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DATA-BASED MECHANISTIC MODELLING OF RAINFALL TO RIVERFLOW OF LARGE NESTED TROPICAL RAINFOREST CATCHMENTS IN GHANA

*Boateng Ampadu[1], Nick A Chappell[2] andWlodek Tych[2]
[1]Department of Earth and Environmental Science, Faculty of Applied Sciences
University for Development Studies, PO. Box 24, Navrongo-Ghana
[2]Lancaster Environment Centre, Lancaster University, Lancaster LA1 4YQ, UK

ABSTRACT

Within the Data Based Mechanistic (DBM) Transfer Function rainfall to riverflow modelling approach a mathematical model in the form of a transfer function rainfall to riverflow is obtained by extracting information from the available time series data. The DBM methodology is able to use the data to identify the model structure in an objective statistical manner using the simplified recursive instrumental variable algorithm (SRIV). The approach requires few spatially-distributed data for the estimation of the models and is, therefore, suitable for data limited regions like West Africa. Within this paper we present a review of the application of the model in hydrological studies in different climatic conditions. The application of the approach to large nested catchments in the humid rainforest zone in Ghana have also been presented. The approach revealed an exponential form of non-linear behaviour for the catchments. The estimated model parameters and the associated dynamic response characteristics (DRCs) of time constant (TC) and steady state gain (SSG) indicates that riverflow generation within the catchments are not flashy. The model identified mathematical relationships which could be used to simulate flows in the catchments.

Keywords: Ghana, dbm model, rain forest, transfer function.

INTRODUCTION

Rainfall-riverflow modelling provides the means, for the investigation of the interaction between climate and riverflow. Understanding the dynamic link between rainfall and riverflow response can also give greater understanding of rainfall-riverflow processes through hydrologic interpretation of the Dynamic Response Characteristics (DRCs) of a catchment. This requires the use of extensive historical records which are generally lacking in Africa, as highlighted by Giles (2005) and Weston and Steven (2005).

In West Africa, the dearth of meteorological data is very common, as pointed out by van de Giessen *et al*. (2002). In Ghana, the situation is not far different from other African countries; hydrological and meteorological data of the country are inadequate and of poor quality, with the exception of a few stations (Adiku *et al*., 1997). Generally, rainfall and riverflow stations in the tropics are of low density and in some areas of interest; the requisite data is simply not available (Douglas, 1999). Some of the recording instruments are no more in existence, while the existing ones are deteriorating. This calls for the use of models that can handle few data inputs, and quantify the effects of sometimes poor data quality on model structures and parameters to be interpreted (Young *et al*., 1999; Young, 2001; Chappell *et al*., 2006). One of such approaches is the relatively new Data-Based Mechanistic (DBM) modelling routines (Young and Minchin, 1991; Young and Lees, 1993; Young and Beven, 1994; Chappell *et al*., 1999; Lees, 2000).

The Data-Based Mechanistic (DBM) modelling approach (Young and Lees, 1993; Young and Beven, 1994; Young, 2001; Chappell *et al*., 1999) involves three steps, which are a) extraction of information from the rainfall and riverflow records by fitting models to the data, b) identification of a range of transfer function models and their associated hydrological system parameters using objective statistical tests and c) selection of the model with the most plausible physical/hydrological explanation of the data. Unlike physics-based and conceptual modelling approaches, it is based on the concept whereby the data is allowed to suggest the type of model which is compatible with the input and output data in a stochastic manner. The mechanistic nature of the final stage of the approach allows the physical interpretation of the resulting model.

The DBM Transfer Function (TF) modelling approach is one of the routines within the DBM-CAPTAIN package (Taylor *et al*., 2007) used in hydrological modelling (e.g. see: Young and Beven, 1994; Young *et al*., 1997; Lees,

*Corresponding author email: ampaduboateng@yahoo.com

2000; Mwakalila *et al*., 2001). The modelling routine is capable of revealing possible hydrological pathways within a catchment (Chappell *et al*., 1999; Young, 2001; Vongtanaboon and Chappell, 2004) and estimates the parameters or DRCs associated with the different flow pathways (Young *et al*., 1997; Chappell *et al*., 1999).

Within the DBM transfer function rainfall to riverflow modelling approach, an optimal mathematical relationship relating rainfall (input) to riverflow (output) in the form of transfer functions with its associated parameters is obtained. Generally, the identified model structure includes nonlinear components due to the effects of nonlinearity as a result of antecedent moisture in the sub-surface. The optimal model is selected from a range of model structures using objective statistical tests and the model structure's consistency with physical/hydrological theory. One of the major advantages of this modelling procedure over conceptual hydrological models is that the structure is objectively identified as part of the modelling procedure. The DBM model is robust and parsimonious as compared to the traditional modelling approaches (e.g. physics-based models), as it minimises the number of parameters while producing models with a high simulation efficiency (Chappell *et al*., 2006). It is, therefore, suitable for data limited region such as Africa in general and Ghana in particular.

The DBM concept has been applied successfully in rainfall-riverflow modelling (e.g. see: Young, 1993, 2001; Young and Beven, 1994; Young *et al*., 1997; Young, 1998; Beven, 2001; Lees, 2000) and flood forecasting (Lees *et al*., 1994; Lees, 2000; Young, 2002, 2006) in humid temperate conditions. In the tropics, Chappell *et al.* (1999, 2004a, 2004b, 2006) report of the application of the approach to the short-term behaviour of the rainfall-riverflow system and rainfall-suspended sediment system of the 0.44 km^2 Baru catchment in Borneo, Malaysia. The model has a 5 minutes resolution and explained 80% and 90% of the variance, respectively. In Thailand, the approach has been applied successfully to model rainfall and riverflow behaviour in large rainforest catchments (Vongtanaboon, 2004; Vongtanaboon and Chappell, 2004). Vongtanaboon and Chappell (2004) report that in the North Western Thailand, within the Mae Chaem catchment (3853 km^2), the output of the DBM model suggests that 97% of water flow to the river travels along with relatively little storage (time constant of 1.2 days). Again within Thailand, recently Vongtanaboon *et al.* (2008) have utilised the technique to model a large monsoon dominated catchment. Chappell *et al.* (2006) have applied the model to simulate the sensitivity of streamflow behaviour to different densities of skidder vehicle trails within a managed rainforest in Borneo, Malaysia.

In reservoir sedimentation analysis, the methodology has been applied successfully by Price *et al*. (2000) and Rowan *et al*. (2001). The maiden application of the model in Africa has been reported by Mwakalila *et al*. (2001). The model was used successfully to predict riverflow generation in a semi-arid environment in Tanzania, East Africa. Recently, in the same country, Vigiak *et al*. (2006) have reported of the successful application of the approach in a humid tropical rainforest catchment to simulate overland flow. The application of the approach in the Volta basin in West Africa has also been reported by Amisigo (2005).

The aim of this study is to apply DBM TF rainfall-riverflow modelling approach to study rainfall to riverflow behaviour within large nested forest catchments in Ghana. The specific objectives are a) to investigate the applicability of the DBM transfer function rainfall to riverflow modelling approach in large nested catchments in tropical rainforest within the River Pra basin in Ghana, using daily time-series, b) to identify the mathematical relationships between the catchment average rainfall and riverflow, and estimate their accompanying parameters with uncertainty and c) to give physical interpretation of the estimated parameters of the identified models in (b) and the accompanying Dynamic Response Characteristics (DRCs).

The study catchment and data series used in the analysis

The study was conducted using rainfall and riverflow data from the River Pra Basin which lies in the forest zone of Ghana within latitude 5º and 7º 30' N and longitude 0º and 2º 30' W, respectively (Fig. 1) with catchment area of 20778 km^2. It is the largest basin in the forest zone of Ghana and has enough water which is capable of generating hydropower (Dickson and Benneh, 1988). The basin lies in the Wet Semi-Equatorial climatic zone with climate that is influenced principally by the tropical maritime (monsoon) and continental (harmattan) air masses. Two distinct seasonal rainfall distributions (i.e. the bi-modal distribution) are normally experienced in the area which usually commences in March peaking around June with dry spell in August, peaking again in September and October. The rainfall pattern generally dictates the riverflow totals (Fig. 2) where most of the rivers in the basin are permanent; flowing throughout the year. The underlain geology of the basin is principally Birimian rocks with a section in the middle underlain by Tarkwain formation with soil cover which is predominantly Acrisols (Forest Ochrosols). The Pra basin is of national and global importance due to cocoa, timber and oil palm production in addition to food crops and mining industries.

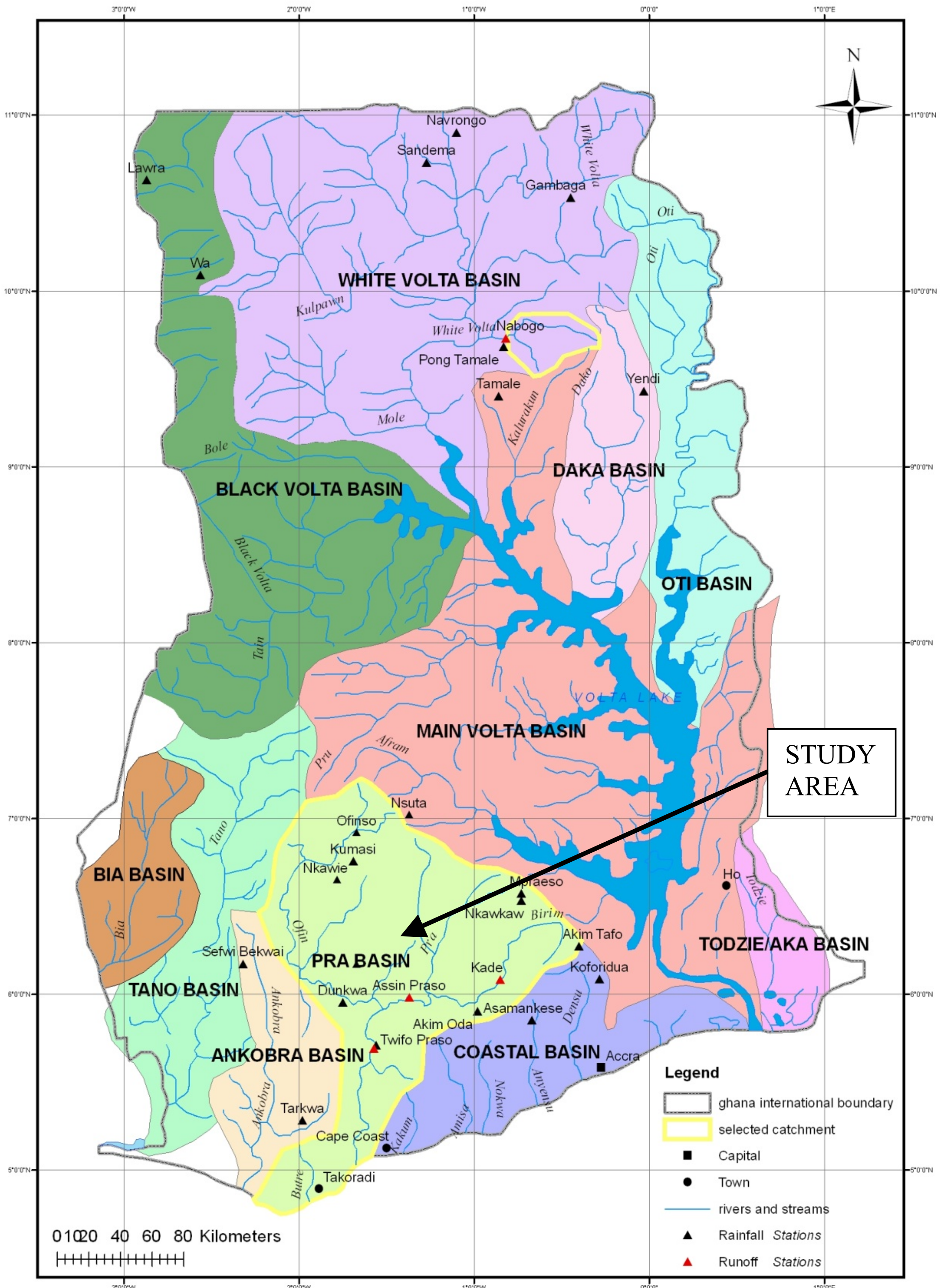


Fig. 1. Map of Ghana showing the location of the selected gauging stations (i.e. small red triangles) in the River Pra basin at Kade on River Birim, and Assin Praso and Twifo Praso on River Pra used in the DBM transfer function rainfall-riverflow modelling and the drainage basins in Ghana.

