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FORECASTING NIGERIAN INFLATION RATES BY A SEASONAL ARIMA MODEL

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ABSTRACT

Time series analysis of Nigerian Inflation Rates (INFL) Data is done. It is observed that it is seasonal. A multiplicative seasonal autoregressive integrated moving average (ARIMA) model, $(0, 1, 1)x(0, 1, 1)_{12}$, is fitted to the series. The model is shown to be adequate and forecasts are obtained on the basis of the model.

Keywords: Inflation rate, seasonal time series, ARIMA model.

INTRODUCTION

A time series is defined as a set of data collected sequentially in time. It has the property that neighbouring values are correlated. This tendency is called *autocorrelation.* A time series is said to be stationary if it has a constant mean and variance. Moreover the autocorrelation is a function of the lag separating the correlated values and called *the autocorrelation function* (ACF).

A stationary time series $\{X_t\}$ is said to follow an *autoregressive moving average model of orders p and q* (designated ARMA(p,q)) if it satisfies the following difference equation

$$X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + ... + \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + ... + \beta_q \varepsilon_{t-q} \quad (1)$$

or

$$\mathrm{A}(B)X_t = \mathrm{B}(B)\varepsilon_t \quad (2)$$

where $\{\varepsilon_t\}$ is a sequence of random variables with zero mean and constant variance, called *a white noise process*, and the α_i's and β_j's constants; $\mathrm{A}(B) = 1 + \alpha_1 B + \alpha_2 B^2 + ... + \alpha_p B^p$ and $\mathrm{B}(B) = 1 + \beta_1 B + \beta_2 B^2 + ... + \beta_q B^q$ and B is the backward shift operator defined by $B^k X_t = X_{t-k}$.

If p=0, model (1) becomes *a moving average model of order q* (designated MA(q)). If, however, q=0 it becomes *an autoregressive process of order p* (designated AR(p)). An AR(p) model of order p may be defined as a model whereby a current value of the time series X_t depends on the immediate past p values: $X_{t-1}, X_{t-2}, ..., X_{t-p}$. On the other hand an MA(q) model of order q is such that the current value X_t is a linear combination of immediate past values of the white noise process: $\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-q}$. Apart from stationarity, invertibility is another important requirement for a time series. It refers to the property whereby the covariance structure of the series is unique (Priestley, 1981). Moreover it allows for meaningful association of current events with the past history of the series (Box and Jenkins, 1976).

An AR(p) model may be more specifically written as $X_t + \alpha_{p1}X_{t-1} + \alpha_{p2}X_{t-2} + ... + \alpha_{pp}X_{t-p} = \varepsilon_t$ Then the sequence of the last coefficients$\{\alpha_{ii}\}$ is called *the partial autocorrelation function*(PACF) of $\{X_t\}$. The ACF of an MA(q) model cuts off after lag q whereas that of an AR(p) model is a combination of sinusoidals dying off slowly. On the other hand the PACF of an MA(q) model dies off slowly whereas that of an AR(p) model cuts off after lag p. AR and MA models are known to have some duality properties. These include:

1. A finite order AR model is equivalent to an infinite order MA model.
2. A finite order MA model is equivalent to an infinite order AR model.
3. The ACF of an AR model exhibits the same behaviour as the PACF of an MA model.
4. The PACF of an AR model exhibits the same behaviour as the ACF of an MA model.
5. An AR model is always invertible but is stationary if $\mathrm{A}(B) = 0$ has zeros outside the unit circle.
6. An MA model is always stationary but is invertible if $\mathrm{B}(B) = 0$ has zeros outside the unit circle.

Parametric parsimony consideration in model building entails preference for the mixed ARMA fit to either the pure AR or the pure MA fit. Stationarity and invertibility conditions for model (1) or (2) are that the equations $\mathrm{A}(B) = 0$ and $\mathrm{B}(B) = 0$ should have roots outside the unit circle respectively.

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Often, in practice, a time series is non-stationary. Box and Jenkins (1976) proposed that differencing of an appropriate order could render a non-stationary series $\{X_t\}$ stationary.

