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A SIMPLE METHOD FOR IMPROVING THE VIBRATION AND ACOUSTIC CHARACTERISTICS OF THIN-WALLED BEAMS

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ABSTRACT

Avoiding resonance and reducing the sound radiation from vibrating beams are crucial objectives in structural design. In this paper, a simple approach for optimizing the natural frequencies and minimizing the radiated acoustic power from vibrating thin-walled beams is presented. The method is basically based on bending the beams at specific key points. The out of plane coordinates of these key points are chosen as design variables. Bending the beam alters its stiffness and its mode shapes and as a result changes its natural frequencies and the radiated sound power. The design method couples the finite element method for modal and harmonic analysis, the Lumped Parameter Model (LPM) for sound power calculation, and Genetic Algorithm (GA) for the optimization process. Several examples using bending technique are provided. The optimization results show the efficiency of bending technique in modifying the natural frequencies and reducing the sound radiation from vibrating clamped beams.

Keywords: Thin-walled beams, natural frequency, sound radiation, genetic algorithm.

INTRODUCTION

Beams are widely used in many mechanical and civil engineering applications. They are frequently subjected to dynamic excitations resulted from unbalanced rotating, oscillating equipment, etc. Thus, improving the vibroacoustic performance of beams is an important requirement in today's structural design. The literature contains many proposals for altering the vibroacoustic characteristics of structures. These proposed methods are known as Structural Dynamic Modification methods (SDM). One objective of these methods is to shift the structure natural frequencies away from the frequency of the excitation force to avoid the resonance phenomenon. Another objective is to reduce the sound power of vibrating structures.

For many years, a number of studies have examined shifting the natural frequencies of beams. For example, by adding internal point supports, Szelag and Mroz (1979) maximized the natural frequencies of beam by changing the support positions and stiffnesses. Wu and Lin (1990) studied the effect of adding concentrated masses to a cantilever beam. They found that the fundamental frequency of a cantilever beam which is carrying a single mass increased when the mass is moved from the free end of the fixed end. Wang and Cheng (2005) used structural patches to shift the natural frequencies of a beam to the designated values. Other authors studied the problem of beams carrying elastically mounted masses, carrying springs and/ or dampers (Wu and Chou, 1998; Zhou and Ji, 2006). Changing the beam geometry is used also for altering the natural frequencies of beams. For example, Ece *et al*. (2007) studied the natural frequencies of a tapered beam in which its width was varied exponentially along its length. They found that this design increased the natural frequencies of clamped beams. Karihaloo and Niordson (1973) maximized the fundamental frequency of a cantilever beam by tapering the cross section and keeping the mass constant. Gupta and Murthy (1978) investigated the optimal design of uniform non-homogeneous beams. They varied the modulus of elasticity distribution through the beams, assuming a constant density, to maximize its fundamental frequency. Recently, Alshabatat and Naghshineh (2012) studied optimizing the natural frequencies of different kinds of beams by forming a series of cylindrical dimples on their surfaces.

In addition to shifting the natural frequencies, the passive SDM methods are used to reduce the radiated sound power from vibrating beams. For example, Naghshineh and Koopmann (1992) used material tailoring to minimize the sound radiation at specific frequency. They enforced the structure to vibrate as a weak radiator at this frequency. In their approach, the radiation problem was decoupled from the structural vibration problem. They found the optimal surface velocity distributions which produce the minimum radiation condition (weak radiator), and then they enforced the structure to vibrate similar to the optimal surface velocity distributions by tailoring the modulus of elasticity and density of the structure. Marburg *et al*. (2006) investigated the minimization of sound radiation from finite beams over a frequency range.

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They optimized the distribution of density and modulus of elasticity to minimize the sound power of vibrating beams over a frequency range containing about eight natural frequencies. Koopmann and Fahnline (1997) used point masses to minimize the radiated sound power of a baffled beam at its second symmetric natural frequency. Later in 2011, Cheng *et al*. minimized the sound radiation of a vibrating beam by patterning the beam with a series of cylindrical dimples such that one or more of the vibration modes have the same shape as the corresponding weak modes.