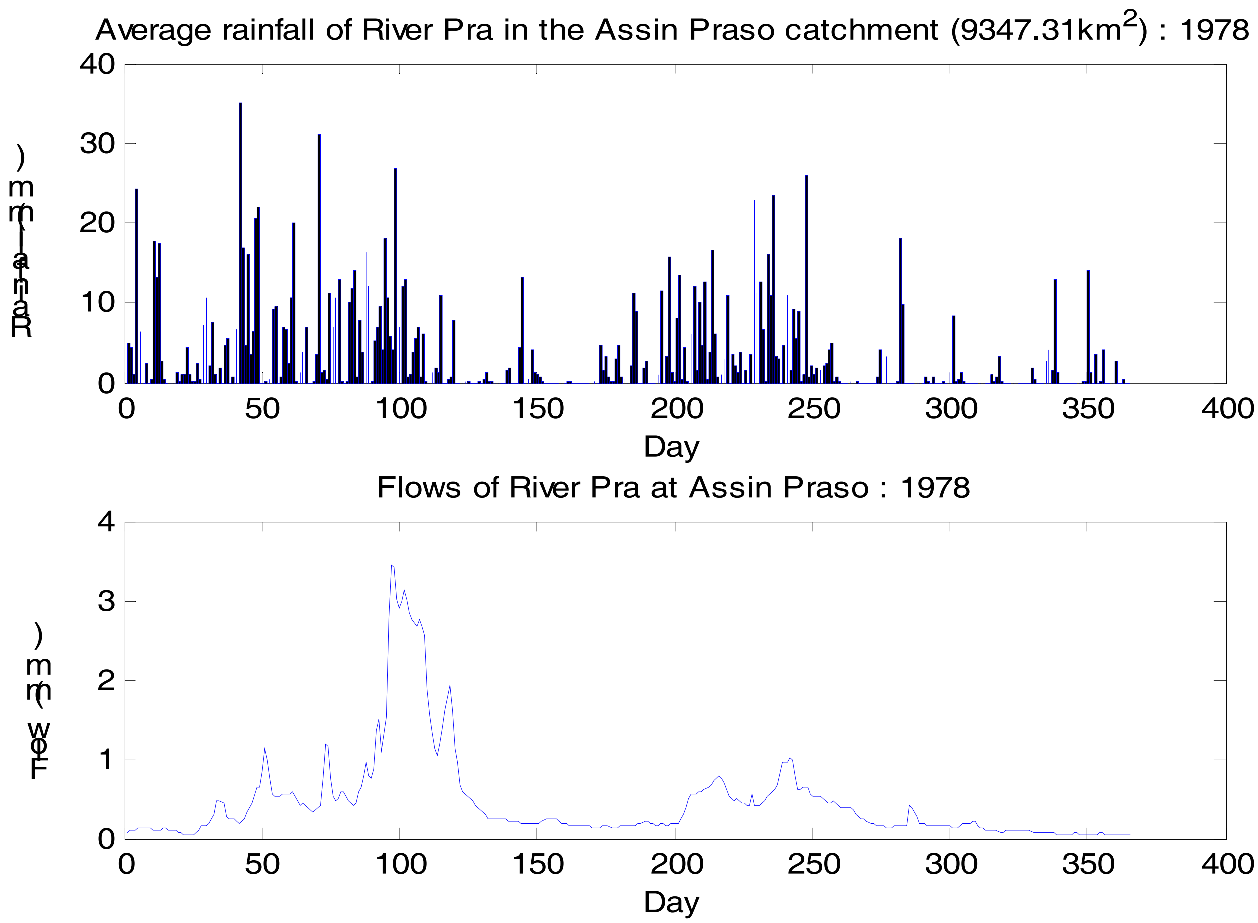


Fig. 2. Daily rainfall and flows of River Pra at Assin Praso for the 1978 water year (from March 1, 1978 to February 28, 1979) showing the bimodal regime of rainfall in the Forest zone which is followed by riverflow.

The data series used were daily riverflow (cumecs) and daily rainfall (mm) obtained from the Hydrological Services Department (HSD) and Meteorological Services Department (MSD), respectively in Accra, Ghana. Flows from River Birim gauged at Kade and River Pra gauged at Assin Praso and Twifo Praso were used (Fig. 1). The riverflow data in cumecs were converted to millimetres per day using the respective catchment areas of the selected gauging stations (Table 1).

Table 1. Catchment sizes for River Birim at Kade and River Pra at Assin Praso and Twifo Praso in the River Pra basin (see Fig. 1 for the location of the gauging stations)

River	Gauging Station	Catchment Area (km^2)
Birim	Kade	2126.67
Pra	Assin Praso	9347.31
Pra	Twifo Praso	20778.00

MATERIALS AND METHODS

Linear transfer function model (LTFM)

The general form of linear transfer function (single input-single output) which forms the basis of the transfer function model (TFM) package is given by

$$y_t = \frac{B(z^{-1})}{A(z^{-1})} U_{t-\delta} + \varepsilon_t \tag{1}$$

where the transfer function polynomials are defined as

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots .. a_N z^{-N} \tag{2}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots .. b_M z^{-M} \tag{3}$$

where y_t is the observed riverflow, U_t is 'effective rainfall', z is a backward shift operator (i.e. $z^{-q} u_t = u_{t-q}$) and δ is the pure time delay (i.e. delay between rainfall and initial river response). The *N* and *M* represent the number of *a* and *b* parameters, respectively. The residual ε_t is defined as

$$\varepsilon_t = y_t - Q_t \tag{4}$$

$$Q_t = \frac{\hat{B}(z^{-1})}{\hat{A}(z^{-1})} U_{t-\hat{\delta}} \tag{5}$$

where Q_t is the model output, $\hat{A}(z^{-1})$ and $\hat{B}(z^{-1})$ are the estimated TF polynomials in z^{-1} of Equation 2 and 3, respectively and $\hat{\delta}$ is the estimated pure time delay. In a first order model the estimates of the TF polynomials are $\hat{B}(z^{-1}) = \hat{b}_0$ and $\hat{A}(z^{-1}) = 1 + \hat{a}_1 z^{-1}$. Term $\hat{b}_0$ is the system production or gain parameter estimate (or 'water balance' term) which scales the difference in total volumes of input and output and $\hat{a}_1$ is the recession or lag parameter estimate which is linked to the 'residence time' of the response in the catchment. The derivation of Equation 1 can be found in Beven (2001a). According to Young (2003) the residual (ε_t; noise term) accounts for all the riverflow not explained by Q_t and includes factors such as the modelling error, noise in the data, the effects of unobserved inputs and spatial heterogeneity in the rainfall data. The order of the transfer function model is defined by the triad [*N, M,* δ]. Where *N* and *M* represent the number of *a* and *b* parameters in Equations 2 and 3, respectively and δ is the pure time delay.

Depending on the nature of the dominant pathways within a catchment, a first-order transfer function model (see: Young, 1992, 1993; Young and Beven, 1994; Chappell *et al*., 1999, 2004b) or a higher-order model (see: Young and Beven, 1994; Young, 1993; Young *et al*., 1997; Lees, 2000; Young, 2001; Vongtanaboon and Chappell, 2004) may best describe the rainfall-riverflow response. A typical first-order transfer function model is given by Young (1992, 2005), Young and Beven (1994) and Chappell *et al*. (1999, 2004b, 2006) as:

$$Q_t = \frac{P}{1 - \Re z^{-1}} U_{t-\delta} \tag{6}$$

where Q_t is the subsurface flow along the dominant flow pathway at time step *t*; *P* the production parameter; $\Re$ the recession parameter; *U* effective rainfall; δ the pure time delay between the effective rainfall and the initial riverflow response and z^{-1} is the backward shift operator. The DRCs which describes the rainfall-riverflow of a catchment are based on the parameters *P* and $\Re$ of Equation 6 and are given by Chappell *et al.* (1999, 2006) as:

$$SSG = \frac{P}{1 - \Re} \tag{7}$$

$$TC = \frac{-t_{base}}{\log_e(\Re)} \tag{8}$$

where *SSG* is the steady state gain (water balance term); *TC* is the time constant (residence time) and t_{base} is the sampling interval (in this study a day). The *SSG* indicates the amount of the rainfall which appears as riverflow following evapo-transpiration and other losses, while *TC* is a measure of the residence time of the rainfall in the catchment.

Modelling non-linearities in hydrological behaviour

The hydrological process of the translation of rainfall into riverflow is inherently nonlinear due to the effects of varying subsurface moisture (FAO, 1981; Young and Beven, 1994). To model this non-linearity, the effective rainfall *U* in Equation 1 is often related to the actual rainfall *R* and the observed flow y by a nonlinear function (e.g. a power law: see: Young and Beven, 1994; Chappell *et al.*, 1999; Beven, 2001).

Power law sub-model (SSSM)

In the power law application, the effective rainfall *U* in (Equation 1) is linked with the actual rainfall *R* and the observed flow y by a power law relationship which is referred to as the store–surrogate sub-model (SSSM) and is defined by Young and Beven (1994) as:

$$U_t = R_t y_t^{\alpha} \tag{9}$$

where α is the estimate of the power law exponent which is a measure of the sensitivity of the catchment to antecedent moisture conditions. The term α, usually ranges between zero and unity (Beven, 2001a) with the value of unity indicating higher sensitivity and zero no sensitivity, i.e. the linear model (Equation 1). Young and Beven (1994) estimated a value of 0.628 for a catchment in Mid-Wales in the U.K. and Young (1998) estimated a value of 0.770 for a catchment in the USA. Chappell *et al*. (1999) also estimated a value of 0.420 for equatorial catchment in East Malaysia. The catchment in the USA is clearly more sensitive to antecedent moisture conditions.

Bedford Ouse sub-model (BOSM)

Within the DBM methodology the Bedford Ouse Sub-Model (BOSM) (Young, 2001;
Chappell *et al*., 2004b, 2006) is also used to model the nonlinear component of the rainfall-riverflow process. The general form of the model is given in Chappell *et al*. (2004b, 2006) as:

$$U_t = R_t \theta_t \tag{10}$$

where

$$\theta_t = \theta_{t-1} + \frac{1}{\tau_u}\{R_t - \theta_{t-1}\} \tag{11}$$

where U_t is the effective rainfall (mm); R_t is the average (gross) rainfall (mm); θ_{t-1} is the unsaturated zone storage variable at the previous time step (mm); τ_u is the dimensionless nonlinearity term for the whole catchment response. The nonlinearity term (τ_u) is obtained by an iterative process applied to the BOSM and transfer function expressions with the objective function set at a higher R_t^2 and a minimum YIC with θ initially set as zero. The IHACRES model (Jakeman *et al.*, 1990; Jakeman and Hornberger, 1993) has also been used in the modelling of nonlinear behaviour in the rainfall-riverflow process (e.g. see: Post and Jakeman, 1996; Sefton and Howarth, 1998; Young, 2001). This model is an extension of the BOSM approach, which includes temperature effects.