Let degree of differencing necessary for stationarity be d. Such a series $\{X_t\}$ may be modelled as

$$(1 + \sum_{i=1}^{p} \alpha_i B^i)\nabla^d X_t = B(B)\varepsilon_t \qquad (3)$$

where $\nabla = 1 - B$ and in which case $A(B) = (1 + \sum_{i=1}^{p} \alpha_i B^i)\nabla^d = 0$ shall have unit roots d times. Then differencing to degree d renders the series stationary. The model (3) is said to be an autoregressive integrated moving average model of orders p, d and q and designated ARIMA(p, d, q).

Seasonal ARIMA Models:
A time series is said to be seasonal of order d if there exists a tendency for the series to exhibit periodic behaviour after every time interval d. Traditional time series methods involve the identification , unscrambling and estimation of the traditional components: secular trend, seasonal component, cyclical component and the irregular movement. For forecasting purpose, they are reintegrated. Such techniques could be quite misleading.

The time series $\{X_t\}$ is said to follow a multiplicative (p, d, q)x(P, D, Q)$_s$ *seasonal ARIMA model* if

$$A(B)\Phi(B^s)\nabla^d\nabla_s^D X_t = B(B)\Theta(B^s)\varepsilon_t \qquad (4)$$

where Φ and Θ are polynomials of order P and Q respectively. That is,

$$\Phi(B^s) = 1 + \phi_1 B^s + ... + \phi_P B^{sP}, \qquad (5)$$

$$\Theta(B^s) = 1 + \theta_1 B^s + ... + \theta_Q B^{sQ}, \qquad (6)$$

where the ϕ_i and θ_j are constants such that the zeros of the equations (5) and (6) are all outside the unit circle for stationarity and invertibility respectively. Equation (5) represents the autoregressive operator whereas (6) represents the moving average operator.

Existence of a seasonal nature is often evident from the time plot. Moreover for a seasonal series the ACF or correlogram exhibits a spike at the seasonal lag. Box and Jenkins (1976) and Madsen (2008) are a few authors that have written extensively on such models. A knowledge of the theoretical properties of the models provides basis for their identification and estimation. The purpose of this paper is to fit a seasonal ARIMA model to Nigerian Inflation Rate (INFL) series.

MATERIALS AND METHODS

The data for this work are inflation rates – All items (Year on Change)- from 2003 to 2011 obtainable from the Data and Statistics publication of Central Bank of Nigeria retrievable from the website http://www.cenbank.org/.

Determination of the orders d, D, p, P, q and Q:
Seasonal differencing is necessary to remove the seasonal trend. If there is secular trend non-seasonal differencing will be necessary. To avoid undue model complexity it has been advised that orders of differencing d and D should add up to at most 2 (i.e. d + D < 3). If the ACF of the differenced series has a positive spike at the seasonal lag then a seasonal AR component is suggestive; if it has a negative spike then a seasonal MA term is suggestive.

As already mentioned above, an AR(p) model has a PACF that truncates at lag p and an MA(q)) has an ACF that truncates at lag q. In practice $\pm 2/\sqrt{n}$ where n is the sample size are the non-significance limits for both functions.

Model Estimation
The involvement of the white noise terms in an ARIMA model entails a nonlinear iterative process in the estimation of the parameters. An optimization criterion like least error of sum of squares, maximum likelihood or maximum entropy is used. An initial estimate is usually used. Each iteration is expected to be an improvement of the last one until the estimate converges to an optimal one. However, for pure AR and pure MA models linear optimization techniques exist [See for example Box and Jenkins (1976), Oyetunji (1985)].

There are attempts to adopt linear methods to estimate ARMA models [See for example, Etuk (1987, 1996)]. We shall use Eviews software which employs the least squares approach involving nonlinear iterative techniques.

Diagnostic Checking
The model that is fitted to the data should be tested for goodness-of-fit. We shall do some analysis of the residuals of the model. If the model is correct, the residuals would be uncorrelated and would follow a normal distribution with mean zero and constant variance.

The autocorrelations of the residuals should not be significantly different from zero.

