume a solution of the form

\[ u(\varphi) = u_0(\varphi) + kv(\varphi) + O(k^2) \]  

(29)

To find \( u_0(\varphi) \) and \( v(\varphi) \). (29) is substituted into (28),

\[ u^* + kv^* + bu + kvb = H - \frac{k}{h^2u_0} + O(k^2) \]  

(29a)

Neglecting all terms containing \( k \), we have a simple case of \( u_0 + bu_0 = H \)  

(29b)

The solution is easily checked to be

\[ u_0 = \frac{H}{b} + kD\cos(\sqrt{b}\varphi + \gamma) \]  

(30)

where \( D \) and \( \gamma \) are arbitrary constants. By an appropriate orientation of the axes we may make \( \gamma \) equal to zero, the familiar equation of an ellipse becomes,

\[ u_0 = \frac{H}{b} + kD\cos(\sqrt{b}\varphi) \]  

(30a)

Similarly, equating the first-order \( k \) terms in (29a), we obtain

\[ v^* + bv = -\frac{1}{h^2u_0^3} \]  

(31)

Substituting (30a) into (31);

\[ v^* + bv = -\frac{b^3}{h^2H^3} + \frac{3b^2kD}{h^2H^3}\cos(\sqrt{b}\varphi) + O(k^2) \]  

(31a)

Now \( v \) can be the sum \( v = v_1 + v_2 \), where \( v_1 \) and \( v_2 \) are solutions of the equations

\[ v_1^* + bv_1 = -\frac{b^3}{h^2H^3} \text{ v_2^* + bv_2 = } \frac{3b^2kD}{h^2H^3}\cos(\sqrt{b}\varphi) \]  

(31b)

whose solutions are
\[ v_i = -\frac{b^2}{h^2H^3}, \quad v_2 = \frac{3b^2kD}{2h^2H^4} \sin b\phi \]  

(31c)

Thus,

\[ v = v_i + v_2 = -\frac{b^2}{h^2H^3} + \frac{3b^2kD}{2h^2H^4} \sin b\phi \]  

(32)

and we get

\[ kv = -\frac{kb^2}{h^2H^3} + O(k^2) \]  

(32a)

Combining this with (30a), the entire solution for the orbit to first order in \( k \) appears as

\[ u = u_0 + kv = \frac{H}{b} + kDCos\sqrt{b}\phi - \frac{kb^2}{h^2H^3} \]  

(32b)

Substituting \( H \) and \( b \) of (28a) into (32b) to obtain

\[ u = \frac{GM}{h^2(1-G^2M^2c^2/2h^2c^2)} + kDCos\sqrt{b}\phi\sqrt{1-G^2M^2/2h^2c^2} - \frac{k(1-G^2M^2c^2/2h^2c^2)}{G^2M^2}h^4 \]  

(33)

The perihelion shift is given by

\[ \delta\phi = 2\pi(1 + \frac{G^2M^2}{4h^2c^2}) \]  

(34)

The spring constant does not appear in (34) but it affects the radial distance. The second term on the right-hand side of (27) affects both the perihelion shift as well as the radial periodic variations. Unfortunately, the latter is very hard to be observed. To estimate the value of the sun’s \( k \), we consider the cosine term in (33) be zero and set the following table 1 (Roman, 1989):

Table 1. Mass M for sun \( 2 \times 10^{30} \text{kg} \), \( h \), \( 2\pi^2/T \)

<table>
<thead>
<tr>
<th>Distance r ( (10^9 \text{ m}) )</th>
<th>Mercury</th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
<th>Jupiter</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>108</td>
<td>149</td>
<td>224.9</td>
<td>778.34</td>
<td></td>
</tr>
<tr>
<td>Period T(days)</td>
<td>89</td>
<td>224.7</td>
<td>365.25</td>
<td>686.98</td>
<td>4332.59</td>
</tr>
<tr>
<td>( k ) of sun (sec(^{-2}))</td>
<td>10(^{-16})</td>
<td>10(^{-16})</td>
<td>10(^{-16})</td>
<td>10(^{-16})</td>
<td>10(^{-19})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Saturn</th>
<th>Uranus</th>
<th>Neptune</th>
<th>Pluto</th>
</tr>
</thead>
<tbody>
<tr>
<td>1427</td>
<td>2869.6</td>
<td>4496.67</td>
<td>5900.2</td>
</tr>
<tr>
<td>10759.2</td>
<td>30684.9</td>
<td>60190.3</td>
<td>90470</td>
</tr>
<tr>
<td>( k ) of sun (sec(^{-2}))</td>
<td>10(^{-21})</td>
<td>10(^{-21})</td>
<td>10(^{-21})</td>
</tr>
</tbody>
</table>