Exponential function sub-model (EFSM)

An exponential function sub-model (EFSM) has also been used to quantify nonlinear component of the rainfall riverflow process (see: Young, 2006). The application of the EFSM within the DBM methodology of this study is probably, the first of its kind in the tropics. The general form of the model is given by

$$U_t = R_t\left(1 - e^{-\beta \cdot y_t}\right) \tag{12}$$

where U_t is the effective rainfall (mm); R_t is the average (gross) rainfall (mm); y_t is the observed riverflow (mm); β is the exponential parameter ($\beta \neq 0$). This approach was successfully used by Young (2006) to model the daily rainfall-flow data from the Leaf River catchment, with an estimated β parameter of 0.0124 and efficiency of 86.0%. This 1944 km^2 catchment is a humid watershed, located in Mississippi, USA.

Normalisation of 'effective rainfall' produced by nonlinear sub-model

In order to maintain mass balance, the effective rainfall from the EFSM and BOSM nonlinear rainfall filters are normalised in relation to the catchment average rainfall. The normalised effective rainfall Ue_t is given in Chappell *et al*. (1999) as:

$$Ue_t = U_t\left(\frac{\sum R_t}{\sum U_t}\right) \tag{13}$$

The nonlinearity term with these models can be incorporated into the triad to give $[N, M, \delta]^P$ where P is the nonlinear term (i.e. BOSM or EFSM filter), N and M represent the number of a and b parameters in Equations 2 and 3, respectively and δ is the pure time delay.

The EFSM model utilises past riverflows to derive the form of the nonlinearity while the BOSM filter requires only rainfall. In this study, the EFSM together with the BOSM are applied.

Model order (complexity) identification

The model identification may result in a range of models $[N, M, \delta]^P$ giving a good fit to the data. A first-order model has one dominant mode water pathway describing the rainfall-riverflow response. A second-order model is normally explained by having two parallel water pathways, a fast pathway and a slow pathway (Young and Beven, 1994; Young, 1993, 2002). Example of fast pathways includes infiltration-excess overland flow (van Loon and Keesman, 2000) or shallow sub-surface flow (Chappell *et al.*, 1998). Slow pathway includes flow deep within rock aquifers (e.g. Sefton and Howarth, 1998). The more pathways the model identifies as plausible, the higher the model order. The best of them is considered based around the coefficient of determination (R_t^2) (or Simplified Nash and Sutcliffe efficiency (1970) criteria in hydrological literature) and the heuristic Young Information Criterion (YIC) (Young and Beven, 1994; Lees, 2000; Young, 2001; Beven, 2001a) which are defined as follows:

$$R_t^2 = 1 - \frac{\sigma^2}{\sigma_0^2} \tag{14}$$

$$\text{YIC} = \ln\left(\frac{\sigma^2}{\sigma_0^2}\right) + \ln\left(NEVN\right);$$

$$NEVN = \frac{1}{np}\sum_{i=1}^{i=np}\frac{\sigma^2 P_{ii}}{\hat{\theta}_i^2} \tag{15}$$

where σ_0^2 is the variance in the observed data; σ^2 is variance of the model residuals; NEVN is the Normalised Error Variance Norm which is a measure of the model's parsimony (i.e. the degree of over-parameterisation in the model); $np = n + m + 1$ is the number of estimated parameters in the θ vector; $\sigma^2 P_{ii}$ is an estimate of the variance of the estimated uncertainty on the ith parameter estimate; and $\hat{\theta}_i^2$ is the square of the ith parameter in the θ vector.

The R_t^2 is a statistical measure of how well the model explains the variance of the data, if it is between zero and unity it is the proportion of output variance explained by the model: as the model fit improves its value approaches unity, thus when the variance of the residuals is low as compared to the variance of the data. If σ^2 and σ_0^2 are of similar magnitude then it tends towards zero and the model fits no better than the mean of the observed data. With particularly bad models (e.g. unstable), residual variance can be larger than that of the output data, which explains negative values of R_t^2.

The YIC is a more complex criterion that provides a measure of the balance between model fit and over-parameterisation (Lees, 2000). The first term of YIC is simply a relative measure of how well the model explains the data. Thus, when the model residuals get smaller and closer to zero the term becomes more negative. The second term quantifies the degree of over-parameterisation in the model, and tends to become larger when the model is over-parameterised and the parameter estimates are poorly defined. Based on the above criteria the approach helps to identify a model which explains the data well with a minimum number of parameters which are statistically well defined.

DBM rainfall to riverflow modelling steps

The procedure for the building of DBM transfer function rainfall to riverflow model (see: Young and Beven, 1994; Lees, 2000; Young, 2001) is as follows:

1) Identify linear transfer function model for the time series data [y_t, U_t] using the Simplified Refined Instrumental Variable (SRIV) algorithm (Young, 1985, 1991). The SRIV method uses a recursive least square algorithm (Young, 1984) followed by the application of the instrumental variable (IV) method (Young, 1985) which removes the bias of the estimates.
2) Examine the model fit by visualisation and investigation of goodness of fit using R_t^2. If the model fit is satisfactory the analysis is complete, proceed to step 8. Otherwise, proceed to step 3.
3) Based on the analysis in steps 1 and 2 plus knowledge of the physical/hydrological system select the simplest transfer function model which appears capable of characterising the behaviour of the output variable (riverflow) in relation to the observed input (rainfall).
4) Obtain initial estimate of time variable parameters (TVPs) in a transfer function model by using fixed interval smoothing estimation (FIS) (Young, 1984; Young 1986; Young, 1998).
5) Investigate state dependent parameter (SDP) relations (e.g. gain versus riverflow) using scatter plots (Young and Beven, 1994; Young, 2001; 2003; 2006).
6) If a single relationship emerges, repeat the TVP estimation with the data processed in order of the ranked dependent state to improve the SDP relation.
7) In case of the presence of gain nonlinearities, reformulate the model as an input nonlinearity combined with a linear transfer function model and estimate the parameters using the SRIV method of system identification.
8) Investigate the physical interpretation of the different resultant models and select the one that explained the data well and has a sensible mechanistic (hydrological) interpretation of the data. This aspect of the approach is the 'heart and soul' of the DBM approach.

Application of the DBM TF model to the data

Initial visual analysis of the rainfall and riverflow data in the catchments revealed that the 1978 water year (i.e. from March 1, 1978 to February 28, 1979) was the only period where data was available at riverflow stations used in the study. The 1978 water year was, therefore, used as the period of analysis for the application of the DBM TF model.

The DBM TF model as outlined in above was applied to riverflows of River Birim at Kade and River Pra at Assin Praso and Twifo Praso within the River Pra Basin (see map: Fig. 1). Average rainfall over the catchments of Kade, Assin Praso and Twifo Praso at each time step of a day was used as input into the model. The averaging process was done by using the Thiessen Polygon approach (Mutreja, 1986; Linsley *et al*., 1988; Shaw, 1994). This approach allows area-weighted integration of rain gauge totals from gauges within and adjacent to the catchment to be used as input into the model.

Using the SRIV identification algorithm and YIC and R_t^2 as model order identification criteria a range of linear transfer function models relating the input (average rainfall) and the output (riverflow) were obtained for the above named riverflow stations. Due to catchment hydrological systems being inherently nonlinear, a time varying parameter (TVP) model was applied to investigate the form of the nonlinear behaviour in the data (e.g. see: Young and Beven, 1994: Chappell *et al*., 1999; Lees, 2000). The TVP model was estimated where the production parameter (P: see Equation 6) was allowed to vary whilst the recession parameter ($\Re$: see Equation 6) was kept constant, followed by State Dependent Parameter (SDP) modelling with the flow representing the dependent state. The SDP analysis quantifies any state dependency in the parameter variations which is associated with nonlinear behaviour in the catchments. More detailed discussion on SDP can be found in Young (2001, 2006).

To show the relationship between the production parameter and the flow, the sorted state (i.e. riverflow) and the sorted SDP estimates (from the SDP function) were plotted. Exponential relationship between them was

Table 2. Purely linear model identification of flows of River Birim at Kade, River Pra at Assin Praso and Twifo Praso in the River Pra basin.

Station	River	Model	YIC	R_t^2 (%)
Kade	Birim	[1 1 1]	-6.082	62.45
Assin Praso	Pra	[1 1 1]	-5.151	50.36
Twifo Praso	Pra	[1 1 2]	5.371	58.41

investigated using an optimisation routine fitted to the sorted SDP parameters (i.e. non-parametric estimate). Following that a separate optimisation routine was then used to estimate the exponential parameter β (Equation 12) for the data (i.e. parametric estimate).

The routine utilises the SRIV algorithm within an iterative procedure to optimise the exponential parameter while the SRIV model residual variance was minimised. The exponential parameter was then used to transform the catchment average rainfall, into catchment-average 'effective rainfall' using the EFSM (see: Equation 12) equation. To ensure mass balance, the catchment average 'effective rainfall' was normalised in relation to the catchment-average rainfall using Equation 13. After the normalisation, the SRIV algorithm was used again to identify a range of transfer function models relating the 'normalised catchment-average effective rainfall' to riverflow with their respective parameters. Using YIC and R_t^2, the model which explained the data well with good physical meaning of the estimated parameters was selected for each riverflow station.

RESULTS AND DISCUSSION

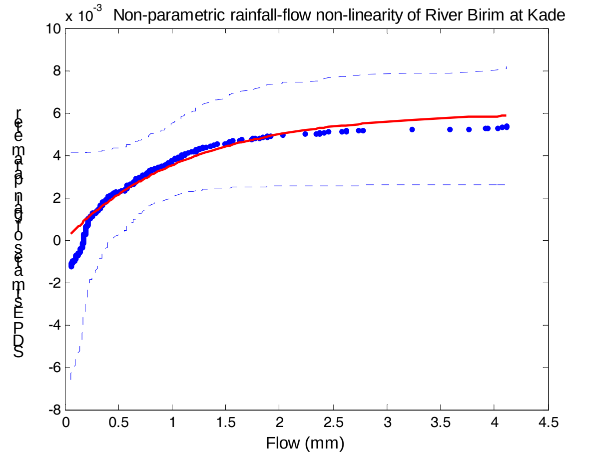
Purely linear TF modelling

The efficiencies (R_t^2) of purely linear transfer function modelling of the data at all the riverflow stations considered within the River Pra basin ranges between 50.36% and 62.45% (Table 2). These efficiencies are low, possibly due to the presence of nonlinearities as a result of variable antecedent moisture conditions (FAO, 1981; Young and Beven, 1994; Chappell *et al*., 2004a).

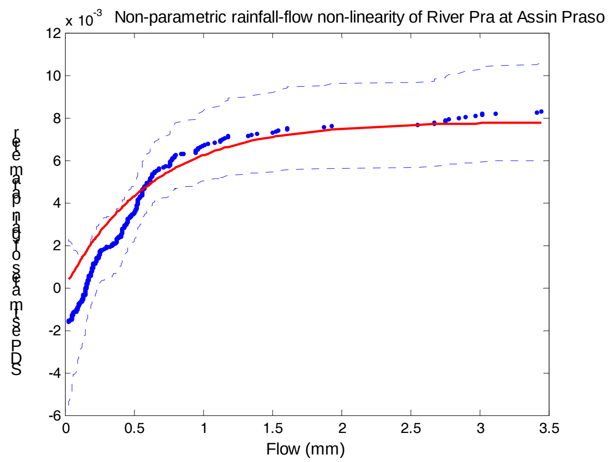
The model structures of all the catchments are first-order (see: Table 2). These structures only give preliminary indication of the likely model orders and time delays because the linear models only have low efficiencies.