RESULTS AND DISCUSSION

The time plot of the original series INFL in figure1 shows no clear secular trend nor seasonality. Seasonal (i.e. 12-month) differencing of the series produces a series SDINFL also with no trend nor clear seasonality (see Fig. 2). Non-seasonal differencing yields a series DSDINFL with no trend and no clear seasonality (see Fig. 3). Its ACF in figure 4 has a negative spike at lag 12 revealing a seasonality of lag 12 and a seasonal MA component to

Table 1. Model Estimation.

Dependent Variable: DSDINFL
Method: Least Squares
Date: 01/25/12 Time: 20:04
Sample(adjusted): 2004:02 2011:12
Included observations: 95 after adjusting endpoints
Convergence achieved after 11 iterations
Backcast: 2003:01 2004:01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	0.184619	0.077003	2.397550	0.0185
MA(12)	-0.641404	0.057709	-11.11443	0.0000
MA(13)	0.077588	0.088573	0.875975	0.3833

R-squared	0.498427	Mean dependent var	-0.140000
Adjusted R-squared	0.487524	S.D. dependent var	3.651272
S.E. of regression	2.613852	Akaike info criterion	4.790597
Sum squared resid	628.5647	Schwarz criterion	4.871246
Log likelihood	-224.5534	F-statistic	45.71156
Durbin-Watson stat	1.752935	Prob(F-statistic)	0.000000

Inverted MA Roots	.94	.81+.48i	.81 -.48i	.46+.83i
	.46 -.83i	.12	-.03 -.96i	-.03+.96i
	-.51 -.83i	-.51+.83i	-.86+.48i	-.86 -.48i
	-.99			

Table 2. Forecasts.

Time	Residuals	DSDINFL	SDINFL	INFL
December 2010	-1.87861	-2.5	-2.1	11.8
January 2011	0.48996	-0.2	-2.3	12.1
February 2011	-1.73901	-2.2	-4.5	11.1
March 2011	2.18770	2.5	-2.0	12.8
April 2011	-1.39480	-1.7	-3.7	11.3
May 2011	1.82048	3.2	-0.5	12.4
June 2011	-2.49961	-3.4	-3.9	10.2
July 2011	-0.00673	0.3	-3.6	9.4
August 2011	-0.19715	-0.8	-4.4	9.3
September 2011	1.44585	1.1	-3.3	10.3
October 2011	-0.91003	0.4	-2.9	10.5
November 2011	0.38172	0.6	-2.3	10.5
December 2011	-0.41409	0.8	-1.5	10.3
January 2012		-0.536	-2.04	10.1
February 2012		1.153	-0.89	10.2
March 2012		-1.538	-2.43	10.4
April 2012		1.064	-1.37	9.9
May 2012		-1.276	-2.64	9.8
June 2012		1.745	-0.90	9.3
July 2012 August 2012		-0.189	-1.09	8.3
September 2012		0.126	-0.96	8.3
October 2012 November 2012		-0.943	-1.90	8.4
December 2012		0.696	-1.20	9.3
		-0.315	-1.52	9.0
		0.295	-1.22	9.1