A limited number of simple cases have analytical equations to calculate the radiated sound power. Different approximate methods are used to estimate the sound power. One such method is the volume velocity (VV) approach which is used to approximate the sound power at low frequencies (Fritze *et al*., 2009). The Equivalent Radiated Power (ERP) which assumes that the radiation efficiency is equal to one, therefore, this method is suitable for high frequencies (Fritze *et al*., 2009). The Lumped Parameter Model (LPM) is based on the approximation of the Rayleigh integral formula (Fahnline and Koopmann, 1996). The computation of the power based on ERP and VV is faster than the sound power computation based on LPM. However, the LPM gives the most accurate results among the three methods (Fritze *et al*., 2009). To this reason, the LPM has been used in this study.

This paper aims to introduce a simple method for optimizing the natural frequencies and for minimizing the radiated sound power from vibrating beams. The method based on bending the beam at specific points. Performing the beam structure by bending alters its stiffness significantly with a negligible increase in its mass. The presented design strategy is as follows: First, the natural frequencies and the vibration response of the beam structure are calculated by using the finite element method. In particular, ANSYS parametric design language is used in modeling and finite element analysis. Then, the radiated sound power from the vibrating beam is calculated based on LPM. Finally, an optimizer based on the method of genetic algorithm (GA) is used to optimally design the beam.

THEORETICAL BACKGROUND

Structural vibration

To calculate the sound power of a vibrating beam, we need to know the velocity distribution throughout the beam. The finite element method is used to predict and to analyze the vibrating motion of the beams. The dynamic equations of motion for the forced harmonic response can be given by the following linear system of equations

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\}e^{j\omega t}, \tag{1}$$

where $[M]$, $[C]$, and $[K]$ represent the mass, damping, and stiffness matrices of the finite element model, respectively, $\{x\}$ and $\{f\}$ represent the nodal displacement vector and nodal excitation vector, ω represents the excitation frequency, and $j = \sqrt{-1}$. The nodal displacement vector can be calculated by the expansion of structural response in terms of eigenvectors. The structural response as a function of eigenvectors is given by (Ewins, 2000)

$$\{x\} = \sum_{i=1}^{N} \frac{\{\emptyset_i\}\{\emptyset_i\}^T\{f\}}{\omega_i^2 - \omega^2 + j\eta_i\omega_i^2}, \tag{2}$$

where $\{\emptyset_i\}$ is the mass normalized eigenvector corresponding to the i^{th} mode, and η_i is the damping loss factor corresponding to the i^{th} mode.

Acoustic analysis

Consider a beam located within an infinite rigid baffle as shown in figure 1. The sound pressure at any observation point $p(\vec{r})$ can be calculated by using the Rayleigh integral (Junger and Feit, 1993)

$$p(\vec{r}) = \frac{j\omega\rho}{2\pi} \int_S \frac{v_n(\vec{r}_s)\, e^{-jkR}}{R} dS, \tag{3}$$

where k is the acoustic wave number ($k = \omega/c$), c is the speed of sound in ambient fluid, ω is the circular frequency of the excitation force, ρ is the density of the surrounding fluid, $\vec{r}$ is the position vector of the observation point, $\vec{r}_s$ is the position vector of the elemental surface dS, $v_n(\vec{r}_s)$ is the normal velocity of surface element dS, and $R = |\vec{r} - \vec{r}_s|$. The acoustic intensity at any point $\vec{r}$ is given by (Junger and Feit, 1993)

$$I(\vec{r}) = \frac{1}{2} Re\{p(\vec{r})v_n^*(\vec{r})\}, \tag{4}$$

where $Re\{\ \}$ is the real part of the number inside the bracket, and the ($*$) represents the complex conjugate. The radiated sound power can be calculated by integrating the intensity over a surface surrounding the radiating beam or on the surface of the beam as

$$W = \frac{1}{2} \int_S Re\{pv_n^*\} dS. \tag{5}$$

In discretized form, Eq. 5 can be written as

$$W = \frac{1}{2} Re\left\{\sum_{i=1}^{N_e} s_i p_i v_{n,i}^*\right\}, \tag{6}$$

where N_e is the number of elements, s_i, p_i, and $v_{n,i}^*$ are the area, the average pressure, and the conjugate average normal velocity at element i, respectively. The sound power can be approximated using the Lumped Parameter Model which is presented by Fahnline and Koopmann

(1996). This method is based on dividing the radiated surface into elements and characterizing the amplitude of the radiation from each element by its volume velocity. The LPM for the radiated power is given by

$$W = \frac{1}{2} \sum_{\mu=1}^{N_e} \sum_{\nu=1}^{N_e} \mathcal{R}_{\mu\nu} u_\mu^* u_\nu, \quad (7)$$

where $\mathcal{R}_{\mu\nu}$ is the acoustic resistance of element μ to element ν, u_μ and u_ν are the volume velocities of elements μ and ν, respectively. The acoustic resistance is given as

$$\mathcal{R}_{\mu\nu} = \frac{k\rho c}{2\pi} \frac{\sin(k r_{\mu\nu})}{r_{\mu\nu}}, \quad (8)$$

where $r_{\mu\nu}$ is the distance between element μ and element ν centers.