TVP and SDP TF modelling

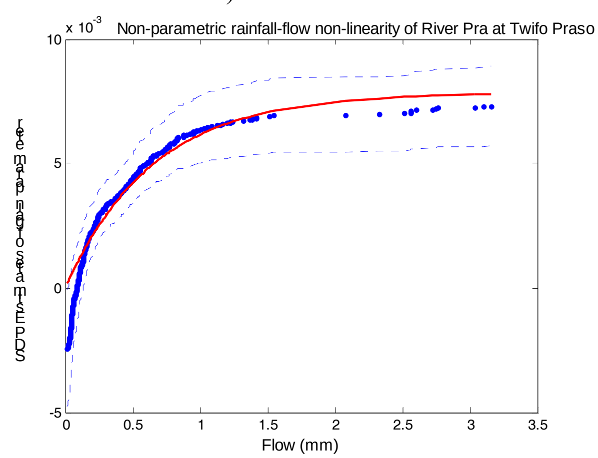
Investigation of the presence of nonlinearities using TVP and SDP modelling as explained in above resulted in an SDP fit of the observed riverflows, which describes the rainfall-riverflow response with efficiency (R_t^2) ranging from 98.11 to 98.78% for all the catchments. These indicate that the SDP model captured almost all of the nonlinearities in the rainfall-riverflow behaviour within the catchments. This suggests that the plot of the SDP parameter estimates would give the full nature of the nonlinear behaviour of the catchments. The plots of the SDP parameter estimates (namely gain or '*P*' in Equation 6) against the riverflows are shown in figure 3. The plots show that the gain parameter increases with increasing flow, which suggests that nonlinearities are present in the translation of rainfall to riverflow in the catchments.



a) Kade



b) Assin Praso



c) Twifo Praso

Fig. 3. Non-parametric estimate of the rainfall flow nonlinearity in gain parameter (see 'P' in Equation 6) as a function of flow (blue dots) and uncertainty (blue dashed lines). a) River Birim at Kade b) River Pra at Assin Praso and c) River Pra at Twifo Praso fitted with exponential curve (solid red line).

The plot of River Birim at Kade (Fig. 3a), River Pra at Assin Praso (Fig. 3b) and Twifo Praso (Fig. 3c) suggest that the nonlinear behaviour of the riverflows within the River Pra Basin follow an exponential relationship between the gain parameter and the riverflow. From the plots it could be seen that as the catchment wets up the instantaneous runoff coefficient (i.e. proportion of rainfall generating riverflow) keeps increasing but gets to a point where it does not change. This means that as the catchment wets up, the runoff coefficient increases, until it reaches a point where the runoff coefficient remains constant, effectively giving a linear relationship between rainfall and riverflow.

The estimated exponential function from the SDP modelling for the flows of River Birim at Kade and River Pra at Assin Praso and Twifo Praso are given as:

$$boKD_t = 0.006\left(1 - e^{-0.8812\, yKD_t}\right) \tag{16}$$

$$boAS_t = 0.0078\left(1 - e^{-1.5999\, yAS_t}\right) \tag{17}$$

$$boTW_t = 0.0078\left(1 - e^{-1.5433\, yTW_t}\right) \tag{18}$$

where *boKD*, *boAS* and *boTW* are the gain parameter estimates and *yKD*, *yAS*, and *yTW* are the riverflows of River Birim at Kade, River Pra at Assin Praso and Twifo Praso, respectively. Structurally, the exponential function is limited to be non-negative and this is a sensible solution in this case – also well contained within the uncertainty bounds of the SDP estimates.

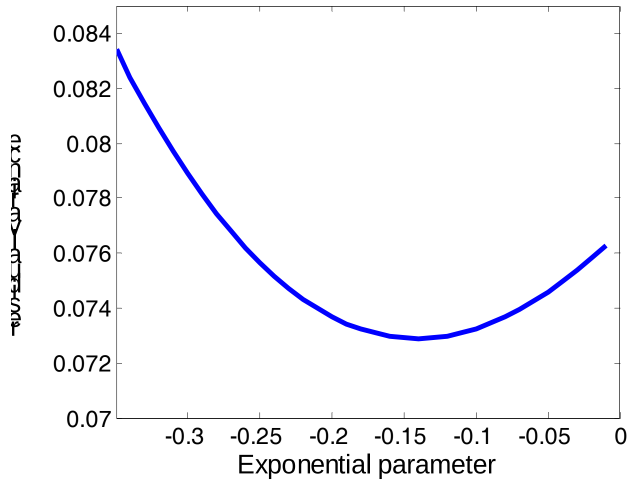
Nonlinear TF modelling

Final optimisation of the exponential parameter β (Equation 12) for the catchments, using iterative routines are shown in figure 4. The optimised values of β are for the estimation of 'effective rainfall' and subsequent modelling of nonlinear behaviour within the catchments. The plots suggest that, riverflow simulation is highly sensitive to β values.

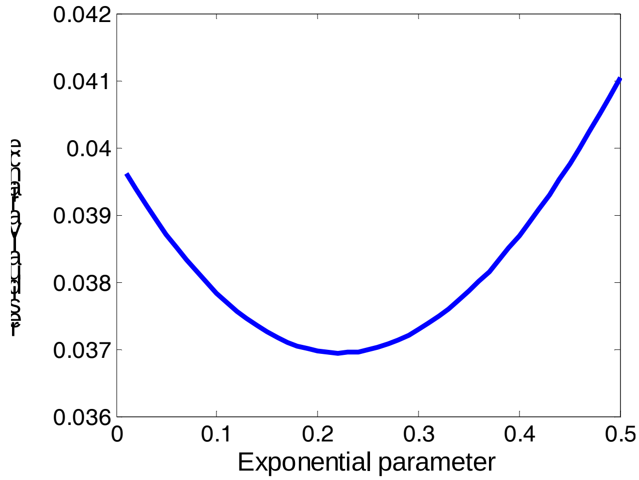
First-order modelling (Single water pathway)

The estimates of the exponential parameter for the catchments, model efficiencies and the resultant optimised first-order nonlinear transfer function model parameters and statistics optimised against YIC and R_t^2 as the objective functions are presented in table 3. The model parameters and statistics estimated for an optimised first-order transfer function model using the BOSM nonlinear filter are shown in table 4.

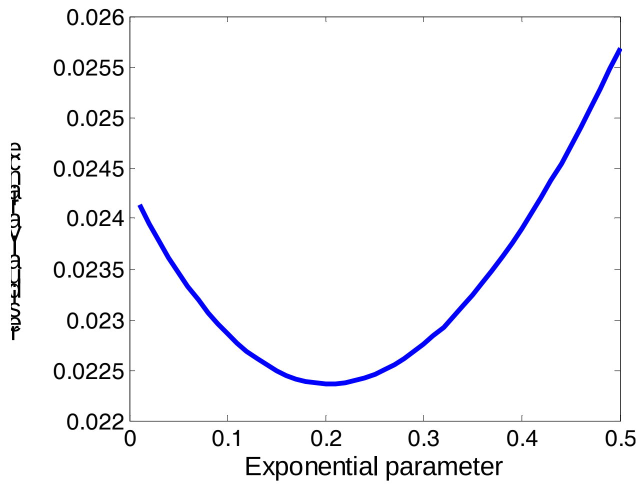
From table 3, the first-order EFSM model provides an excellent fit for riverflows of River Birim at Kade, River Pra at Assin Praso and Twifo Praso with efficiencies (R_t^2) of 88.26, 89.55 and 92.94%, respectively. The BOSM model (Table 4) gives efficiencies of 72.53, 69.71 and 77.94%, respectively, for the same stations in the basin.



a) Kade (EFSM)



b) Assin Praso (EFSM)



c) Twifo Praso (EFSM)

Fig. 4. Final optimisation of the exponential parameter of the EFSM (see 'β' in Equation 12) for the catchments for the estimation of effective rainfall and modelling of nonlinear behaviour (see: Fig. 1 for the location of the gauging stations).

Table 3. First-order nonlinear DBM model parameters identified for the catchments within the River Pra basin for the 1978 water year using EFSM as the nonlinearity filter.

Parameters and statistics	Catchments within the River Pra Basin		
	Kade	Assin Praso	Twifo Praso
Area (km^2)	2126.67	9347.31	20778.0
R_t^2 (%)	**88.26**	**89.55**	**92.95**
Model order	[1 1 0]	[1 1 0]	[1 1 0]
YIC	**-8.740**	**-9.021**	**-9.738**
β	-0.1409	0.2226	0.2037
$\Re$	-0.8947	-0.8860	-0.8868
$\sigma(\Re)$	0.0043	0.0042	0.0035
P	0.0195	0.0132	0.0111
σ(P)	0.0007	0.0004	0.0003
TC (days)	8.9868	8.2618	8.3234
σ(TC)	0.3822	0.3275	0.2780
SSG	0.1853	0.1158	0.09828
σ(SSG)	0.0030	0.0019	0.0013

Note: R_t^2: Simplified Nash and Sutcliffe efficiency for model; Model order: [No. of denominators, numerators, pure time delays]; YIC: Young Information Criterion; β: exponential parameter; $\Re$: recession parameter; P: production parameter; TC: time constant; SSG: steady state gain of the transfer function; $\sigma(\Re)$, σ(P), σ(TC) and σ(SSG): standard deviation of parameter in the parenthesis. See Equation 12 (EFSM).

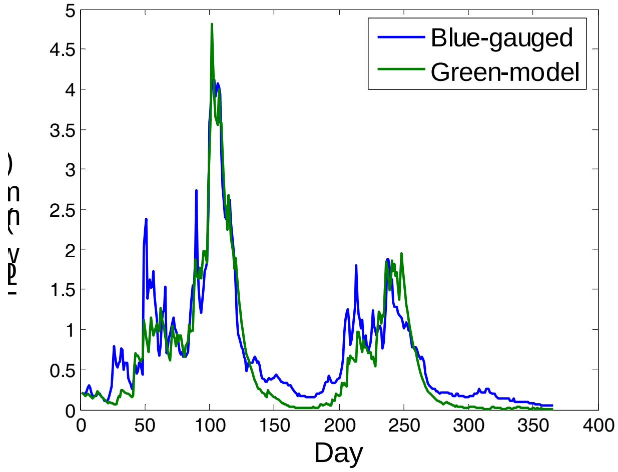
Table 4. First-order nonlinear DBM model parameters identified for the catchments within the River Pra basin using BOSM as nonlinearity filter to model rainfall to riverflow for 1978 water year.

Parameters and statistics	Catchments within the River Pra Basin		
	Kade	Assin Praso	Twifo Praso
Area (km^2)	2126.67	9347.31	20778.0
R_t^2 (%)	**72.53**	**69.71**	**77.94**
Model order	[1 1 1]	[1 1 1]	[1 1 1]
YIC	**-6.873**	**-6.315**	**-6.918**
τ_u	55	50	30
$\Re$	-0.9221	-0.9145	-0.9051
$\sigma(\Re)$	0.0051	0.0069	0.0066
P	0.0171	0.0123	0.0114
σ(P)	0.0010	0.0009	0.0008
TC (days)	12.3271	11.191	10.0316
σ(TC)	0.8563	0.9827	0.7566
SSG	0.2197	0.1434	0.1201
σ(SSG)	0.0052	0.0040	0.0027

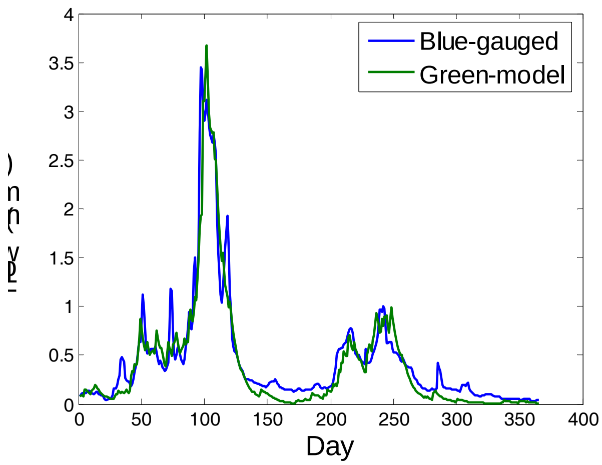
Note: τ_u : BOSM nonlinearity term. See Equation 11 (BOSM).