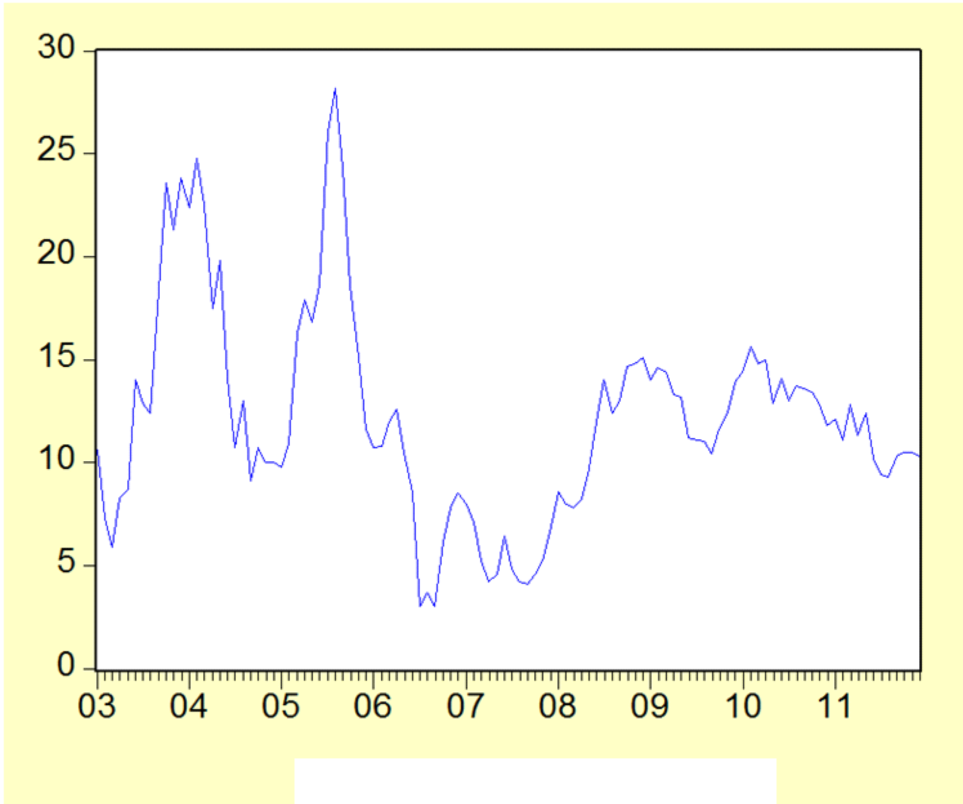


Fig. 1. INFL.

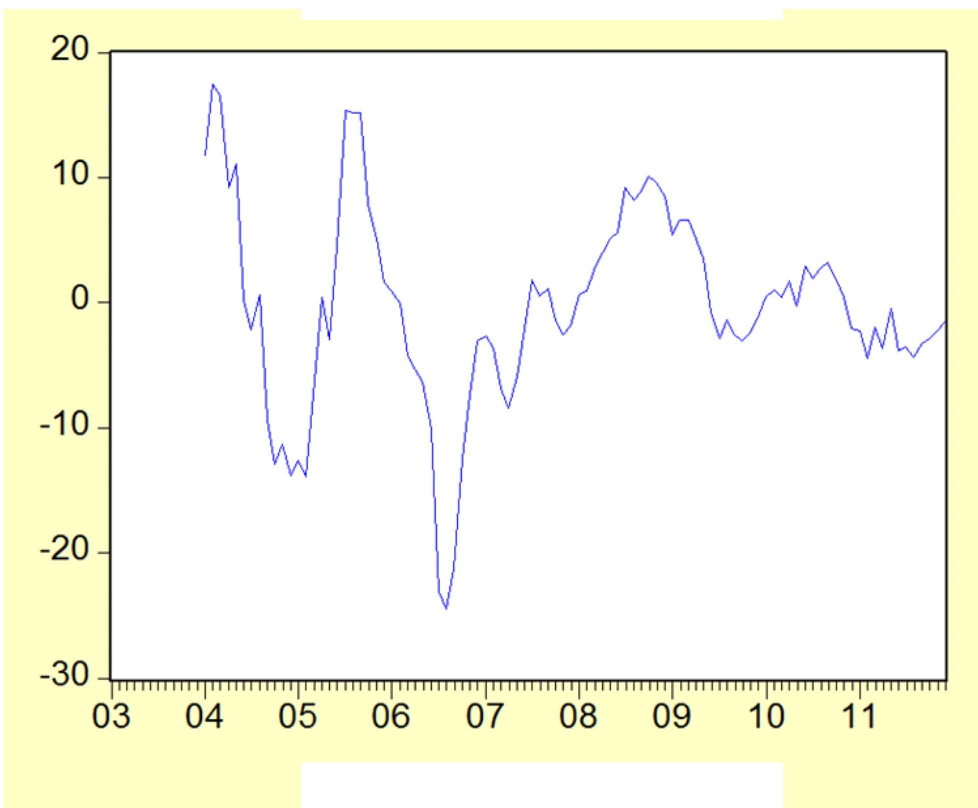


Fig. 2. SDINFL.