The sun’s \( k \) is \( 10^{-16} \text{ sec}^{-2} \) to \( 10^{-21} \text{ sec}^{-2} \). (35)

Discrepancies appearing in (35) are expected. One reason is that the spring loses its elasticity at very large distances. Moreover, (33) is not an accurate solution of (27). Taking the value \( 10^{-21} \text{ sec}^{-2} \), \( a = 0 \), the sun’s effective range can be obtained from (23):

\[ r = 10^{13} \text{ m} \]  

(36)

The bending of light under the sun’s gravitational field

Let the sun’s location at \( x = 0 \). Light path is traveling from \( x = +\infty \) to \( x = -\infty \). The closest distance from the sun is at \( x = 0, y = r_0 \) and \( r \) is the distance between the light and the sun. The deflection \( \delta \) is the angle between the light path and the horizontal line \( y = 0 \). Light path should be described as the motion of an ordinary particle and hence the \( v_1 \) and \( v_0 \) denote different light speeds. The work done by a photon can be calculated in this way:

\[ dW = f \cdot dr = ma \cdot dr = m \frac{dv}{dt} dr = mv \cdot dv \]

\[ W = \int_{r_0}^{r} dW = \int_{r_0}^{r} mv \cdot dv = \frac{1}{2} mv_1^2 - \frac{1}{2} mv_0^2 = \Delta E \]

We select only the first and third terms of (23),

\[ W = \int_{r_0}^{r} Fdr = \int_{r_0}^{r} \left( \frac{GMm}{r^2} - mkr \right) dr \]

As \( r \rightarrow \infty, \ k = 0 \), we get the potential
The change of kinetic energy per unit mass is
\[ \Delta E = \frac{1}{2} c^2 \Delta v^2 - \frac{1}{2} v^2 \]  
(38)

To estimate the approximate value of \( v \), we ignore the spring term in (37) and compare with Newton's result of
\[ \delta = \frac{GM}{c^2 r_0} \]
(39)

which seems to be reasonable but we need to point out that the speed of light in (38) is not \( c \) since there is no vacuum due to the second term of (23) [see appendix 2]. With the presence of a spring term,
\[ \delta = \frac{GM}{c^2 r_0} + \frac{kr_0^2}{2c^2} \]
(40)

The first term is only half the result of general relativity as expected but there is an additional term containing the spring. The total deflection of the light ray or the angle between the asymptotes is
\[ \Delta = 2\delta = \frac{2GM}{r_0 c^2} + \frac{kr_0^2}{c^2} \]

Using the results of (36), once light is so far away such that at \( r = 10^{13} \) m, the spring breaks but light continues to bend due to Newton’s effect only.

**Spring theory in short range interaction**

The basic assumption of Spring theory is that when a source is placed in space, its external field is occupied by two constituents, namely, the negative energy of the source and tentacles (or springs) attaching to the source. The latter have a range limit, as they will break once being extended to a certain distance. The external field of negative energy is governed by (20)
\[ GM \int_0^\infty \frac{1}{r^3} \left(1 + \frac{B}{r}\right) dr = -M c^2 \]  
\[ \frac{GM^2}{2} \int_0^\infty \frac{1}{r^2} \left(1 + \frac{B}{r}\right) d r = -M c^2 \]  
(20)

**DISCUSSION**

Treder (1975) pointed out that once the mass \( M \) is reduced to \( M = \left( \frac{hc}{G} \right)^{1/2} = 10^{-8} \) kg, gravitation will be converted into a short range strong interaction. In this case, the Newton format on the left side of (20) can be converted into the quantum format, i.e. convert \( GM^2 \) into \( hc \).

In the case of short range, integration takes place from zero to \( \lambda \) inside a small domain instead of from zero to \( \infty \).

The right side of (20) can be written as \( Mc^2 = hc/\lambda \). The negative sign disappears since interaction takes place inside the domain of range \( \lambda \), but not the external field of the source.

**Axiom:**
"If the mass of each of the two interacting particles is less than \( 10^{-8} \) kg, Newton’s inverse square law \( GMm/r^2 \) should be replaced by the quantum gravity format of \( hc/\lambda^2 \), where \( M, m \) each is less than \( 10^{-8} \) kg and \( \lambda \) is a short range”.

Eq(20) can now be re-written as
\[ \frac{hc}{2} \int_0^\infty \frac{1}{r^3} \left(1 + \frac{B}{r}\right)^{-2} d r = hc/\lambda \]
(40)

and upon integration, the constant \( B = 0.366 \lambda \).