The EFSM is expected to perform better than the BOSM model, because in the evaluation of the nonlinear behaviour of the catchments the EFSM model uses riverflow as a surrogate of sub-surface moisture, unlike BOSM which *a priori* fixes the form of the non-linearity. Figure 5 and 6 shows the ability of the DBM model to capture the key dynamics inherent in the relationship between the incoming rainfall and the outgoing riverflow within the catchments, using EFSM and BOSM models as nonlinear filters, respectively.

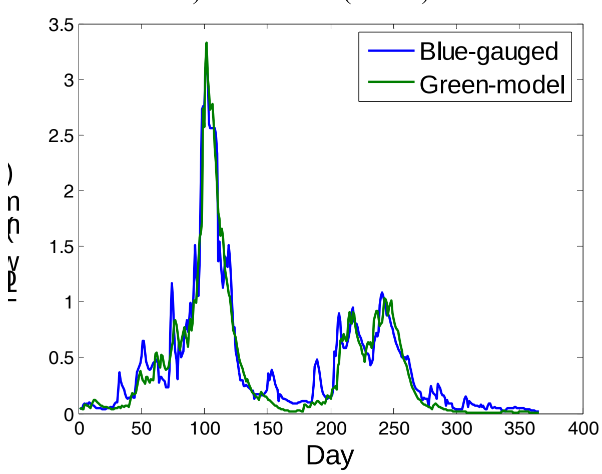
Generally, the performance of the models, in terms of explanation of the model output variance is excellent for all the models (Table 3 and 4) but the model fit shows that peak flows during the major rainfall season (i.e. May to June) were underestimated by the BOSM model (Fig. 6). However, the BOSM model predicted the recession flows very well as compared to the EFSM model at all the stations. (Figs. 6a, b and c). Thus, within the River Pra basin the BOSM and EFSM sub-models are recommended for low and high flow studies, respectively, based on their performance (Figs. 5 and 6).



a) Kade (EFSM)

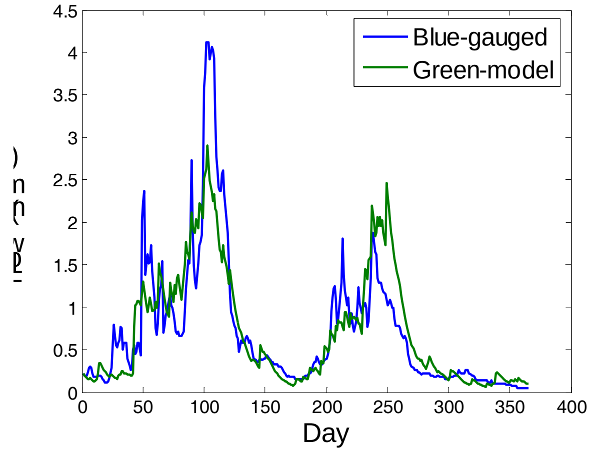


b) Assin Praso (EFSM)

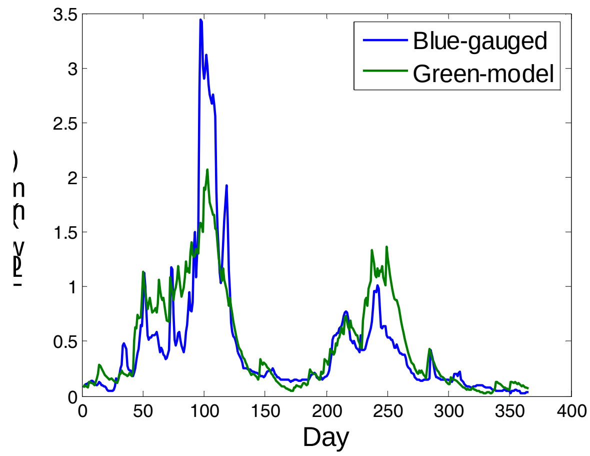


c) Twifo Praso (EFSM)

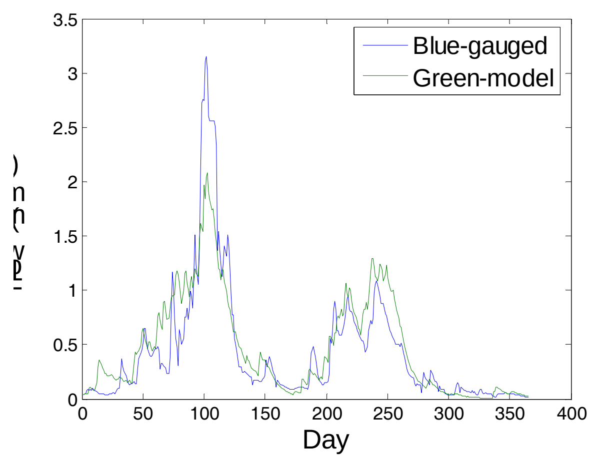
Fig. 5. Daily flows predicted by optimum first-order nonlinear transfer function EFSM model (green) against observed flows (blue) showing the DBM model's ability to capture the dynamics of the rainfall to riverflow generating mechanism in the catchments within the River Pra basin (i.e. from March 1, 1978 to February 28, 1979) (See: Table 3 for the models).



a) Kade (BOSM)



b) Assin Praso (BOSM)

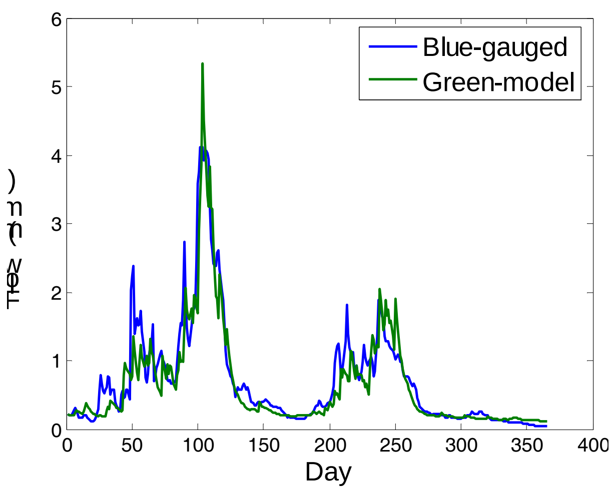


c) Twifo Praso (BOSM)

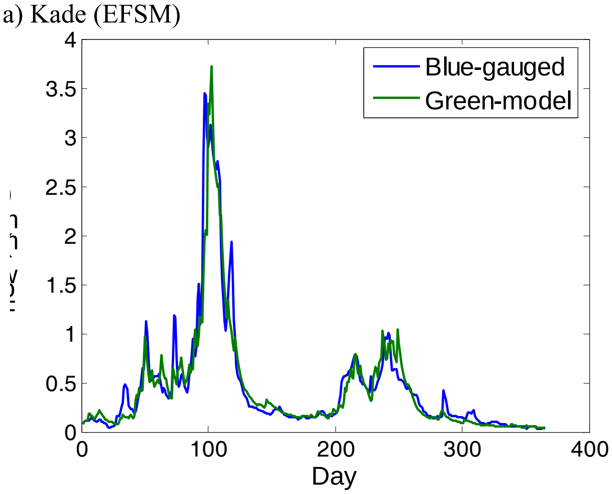
Fig. 6. Daily flows predicted by optimum first-order nonlinear transfer function BOSM model (green) against observed flows (blue) showing the DBM model's ability to capture the dynamics of the rainfall to riverflow generating mechanism in the catchments within the River Pra basin for the 1978 water year (i.e. from 1st March, 1978 to 28th February, 1979). (See: Table 4 for the models).

Higher-order modelling (multiple water pathways)

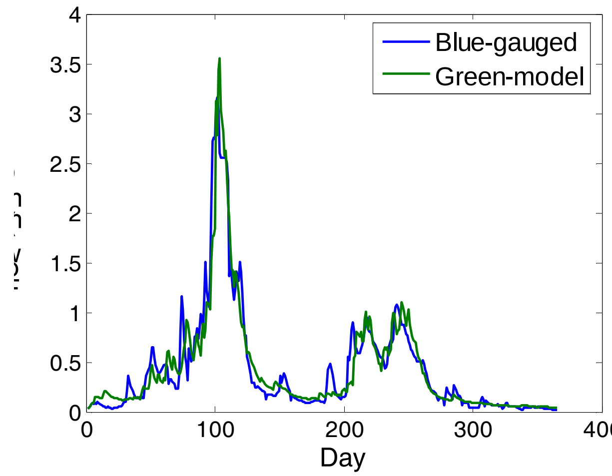
The presence of multiple runoff pathways in the catchments was also investigated using YIC and R_t^2 as the objective functions in higher-order modelling (up to fourth order). The results are shown in table 5 and 6 for the EFSM and BOSM models, respectively. From the tables, comparison of the R_t^2 and YIC of the higher-order models to those of the first-order models in the basin show reduced R_t^2 values and higher YIC values of the higher-order models. For instance, at Kade, R_t^2 and YIC of the first-order model reduced from 88.26% and -8.740 to 84.51% and -7.869, at Assin Praso from 89.55% and -9.021 to 88.45% and -7.302 and at Twifo Praso from 92.94% and -9.738 to 89.27% and -7.41, respectively, for the higher-order models. Similarly, the BOSM also shows reduction in R_t^2 and less negative YIC values at all the stations (Table 6). Thus, comparison of the model efficiencies and YICs of the first-order models to those of the higher-order models indicate that higher-order models could not be justified for the catchments despite the improvement in the fit of mid and late recessions (Fig. 7). Thus, a single pathway dominates the catchments behaviour in routing rainfall to riverflow in the basin.



a) Kade (EFSM)



b) Assin Praso (EFSM)



c) Twifo Praso (EFSM)

Fig. 7. Daily flows predicted by optimum second order nonlinear transfer function model (green) against observed flows (blue) showing the DBM model's ability to capture the dynamics of the rainfall riverflow generating mechanism in the catchments within the River Pra basin for the 1978 water year (i.e. from March 1, 1978 to February 28, 1979) (see: Table 5 for the models).

Final first-order models identified for the catchments

Based on the EFSM parameterisation of the nonlinearity (see: Table 3), mathematical relationships between rainfall input and riverflow output with no initial pure time delay were identified for the catchments i.e. Kade, Assin Praso and Twifo Praso. These are as follows:

Kade: $$Q_t = \frac{0.0195}{1-0.8947z^{-1}} UeKD_t \tag{19}$$

Assin Praso: $$Q_t = \frac{0.0132}{1-0.8860z^{-1}} UeAS_t \tag{20}$$

Twifo Praso: $$Q_t = \frac{0.0111}{1-0.8868z^{-1}} UeTW_t \tag{21}$$

where $UeKD_t$, $UeAS_t$, and $UeTW_t$ are 'normalised catchment effective rainfall' inputs for Kade, Assin Praso, and Twifo Praso, respectively and z^{-1} is the backward shift operator. The no pure time delay for the flows suggests that rainfall is more rapidly seen as riverflow in the basin.

Table 7 shows the performance of the DBM models compared with conceptual and physics-based models which have been applied in Ghana and the neighbouring countries. The Table shows that the performance of the simple first-order DBM TF models which require only four parameters, namely exponential parameter (β), recession parameter ($\Re$), production parameter (P), pure time delay (δ) gives efficiencies for similar African catchments (of a range of sizes) which are no smaller than those of complex conceptual or physics-based models.