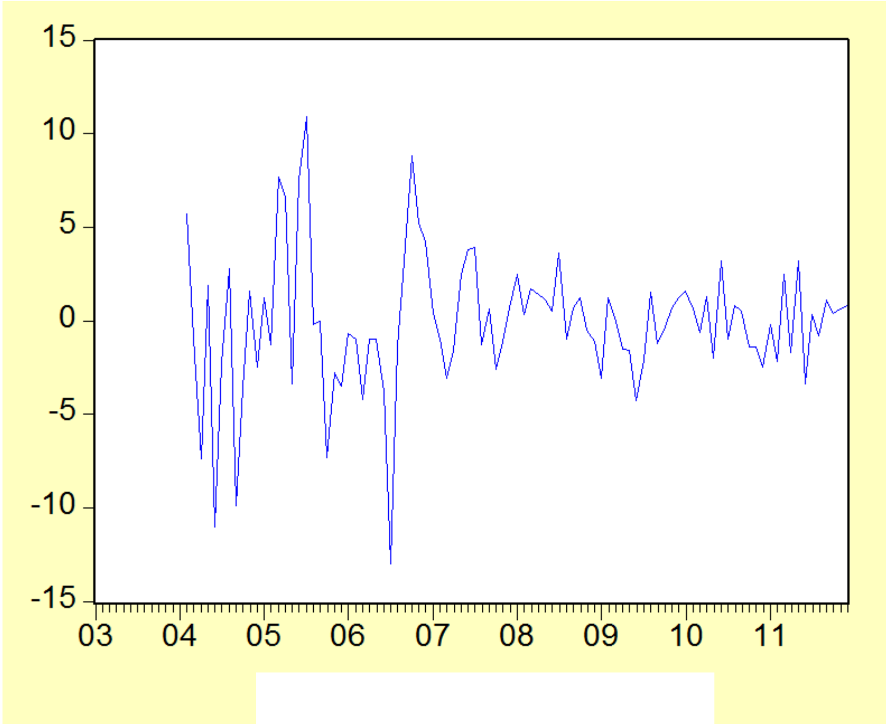


Fig. 3. DSDINFL.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.212	0.212	4.4230	0.035
		2	0.081	0.037	5.0671	0.079
		3	0.079	0.057	5.6852	0.128
		4	-0.132	-0.171	7.4598	0.113
		5	-0.050	0.005	7.7181	0.172
		6	-0.080	-0.064	8.3772	0.212
		7	-0.088	-0.034	9.1877	0.239
		8	0.068	0.088	9.6801	0.288
		9	-0.139	-0.176	11.759	0.227
		10	-0.185	-0.152	15.464	0.116
		11	-0.181	-0.155	19.064	0.060
		12	-0.461	-0.406	42.662	0.000
		13	-0.089	0.046	43.549	0.000
		14	0.012	0.008	43.566	0.000
		15	0.007	-0.007	43.571	0.000
		16	0.137	-0.039	45.757	0.000
		17	0.178	0.120	49.496	0.000
		18	0.157	0.056	52.464	0.000
		19	0.100	-0.015	53.677	0.000
		20	0.007	0.020	53.684	0.000
		21	0.091	-0.021	54.707	0.000
		22	0.088	-0.042	55.683	0.000
		23	0.038	-0.045	55.866	0.000
		24	-0.015	-0.242	55.894	0.000
		25	0.015	0.063	55.923	0.000
		26	-0.046	-0.011	56.203	0.001
		27	-0.012	0.106	56.224	0.001
		28	-0.125	-0.077	58.377	0.001
		29	-0.113	0.085	60.145	0.001
		30	-0.052	0.051	60.522	0.001
		31	-0.086	-0.059	61.587	0.001
		32	-0.041	-0.006	61.830	0.001
		33	0.011	0.009	61.847	0.002
		34	0.072	0.065	62.626	0.002
		35	0.052	-0.064	63.042	0.003
		36	0.062	-0.086	63.635	0.003

Fig. 4. Correlogram of DSDINFL.