As previously mentioned, the acceleration of a falling object under the influence of both the field intensity plus the spring force of the source is in the form of
\[ \frac{GM}{r^2} \left(1 + \frac{B}{r}\right)^{-1} - kr = \text{acceleration} \]  
(41)

where \( k \) is the spring constant of the source, for instance, the proton. However, in short range, there is no acceleration as the interacting particles are confined in a small domain. Using the obtained value of \( B = 0.366 \lambda \), the above equation can be reduced to
\[ k = \frac{\sqrt{hcG}}{1.366 \lambda^3} \text{ sec}^{-2} \]

Since
\[ \text{total energy} = \text{potential energy} + \left( \text{strain energy of the spring} \right) \]

with the help of (41)

\[ \text{total energy} = \frac{hc}{0.366 \lambda} \ln \left(1 + \frac{B}{\lambda}\right) + \frac{1}{2} \sqrt{\frac{hc}{G}} \frac{\sqrt{hcG}}{1.366 \lambda^3} \lambda^2 \]  
(42)

The above yields the total energy stored inside the domain = 1.3 GeV for a short range of, say, 1.1 fm.

The second term of (42) represents the harmonic oscillating energy of a meson which equals \( m_\pi c^2 \). Therefore, the energy of the Yukawa \( \pi \) meson is found to be 370 MeV using \( \lambda = 1.1 \) fm.
Appendix 1

Eq(23) can also be modified as (Amots, 2007)

\[ \frac{GMm}{r^2} + \frac{G^2 M^2 m}{2 r^2 c^2} - kr = \frac{d(mc^2)}{dr}. \]  

(43)

The following equation will explain the gravitational red-shift:

\[ \frac{d(mc^2)}{dr} = h \cdot \frac{dv}{dr}. \]  

(44)

The Jefferson Physical Laboratory at Harvard used a $^{57}$Fe source being placed at a height of 22.6m above the detector. Gamma photons dropped to the detector. The original purpose was to demonstrate the Mössbauer effect (see the famous Pound-Rebka experiment). The data were

- $\Delta E$ the energy gain $3.5 \times 10^{-11}$ eV
- $\Delta r$ height dropped 22.6m
- $E$ the source energy 14.4keV

Hence,

\[ \frac{c^2 \Delta E}{E \Delta r} = 9.679 \text{ m/s}^2 \]

Appendix 2

Using the same experiment as in appendix 1, we re-calculate the earth’s gravity based on the condition that light speed varies along the gravity distance. In fact, some physicists suggested that mass of a photon is non-zero and light speed may not be constant everywhere. They included Jackson (1987,1999); Goldhaber (1971); Ugarov (1979) and Kan (2008). Based on their proposals, like any other particles, a falling photon changes its frequency and velocity in the form of

\[ h \nu_0 \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} = h \nu, \]  

(45)

and the acceleration

\[ a = v \frac{dv}{dr}. \]  

(46)

Solving the above two equations, we obtain

\[ a = \left( c^2 / 2 \Delta r \right) [ v_0^2 / v^2 - 1 ] \]  

(47)

where $v \cdot v_0$ = change of frequency after the drop as given in appendix 1. The maximum speed of light is $c = 3 \times 10^8$ m/s. The rest mass of a photon cannot be zero, indicating $v_0$ is non-zero. Furthermore, no photon can reach its maximum speed c. Hence, using the experimental data from appendix 1 and substitute into(47), we obtain

\[ a = 9.679 \text{ m/s}^2 \]

the same result as in appendix 1.

CONCLUSION

We admit that it is difficult to calculate the exact value of the spring term for each source, especially the sun. By comparing (24), (26) and (35), we discover that the larger the mass, the lesser the value of $\kappa$. The cosmological spring term as abandoned must be extremely small (Weinberg, 1987): $10^{-35}$ sec$^{-2}$, or $10^{-47}$ GeV$^4$, or $10^{-26}$ kg/m$^3$ (Tegmark, 2004). This also explains why the spring inside the quark confinement is so strong. We have no intention to abandon relativity but to suggest that this cosmological (or spring) term needs to be restored. Eq (19) is an attempt to combine electromagnetism into general relativity by splitting the energy-momentum tensors of a charged particle into a real plus a complex part. The most striking result is that the effective range of the earth's spring is 10$^4$m whereas the earth-moon distance is 10$^5$m. A gravity-free spherical surface is predicted which allows artificial satellites orbiting economically around the earth.

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