Table 5. Comparison of YIC and R_t^2 of identified first-order and high-order nonlinear models using EFSM as the nonlinearity filter for the flows within the River Pra Basin.

Station	First-order model			Higher-order model		
	R_t^2 (%)	YIC	model order	R_t^2 (%)	YIC	model order
Kade	**88.26**	**-8.740**	**[1 1 0]**	84.51	-7.869	[2 2 2]
Assin Praso	**89.55**	**-9.021**	**[1 1 0]**	88.45	-7.302	[2 2 1]
Twifo Praso	**92.94**	**-9.738**	**[1 1 0]**	89.27	-7.41	[2 2 2]

Table 6. Comparison of YIC and R_t^2 of identified first-order and high-order nonlinear models using BOSM as the nonlinearity filter for all the gauging stations.

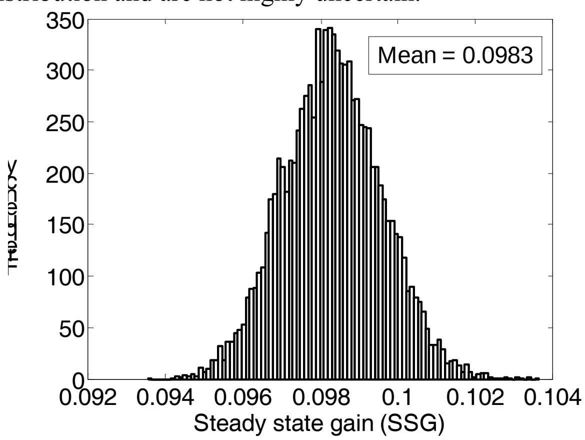
	First-order model			Higher-order model		
Station	R_t^2 (%)	YIC	model order	R_t^2 (%)	YIC	model order
Kade	**72.53**	**-6.875**	**[1 1 1]**	70.99	-5.956	[3 1 2]
Assin Praso	**69.71**	**-6.349**	**[1 1 1]**	61.65	-5.423	[3 1 0]
Twifo Praso	**78.44**	**-6.928**	**[1 1 2]**	76.83	-6.135	[3 1 0]

Other studies which demonstrates that the DBM TF rainfall-riverflow modelling technique performed efficiently with smaller number of parameters and data inputs can be found in Young and Beven (1994), Young *et al*. (1997), Chappell *et al*. (1999, 2004a, 2004b, 2006), Lees (2000), Young (1992, 1993, 1998, 2001, 2002, 2005), Mwakalila *et al*. (2001), Vongtanaboon (2004), Vongtanaboon and Chappell (2004), Romanowicz *et al*. (2006), among others.

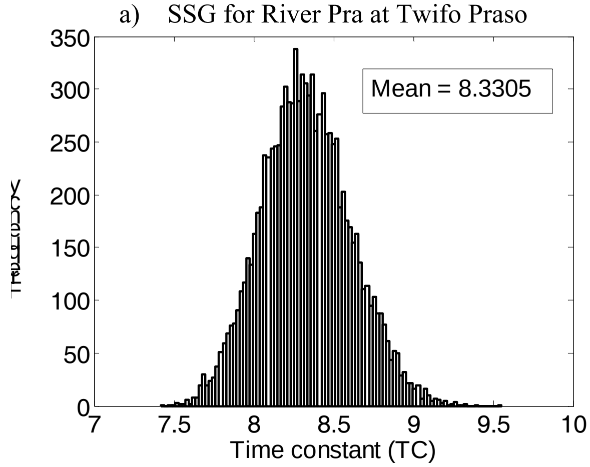
Results: Uncertainty analysis of derived parameters Time Constant (TC) and Steady State Gain (SSG)

In order to compare DRCs between the catchments (or with published data) the uncertainty in the estimated DRCs must first be investigated. The uncertainty on the DBM TF model parameters (i.e. recession parameter; $\Re$ and production parameter; *P*) was determined by assuming that the residuals follow a normal distribution (Young, 2003). It was necessary to quantify the uncertainty in the TC and SSG using Monte Carlo Simulation (MCS: see Young, 1998, 2001, 2003) analysis. MCS analysis is the simulation of a model where the model is run several times (in this study with 10,000 realisations) using different sets of parameters (here TC and SSG) which were selected randomly from the model-predicted standard error about the gain or production parameter (*P*) and the recession parameter ($\Re$).

Figure 8 shows the results of the analysis using 10,000 random realisations for River Pra at Twifo Praso. The distribution of the TC and SSG of the models for the other stations in the basin (not presented) were similar to that of Twifo Praso. The Figure suggests that the SSGs and TCs are symmetric about their means and so comparable with the mean value derived earlier. For instance, for River Pra at Twifo Praso mean SSG is 0.0983, estimated SSG = 0.0983, mean TC = 8.3501 days, estimated TC = 8.3234 days. The above indicates that the derived SSGs and TCs from the DBM TF parameter estimates follow normal distribution and are not highly uncertain.



a) SSG for River Pra at Twifo Praso



b) TC for River Pra at Twifo Praso

Fig. 8. Histogram of Monte Carlo analysis to evaluate the uncertainty associated with the derived parameters; steady state gain (see Equation 7) and time constant (days: see Equation 8) of the TF DBM model of River Pra at Twifo Praso showing well defined distribution about the means.

Table 7. Model efficiency (R_t^2: see Table 3) of identified DBM models compared with that of conceptual and physics-based models which have been applied to catchments in Ghana and neighbouring countries i.e. Ivory Coast, Burkina Faso and Benin based on model estimation.

Model	Type	Catchment	River	Area (km^2)	Country	R_t^2 (%)	Reference
GR2M	CCM	Samien	Sasandra	29,300.0	Ivory Coast	89.0	Paturel *et al.*(2003)
WBM	CCM	Samien	Sasandra	29,300.0	Ivory Coast	46.0	Paturel *et al*. (2003)
GR2M	CCM	Bada	Bandama	24,075.0	Ivory Coast	81.0	Paturel *et al*. (2003)
WBM	CCM	Bada	Bandama	24,075.0	Ivory Coast	69.0	Paturel *et al*. (2003)
GR2M	CCM	Samandeni	Moohoun	4,575.0	Burkina Faso	84.0	Paturel *et al*. (2003)
WBM	CCM	Samandeni	Moohoun	4,575.0	Burkina Faso	68.0	Paturel *et al*. (2003)
SAMULAT-H	PBM	Upper Aguima	Queme	3.2	Benin	82.0	Geirtz *et al*. (2006)
SAMULAT-H	PBM	Upper Niao	Queme	3.1	Benin	67.0	Geirtz *et al*. (2006)
ACRU	CCM	Manhia	Densu	2100.0	Ghana	82.0	Bekoe (2005)
EFSM	DBM	Kade	Birim	2126.67	Ghana	88.3	This study
EFSM	DBM	Assin Praso	Pra	9347.31	Ghana	89.6	This study
EFSM	DBM	Twifo Praso	Pra	20778.0	Ghana	93.0	This study

CCM: Conceptual model, PBM: Physics based distributed model, DBM: Data-based mechanistic, EFSM: Exponential function sub-model.

Results: Hydrological interpretation of estimated model parameters

Within the DBM methodology, hydrological/physical interpretation of the identified parameters associated with the model is very important and cannot be over-emphasised (Young, 2005). DBM models using the EFSM sub-model produced the most statistically sound models, and so it is these models that are interpreted physically/hydrologically. The parameters in the TF equations (Equations 19 - 21) to be considered are: exponential parameter: β, recession parameter: $\Re$, production parameter: *P*, pure time delay: δ, and the associated dynamic response characteristics (DRCs) i.e. the steady state gain: SSG, and time constant: TC.

Pure time delay (δ)

The pure time delay is defined as the response time for rainfall to be first seen as riverflow. The value for a catchment is large if a) the rainfall is disconnected from the water table, i.e. it includes large unsaturated zone storage or b) rainfall is located only in its headwater sub-catchments. No pure time delay was identified for all the catchments (Table 3). This means rainfall is rapidly seen as riverflow.

In Thailand, a humid tropical region like Ghana, within the X113, P47 and P14 catchments which are of size, 129, 521 and 3853 km^2, respectively, Vongtanaboon (2004) estimated no pure time delay (Table 8). Similarly, in the 0.133 km^2 C1 Bukit Berembum catchment in Malaysia, Chappell *et al*. (2004a) estimated zero pure time delay. Again in Malaysia, Chappell *et al*. (2006) obtained a value of 15 minutes, for the 0.44 km^2 Baru catchment. The data series were in 5 minutes time-step implying time delay of 3. Examples, of estimates of pure time delay of catchments in temperate conditions can be seen in table 8.

One would have expected that, the large catchments in the River Pra basin with long rivers would have had long pure time delays if rainfall fell only in the headwaters. Thus, perhaps the catchments have similar rainfall in the downstream areas. The small catchment of River Nabogo in the North of the same country (see: Fig. 1 for location) has a delay of a day (see: Table 8) possibly due to the relative dryness of the catchment (Acheampong, 1988; Kranjac-Berisavljevic, 1999; FAO, 2005; Ahenkorah *et al*., 1994) or disconnected deep groundwater storage (Bates, 1962b).

This study and the above observations suggest that, perhaps, pure time delay may not solely depend on catchment size.

Exponential parameter (β)

The exponential parameter β estimated for the catchments in the River Pra basin are River Birim at Kade: -0.1409 and River Pra at Assin Praso: 0.2226, and Twifo Praso: 0.2037 (Fig. 4). A value of 0.0124 was obtained for the Leaf River catchment located in Collins, Mississippi in the USA by Young (2006). Hydrological interpretation of this parameter is that the higher the β value, the more quickly the runoff coefficient increases with increasing storage, and the greater the resistance to antecedent moisture conditions.

Time constant (TC)

Time constant (TC) is a measure of the ‘residence time’ of rainfall in the catchments (Young, 2003, 2005; Chappell *et al*., 2006), calculated by using Equation 8. TC for the catchments within the River Pra basin is River Birim at Kade, approximately: 9.0 days [7.78 – 10.74 days] and River Pra at Assin Praso: 8.26 days [7.22 – 9.75 days] and Twifo Praso: 8.32 days [7.42 – 9.55 days] with

Table 8. Comparison of time constants of first-order DBM models of catchments in different climatic regions ranked by size

Catchment	Area (km^2)	Climate regime	Geology	TFM	NLF	Time constant	Reference
Plot	0.000015	Temperate	Acid soil	[1 1 0]	SSSM	14.5 minutes	Fawcett *et al*. (1997)
C1	0.133	Humid tropical	Saprolite	[1 1 0]	SSSM	23 days	Chappell *et al*. (2004a)
Baru	0.44	Humid tropical	Mudstone	[1 1 3]	BOSM	37 minutes	Chappell *et al*. (2006)
Coalburn	1.50	Temperate	Mudstone	[1 1 0]	BOSM	8.6 hours	Chappell *et al*. (2006)
Bottoms	10.60	Temperate	Limestone	[1 1 1]	BOSM	8.3 hours	Chappell *et al*. (2006)
X113	129	Humid tropical	Sedimentary	[1 1 0]	SSSM	2.3 days	Vongtaboon (2004)
P47	521	Humid tropical	Metamorphic	[1 1 0]	SSSM	6.14 days	Vongtaboon (2004)
P14	3853	Humid tropical	Granite/Metamorphic	[1 1 0]	SSSM	7.28 days	Vongtaboon (2004)
Nabogo	1950.00	Tropical continental	Voltain	[1 1 1]	SSSM	10.13 days	Ampadu (2007)
Kade	2126.67	Humid tropical	Birimian	[1 1 0]	EFSM	8.99 days	This study
Koumangou	6070.00	Tropical continental	Voltain	[1 2 1]	SSSM	12.10 days	Amisigo (2005)
Assin Praso	9347.31	Humid tropical	Birimian	[1 1 0]	EFSM	8.26 days	This study
Twifo Praso	20778.00	Humid tropical	Birimian/Tarkwain	[1 1 0]	EFSM	8.32 days	This study
Porga	27197.00	Tropical continental	Voltain	[1 2 0]	SSSM	8.21 days	Amisigo (2005)

Note: TFM: Transfer function model structure; NLF: Non linear filter; SSSM: Store surrogate sub-model Equation 9; BOSM: Bedford Ouse sub-model Equation 10 and 11; EFSM: Exponential function sub-model Equation 12. [No. of denominators, numerators, pure time delays].

the uncertainty on the estimated values given in the brackets. These indicate that within the River Pra basin all the catchments (ranging in size from 2126 to 20778 km^2) have similar residence time for rainfall to appear as riverflow.