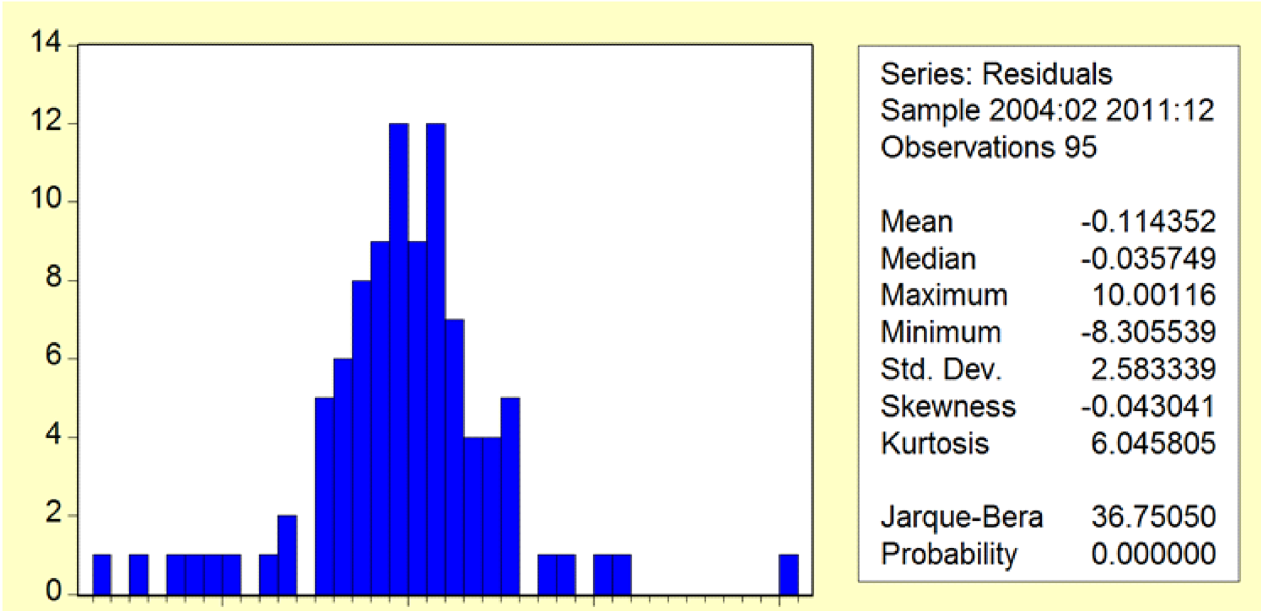
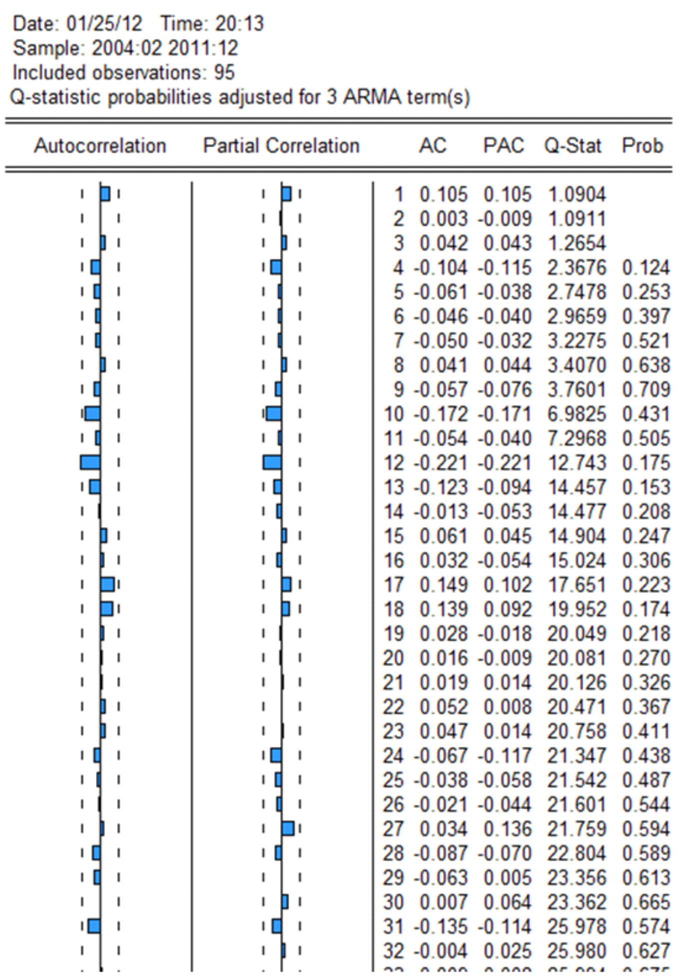


Fig. 5. Histogram of Residuals.



Date: 01/25/12 Time: 20:13
Sample: 2004:02 2011:12
Included observations: 95
Q-statistic probabilities adjusted for 3 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.105	0.105	1.0904	
		2	0.003	-0.009	1.0911	
		3	0.042	0.043	1.2654	
		4	-0.104	-0.115	2.3676	0.124
		5	-0.061	-0.038	2.7478	0.253
		6	-0.046	-0.040	2.9659	0.397
		7	-0.050	-0.032	3.2275	0.521
		8	0.041	0.044	3.4070	0.638
		9	-0.057	-0.076	3.7601	0.709
		10	-0.172	-0.171	6.9825	0.431
		11	-0.054	-0.040	7.2968	0.505
		12	-0.221	-0.221	12.743	0.175
		13	-0.123	-0.094	14.457	0.153
		14	-0.013	-0.053	14.477	0.208
		15	0.061	0.045	14.904	0.247
		16	0.032	-0.054	15.024	0.306
		17	0.149	0.102	17.651	0.223
		18	0.139	0.092	19.952	0.174
		19	0.028	-0.018	20.049	0.218
		20	0.016	-0.009	20.081	0.270
		21	0.019	0.014	20.126	0.326
		22	0.052	0.008	20.471	0.367
		23	0.047	0.014	20.758	0.411
		24	-0.067	-0.117	21.347	0.438
		25	-0.038	-0.058	21.542	0.487
		26	-0.021	-0.044	21.601	0.544
		27	0.034	0.136	21.759	0.594
		28	-0.087	-0.070	22.804	0.589
		29	-0.063	0.005	23.356	0.613
		30	0.007	0.064	23.362	0.665
		31	-0.135	-0.114	25.978	0.574
		32	-0.004	0.025	25.980	0.627

Fig. 6. Correlogram of Residuals.

the model. The PACF shows no spike in the early lags (1, 2, 3, ...) suggesting a non-seasonal MA component. We therefore propose a (0, 1, 1)x(0, 1, 1)$_{12}$ seasonal model.

That means $DSDINFL_t = \beta_1\varepsilon_{t-1} + \beta_{12}\varepsilon_{t-12} + \beta_{13}\varepsilon_{t-13} + \varepsilon_t$ (7)

The estimation of the model is summarized in table 1. The fitted model is given by $DSDINFL_t - 0.184619\varepsilon_{t-1} + 0.641404\varepsilon_{t-12} - 0.077588\varepsilon_{t-13} = \varepsilon_t$ (8)
(±0.077003) (±0.057709) (±0.088573)

The estimation involved 11 iterations. Clearly only β_{13} is not significantly different from zero, being lesser than twice its standard error. The histogram of the residuals in figure 5 shows that the residuals are normally distributed with zero mean indicating model adequacy.

Moreover the correlogram of the residuals in figure 6 depicts the adequacy of the model. Virtually all the residual autocorrelations are not significantly different from zero.

FORECASTING:
For the model (7) at time t+k we have
$X_{t+k} = \beta_1\varepsilon_{t+k-1} + \beta_{12}\varepsilon_{t+k-12} + \beta_{13}\varepsilon_{t+k-13}$

Obtaining conditional expectations given the series up to time t, we have

$$\hat{X}_t(1) = \beta_1\varepsilon_t + \beta_{12}\varepsilon_{t-11} + \beta_{13}\varepsilon_{t-12} + \varepsilon_t$$

$$\hat{X}_t(k) = \beta_{12}\varepsilon_{t+k-12} + \beta_{13}\varepsilon_{t+k-13}, \ k \geq 2, 3, ..., 12$$

where $\hat{X}_t(k)$ is the k-step ahead forecast from time t.

CONCLUSION

The INFL series has been shown to follow a (0, 1, 1)x(0, 1, 1)$_{12}$ model. This model has been shown to be adequate. On the basis of the model 2012 forecasts have been obtained.

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