Within the Malaysian rainforest (i.e. in similar climatic conditions), using the same DBM methodology (Chappell *et al*., 2004a, 2006), entirely different time constants were obtained for the 0.44 km^2 Baru and the 0.133 km^2 Bukit Berembun C1 catchment (see: Table 8). Chappell *et al*. (2006) attributed the vast difference in the time constant to the different geological formation underlying each catchment. In Northern Thailand, in the P14 and P47 catchments, located in the same climatic conditions (Boochabun *et al*., 2004; Vongtanaboon, 2004) and underlain by similar geology (Table 8), Vongtanaboon (2004) estimated similar time constants for these catchments using the same DBM approach (Table 8).

The similar time constants identified for the catchments in the River Pra basin are possible, because the catchments lie within the same climatic condition (i.e. wet semi equatorial) and vegetational zone (i.e. forest zone) and have soil cover which is predominantly Acrisols (i.e. Forest Ochrosol). The whole basin is also principally underlain by the same geological formation (i.e. the Birimian formation) with a small section in the middle of the basin underlain by the Tarkwain formation (Bates, 1962a; Dickson and Benneh, 1988; Atta-Qauyson, 1999). The Birimian formation consists of mainly granitoids (Ahenkorah *et al*., 1994) whilst the Tarkwain formation consists of sandstones, schists, quartzite, and phyllites (Dickson and Benneh, 1988; Bates, 1962a). The catchments also have similar topography, which stretches through a sequence of gently rolling hills with general elevation of between 250m and 300 m above sea level (Dickson and Benneh, 1988).

The time constant of River Nabogo located in the northern part of the same country estimated by Ampadu (2007) using daily time series of rainfall and riverflow and that of the catchments estimated by Amisigo (2005) compared with that of the catchments in the River Pra basin (Table 8) indicates that they are similar in residence time. The climate, vegetation and geological formation underlying these catchments are different resulting in different types of soil cover through weathering. The soil cover in the River Pra basin is predominantly Acrisols (locally called 'Forest Ochrosol') which is deeply weathered and well drained (Brammer, 1962; Ahenkoral *et al*., 1994; Attah-Quayson, 1999). It is possible rainfall within the catchments percolates much deeper into and through the soil before it ends up as riverflow. Deep movement of water in regolith beneath Acrisol on granite and its impacts on rainfall-riverflow processes have been observed by Chappell *et al*. (2007), in catchments within the South East Asia.

The Nabogo, Koumangou and Porga catchment which are located in the northern part of the country are underlain by the Voltain formation which consists of sandstone, shale, mudstones, and limestone (Bates, 1962a; Boateng, 1966; Dickson and Benneh, 1988). The soil cover is predominantly Plinthosol (locally called 'Groundwater Laterites') which is poorly drained and shallow

Table 9. Comparison of steady state gain (SSG) and riverflow coefficient (RC) of the catchments in the River Pra basin for the 1978 water year (i.e. from March 1, 1978 to February 28, 1979)

Catchment	Area (km^2)	AR (mm)	AF (mm)	RC=AF/AR	SSG
Kade	2126.67	1187.40	263.97	0.2223	0.1853
Assin Praso	9347.30	1236.30	164.32	0.1329	0.1158
Twifo Praso	20778.00	1425.20	157.70	0.1107	0.0983

AR: Annual catchment average rainfall, AF: Annual riverflow leaving the catchment.

Table 10. Comparison of observed (ET_O) and DBM estimate (ET_M) of evapo-transpiration losses and possible catchment leakages of the catchments in the River Pra basin for the 1978 water year (i.e. from March1, 1978 to February 28, 1979).

Catchment	AR (mm)	AF (mm)	SSG	ET_O=AR-AF (mm)	ET_M= (1-SSG)×AR (mm)
Kade	1187.4	263.97	0.1853	923.43	967.37
Assin Praso	1236.3	164.32	0.1158	1071.98	1093.14
Twifo Praso	1425.2	157.70	0.0983	1267.50	1285.10

AR: Annual catchment average rainfall, AF: Annual riverflow leaving the catchment, SSG: DBM model estimate of steady state gain.

(Brammer, 1962; Attah-Quayson, 1999). One would have, therefore, expected that the shallow and poorly drained catchment in the North would be flashier (i.e. shorter residence time) than the catchments in the forest area but this is not the case. The catchments in the North does, have a rock aquifer beneath (Bate, 1962b) increasing the time constant to about 10 days. Declining groundwater levels attributed to the numerous (3000) abstraction boreholes drilled in the North have been reported by Gyau-Boakye and Tumbulto (2000) and FAO (2005). Over time, this might lead to a longer time constant.

The observations in Malaysia and Thailand coupled with the results in the River Pra Basin and the studies from other climatic regions which are shown in Table 8, suggest that time constant may be highly influenced by the nature of the geological formation and regolith underlying a catchment. Time constant may be used to predict the type of geological formation and regolith underlying a catchment, especially in catchments located in similar climatic conditions with similar topography and soil cover.

Steady state gain (SSG)

The steady state gain (SSG) calculated by using Equation 7, demonstrates the relationship between the equilibrium input (rainfall) and output (riverflow) of the DBM TF model and indicates physical losses (i.e. SSG < 1) or gain (SSG > 1) in the system (catchment) (Young, 2005). This DRC is analogous to runoff coefficient (RC) which is a measure of how much of the gross total rainfall (for example in a water year) appears as riverflow after evaporation and transpiration losses. For instance, RC of say 0.2 of a catchment over one year or more indicates that 80% of the rainfall has been lost through evaporation and transpiration with the remaining 20% appearing as riverflow.

The SSGs obtained for the catchments in the River Pra Basin (Table 9) are Kade: 0.1853; Assin Praso: 0.1158; and Twifo Praso: 0.0983 for the 1978 water year (i.e. from 1st March, 1978 to 28th February, 1979). These estimates are comparable to those estimated in the tropics, by Vongtaboon (2004), for the 3853 km^2 P14, 521 km^2 P47 and 129 km^2 X113 catchments which are 0.2447, 0.1211 and 0.1998, respectively. These indicate, as expected, that there are high losses within all the catchments which is typical of tropical conditions due to high rates of evaporation. For example, from table 10, SSG of 0.1853 of River Birim at Kade indicates that with average annual rainfall of 1187.4 mm, during the 1978 water year, about 81.47% which is 967.37 mm is lost through evaporation and transpiration leaving only 18.53% (i.e. 220 mm), to appear as riverflow.

The annual evapo-transpiration rates in the forest zone of Ivory Coast for the Tai II and Banco II catchments are 1363mm/year and 1195mm/year respectively, and that of the Guma catchment, in Sierra Leone is 1146 mm/year, all in West Africa (Bruijnzeel, 1990). In the forest zone of East Africa Bruijnzeel (1990) reports of similar evapo-transpiration rates of 1337mm/yr for Kericho catchment in Kenya and 1381mm/yr for Mbeya catchment in Tanzania and in South East Asia, 1170mm/yr for the Ciwidey catchment in Indonesia. These values are comparable to the observed and DBM estimates for the catchments in the forest zone of Ghana. This indicates

that the estimates are reasonable and probably, there are no leakages in the catchments.
In Ghana, according to Bates (1962b) generally, only about 1 to 10 per cent of rainfall ends up as riverflow in the rivers. This observation was based on few streams which were gauged around that time. However, the DBM model estimate of 18.53, 11.58 and 9.83% of the rainfall to appear as riverflow for Kade, Assin Praso and Twifo Praso catchments, respectively, are in agreement with the observation by Bates (1962b).

Table 9 reveals that, there is no significant difference between the catchments RCs (i.e. actual water balance term) and their SSGs (i.e. model water balance term). The slight difference between them might be due to modelling error. This suggests that the SSGs from the DBM TF model could be used as a sufficient representation of catchments actual water balance. However, recently, Chappell *et al.* (2006) have introduced a procedure where the effective rainfall is normalised to give SSG the same as the RC.

CONCLUSION

The DBM transfer function rainfall and riverflow modelling approach is very robust (Young, 1998, 2001; Lees, 2000) and has been used effectively to model rainfall and riverflow behaviour of large catchments in the forest zone of southern Ghana. The approach was applied to catchments of size between 2126.67- 20778km^2 and the following conclusions can be drawn:

1. The DBM TF modelling process through SDP analysis has revealed the nature of nonlinear behaviour for the riverflow generation process in the forest zone of Ghana. Thus, exponential distribution which implies that within the forest zone, as the catchment wets up the instantaneous runoff coefficient (i.e. proportion of rainfall generating riverflow) increases up to a point and remain constant where riverflow generation becomes a linear relationship between rainfall.
2. The estimated parameters exponential parameter (β), recession parameter ($\Re$), production parameter (P), pure time delay (δ) and the associated dynamic response characteristics (DRC) of time constant (TC) and steady state gain (SSG) suggest that riverflow generation within the catchments were not flashy and that their response is dominated by single water pathway.
3. Analysis of the time constants suggests that the riverflow behaviour within the catchments is similar, with all of the catchments having high storages averaging about 8.5 days. The similar storages within the catchments have been linked to the similar geologies underlain them (Bates, 1962a; Boateng, 1966; Dickson and Benneh, 1988).
4. Comparison of the estimated SSG (i.e. the model water balance) with the RC (i.e. catchments actual water balance term) showed no significance difference between the two parameters thus, SSG could be used as a sufficient representation of the catchment water balance.
5. The analyses of the estimated TCs coupled with estimates from other climatic regions indicate that riverflow generation processes within a catchment is highly influenced by the geological formation underlying the catchment and that with a known time constant it may be possible to predict the nature of the geology underlain a catchment if the catchment is located within the same climatic conditions with similar vegetation, soil cover and topography.
6. The DBM modelling has led to development of mathematical relationships between rainfall and riverflow which could be used in simulating flows in the basin. The approach is recommended for the forecasting of riverflow in the country which would greatly improve the government's planning of water supply provision in the country.

REFERENCES

Acheampong, PK (988. Water Balance Analysis for Ghana. Geography. 125-131.

Adiku, SGK., Dayananda, PWA., Rose, CW. and Dowuona, GNN. 1997. An analysis of the within-season rainfall characteristics and simulation of the daily rainfall in two savannah zones in Ghana. Agricultural and Forest Meteorology. 86:51-62.

Ahenkorah, Y., Amatekpor, JK., Dowuona, GNN. and Yentumi, SD. 1994. Soil and water resources of Ghana: their conservation, management and constraints to their utilization for sustainable development. United Nations University/Institute of Natural Resources for Africa. pp131.

Amisigo, BA. 2005. Modelling riverflow in the Volta Basin of West Africa: A data driven framework.

Ampadu, B. 2007. Simulating Rainfall and Riverflow Dynamics in Ghana. PhD Thesis, Lancaster University, UK.

Attah-Quayson, J. 1999. Macmillan atlas for Ghana (3rd edi.). Unimax Publishers Ltd, UK.

pp65.

Bates, DA. 1962a. Geology. In: Agricultural and land use in Ghana. Ed. Willis, JB. Oxford University Press, UK. 51-61.

Bates, DA. 1962[b]. Rural water supplies. In: Agricultural and land use in Ghana. Ed. Willis, JB. Oxford University Press, UK. 62-76.

Beven, KJ. 2001. Rainfall-runoff modelling. The Primer, Wiley, Chichester, UK. pp360.

Boateng, EA. 1961. A Geography of Ghana (2nd edi.). Cambridge University Press, UK. pp212.

Boochabun, K., Tych, W., Chappell, NA., Carling, PA., Lorsirirat, K. and Pa-Obsaeng, S. 2004. Statistical Modelling of Rainfall and River Flow in Thailand. Journal of Geological Society of India. 64: 503-513.

Brammer, H. 1962. Soils In: Agriculture and land use. Ed. Wills JB. Oxford University Press, UK. 88-125.

Bruijnzeel, LA. 1990. Hydrology of moist tropical forests and effects of conversion: a state of knowledge review, UNESCO, Paris. pp224.

Chappell, NA., Tych, W., Yusop, Z., Rahim, NA. and Kasran, B. 2004[a]. Spatial significant effects of selective tropical forestry on water, nutrient and sediment flows: a modelling supported review. In: Forest, Water and People in the Humid Tropics. Eds. Bonell, M. and Brujinzeel, LA. (Eds).513-533.

Chappell, NA., Bidin, K., Sherlock, MD. and Lancaster, JW. 2004[b]. Parsimonious spatial representation of tropical soils within dynamic rainfall-runoff models. In: Forest, Water and People in the Humid Tropics. Eds. M. Bonell and Brujinzeel LA. 756-769.

Chappell, NA., McKenna, P., Bidin, K., Douglas, I. and Walsh, RPD. 1999. Parsimonious modelling of water and suspended sediment flux from nested catchments affected by selective tropical forestry. Phil. Trans. R. Soc. London. B**.** 354:1831-1846.

Chappell, NA., Sherlock, M., Macdonald, R., Najman, Y. and Bidin, K. 2007. Runoff processes in South East Asia: role of soil, regolith and rock type. In: Forest Environments in the Mekong River Basin. Eds. Sawada, H., Araki, M., Chappell, NA., LaFrankie, JV. and Shimizu, A. Springer Velag, Tokyo. 3-23.

Chappell, NA., Tych, W., Choatai, A., Bidin, K., Sinun, W. and Chiew, TH. 2006. BARUMODEL: Combined data based mechanistic models of runoff response in a managed rainforest catchment. Forest Ecology and Management. 224:58-80.

Chappell, NA., Franks, SW. and Larenus, J. 1991. Multi-scale permeability estimation for a tropical catchment. Hydrological Processes. 12:1507-1523.

Douglas, I. 1999. Hydrological investigations of forest disturbance and land cover impacts in South-East Asia: a review. Philosophical Transactions of the Royal Society. B354:1725-1738.

Dickson, KB. and Benneh, G. 1988. A New geography of Ghana, Longman, UK. pp170.

FAO. 1981. Arid zone hydrology, FAO Irrigation and drainage paper. 37. pp96.

FAO. 2005. Irrigation in Africa in figures-Aquastat survey (Country profile-Ghana). FAO Water reports, 29. pp89.

Fawcett, CP., Young, PC. and Feyen, H. 1997. Validation of Data-Based Mechanistic nonlinear rainfall-flow model. In: Proceedings of the Twelfth International Conference on Systems Engineering, ICSE 1997, Coventry University, UK, 9-11 September, Coventry. 252-257.

Giles, J. 2005. Solving Africa's Climate-data problem. Nature. 435:863.

Gyau-Boakye, P. and Tumbulto, JW. 2000. The Volta Lake and declining rainfall and streamflows in the Volta River Basin. Environment, Development and Sustainability. 2:1-10.

Jakeman, A., Littlewood, IG. and Whitehead, PG. 1990. Computation of the instantaneous unit hydrograph and identifiable component flows with application to two small upland catchments. Journal of Hydrology. 117:275-300.

Jakeman, AJ. and Hornberger, GM. 1993. How much complexity is warranted in a rainfall -runoff model?, Water Resources Research. 29:2637-2649.

Kranjac-Berisavljevic, G. 1999. Recent climatic trends in northern interior savannah zone of Ghana. International Hydrological Programme, Proceedings of the International conference on Integrated Drought Management. Lessons for Sub-Saharan Africa, 20-22 September, 1999. Pretoria, South Africa, IHP-V/ Technical Documents in Hydrology, No. 35, UNESCO. 157-165.

Lees, MJ. 2000. Data-based mechanistic modelling and forecasting of hydrological systems. Journal of Hydroinformatics. 2(1):15-34.

Lees, M., Young, PC., Ferguson, S., Beven, KJ. and Burns, J. 1994. An adaptive flood warning scheme for the River Nith at Dumfries. I: River flood hydraulics. Eds. White WR. and Watts, J. John Wiley, Chichester. 65-75.

Linsley, RK., Kohler, MA. and Paulhus, JLH. 1988. Hydrology for Engineers. McGraw-Hill, UK. pp492.

Mutreja, KN. 1986. Applied Hydrology. New Delhi, Tata McGraw-Hill.

Mwakalila, S., Campling, P., Feyen, J., Wyseure, G. and Beven, K. 2001. Application of data-based mechanistic modelling (DBM) approach for predicting runoff generation in semi-arid regions." Hydrological Processes. 15:2281-2295.

Nash, JE. and Sutcliffe, J. 1970. Riverflow forecasting through conceptual models, part I. A discussion of principles. Journal of Hydrology. 10:282-290.

Post, DA. and Jakeman, AJ. 1996. "Relationships between catchment attributes and hydrological response characteristics in small Australian mountain ash catchments." Hydrological Processes. 10: 877-892.

Price, LE., Goodwill, P., Young, PC. and Rowan, JS. 2000. A data-based mechanistic modelling (DBM) approach to understanding dynamic sediment transmission through Wyredale Park Reservoir, Lancashire, UK. Hydrological Processes. 14:63-78.

Romanowicz, RJ., Young, PC. and Beven, K. 2006. Data Assimilation and adaptive forecasting of water levels in the river Severn catchment, United Kingdom. Water Resources Research. 42:1-12.

Rowan, JS., Price, LE., Fawcett, CP. and. Young, PC. 2001. Reconstructing historic reservoir sedimentation rates using data-based mechanistic modelling. Physics and Chemistry of the Earth (B), 26(1):77-82.

Sefton, CE. M. and Howarth, SM. 1998. "Relationships between dynamic response characteristics and physical descriptors in England and Wales." Journal of Hydrology. 211:1-16.

Shaw, EM. 1994. Hydrology in Practice, Chapman and Hall, UK. pp569.

Taylor, CJ., Pedregal, DJ., Young, PC. and Tych, W. 2007. "Environmental time series analysis and forecasting with the Captain toolbox." Environment Modelling and Software. 22:797-814.

Tsang, FC. 1995. Advances in Flood Forecasting using Radar rainfalls and Time series Analysis. PhD Thesis, Lancaster University, UK.

Van de Giesen, N., Kunstmann, H., Jung, G., Liebe, J., Andreini, M. and Vlek, PL. G. 2002. The GLOWA Volta Project: Integrated assessment of feedback mechanismsbetween climate, landuse and hydrology. In: Climatic Change. Implications for the Hydrological Cycle and Water Management. Ed. Beniston M. Advances in Global Research. 10:151-170.

Van Loon, EE. and Keesman, KJ. 2000. Identifying scale-dependent models: The case of overland flow at the hillslope scale. Water Resources Research. 36(1):243-254.

Vigiak, O., R. J. Romanowicz, EE. van-Loon, G. Sterk and KJ. Beven. 2006. A disaggregating approach to describe overland flow occurrence within a catchment. Journal of Hydrology. 323:22-40.

Vongtanaboon, S., Lim, HS. and Richards, K. 2008. Databased-Mechanistic Rainfall-Runoff modelling for a large Monsoon-Dominated catchment in Thailand.

Vongtanaboon, S. and Chappell, NA. 2004. DBM rainfall-runoff modelling of large rainforest catchments in Thailand. In: Forests and Water in Warm Humid Asia. Eds. Sidle, RC., Tani, M., Nik, AR. and Tadese, TA. Disaster Prevention Research Institute, Uji, Japan. 252-255.

Vongtanaboon, S. 2004. Parsimonious modelling of the rainfall-runoff behaviour of large catchments in Thailand. PhD Thesis, Lancaster University, UK.

Weston, M. and Steven, D. 2005. Climate Proofing Africa: Climate and Africa's Development Challenge, DFID/DEFRA, UK. pp26.

Young, P. 2006. Updating algorithms in flood forecasting. Flood Risk Management Research Consortium (FRMRC) Research Report. UR5. pp80.

Young, PC. 198). Recursive estimation and time series analysis. Berlin, Springer-Verlag. 69-98.

Young, PC. 1985. The instrumental variable method: a practical approach to identification and system parameter estimation. In: Identification and system parameter estimation. Eds. Barker, HA. and Young, PC. Pergamon Oxford. 1-16.

Young, PC. 1991. Simplified refined instrumental variable (SRIV) estimation and true digital control. In: Proceedings of the first European Control Conference, Grenoble. pp1295-1306: pp1295-1306.

Young, PC. 1992. Parallel Processes in Hydrology and Water Quality: A Unified Time-Series Approach. Journal of Institution of Water and Environmental Management. 6:598-612.

Young, PC. 1993. Time variable and state dependant modelling of non-stationary and nonlinear time series: In: Developments in time series analysis. Ed. Subba, RT. Chapman and Hall, London. 374- 413.

Young, PC. 1998. Data-based mechanistic modelling of environmental, ecological, economic and engineering systems. Environmental Modelling and Software. 13:105-122.

Young, PC. 2001. Data-based mechanistic modelling and validation of rainfall-flowprocess. In Model validation: Perspectives in hydrological science. Eds. Anderson, MG. and Bates, PD. Chichester, J. Wiley. 117-161.

Young, PC. 2002. Advances in real-time flood forecasting. Philosophical Transactions of the Royal Society London A. 360:1433-1450.

Young, PC. 2003. Top-down and data-based mechanistic modelling of rainfall- flow dynamics at the catchment scale. Hydrological Processes. 17:2195-2217.

Young, PC. 2005. "Rainfall - runoff modelling: Transfer function models." In: Encyclopaedia of hydrological sciences. Anderson, MG. John Wily and Sons, Ltd. 1985-2000.

Young, PC. and Beven, KJ. 1994. "Data-based mechanistic modelling and the rainfall flow non-linearity." Environmetrics. 5:335-363.

Young, PC. and Minchin, P. 1991. Environmetric time-series analysis: modelling natural systems from experimental time-series data. International Journal of Biological Macromolecules. 13:190-201.

Young, PC., Pedregal, DJ. and Tych, W. 1999. Dynamic harmonic regression. Journal of forecasting. 18:369-394.

Young, PC., Jakeman, AJ. and Post, DA. 1997. Recent advances in the data-based modelling and analysis of hydrological systems. Water Science and Technology. 36(5):99-116.

Young, PC. and Lees, MJ. 1993. The active mixing volume (AMV): A new concept in modelling environmental systems. In: Statistics for the environment. Eds. Barnett, V. and Turkman, KF. John Wiley, Chichester. 3-44.

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