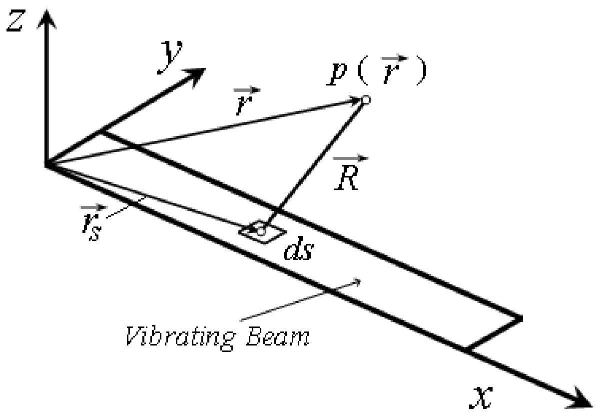

Fig. 1. Coordinate System of a Beam radiator.

MODELING OF A BENT BEAM

The goal of beam bending is to alter its local stiffnesses with negligible increase in its mass. The bent beam is modeled as shown in figure 2. The beam is bent along nine key points; where the design variables are the heights of these key points (the z-coordinates). The bent beam is modeled by ANSYS Parametric Design Language (APDL). In this scripting language, the model can be built in terms of parameters, so the locations of the key points can be modified systematically in order to enable the optimization process to take place. A beam element, with two nodes and three degrees of freedom at each node, is used for meshing the solid model. The degrees of freedom of each node are two translations in the *x*- and z-directions and one rotation about the y-axis. The use of the finite element technique allows us to extract the mode shapes, to calculate the vibration response, and to calculate the sound radiation from such vibrating beams.

The objective of the beam bending design is to find the optimal values of the bending parameters to optimize the fundamental frequency or combination of natural frequencies and to minimize the sound radiation from a vibrating beam at a specific frequency or a broad frequency band.

OPTIMIZING THE NATURAL FREQUENCIES OF BEAMS BY BENDING TECHNIQUE

In this section, two examples are presented to show the efficiency of bending technique to shift the natural frequencies of clamped-clamped beams. In the first example, the fundamental frequency of a clamped-clamped beam is maximized by bending the beam at nine points. In the second example, the bending technique is used to maximize the gap between two adjacent natural frequencies. In both examples, the beam under consideration is a clamped-clamped beam with length L = 0.3 m, width b = 0.025 m, thickness h = 0.00116 m, modulus of elasticity E=190 GPa, and density ρ = 7600 kg/m^3. The first five natural frequencies of this beam before bending are 66.2, 182.6, 358.0, 591.7, and 883.8 Hz, respectively.

In the first example, we seek for the design of a clamped beam that yields the maximum fundamental frequency. The GA is used to find the heights of the key points which give the maximum fundamental frequency. The population size in each generation is assumed 100. The stopping criterion is chosen as the maximum number of generations is 150 generations or the objective function tolerance is less than 10^{-4}. In general, our optimization problem can be written as

Maximize f_1,

Subject to $z_{lb} \leq z_i \leq z_{ub}$, i= 1, 2,..., 9, (9)

where lb, and ub are the lower and upper bounds of key points heights, respectively. The feasible heights of bending points vary between -5 mm and 5 mm.

The optimal clamped bent beam which maximizes the fundamental frequency is shown in figure 3. The optimal design variables are summarized in table 1. By creating the optimal design, the fundamental frequency can be increased from 66.2 Hz to 182.5 Hz, which is 175.7% greater than the fundamental frequency of the original beam. As shown in Fig. 3, the optimal beam design which maximizes the first natural frequency is similar to a half wave. In other words, the optimal design is similar to the mode shape which corresponds to the first natural frequency. This result agrees with the results of Kelly *et al*. (1991) which explains the possibility of avoiding an unwanted vibration mode of a beam by preforming the beam into the shape of unwanted mode of vibration.

When the excitation frequencies are limited within a range of upper and lower bounds, the suitable design of the beam may be obtained by maximizing the gap

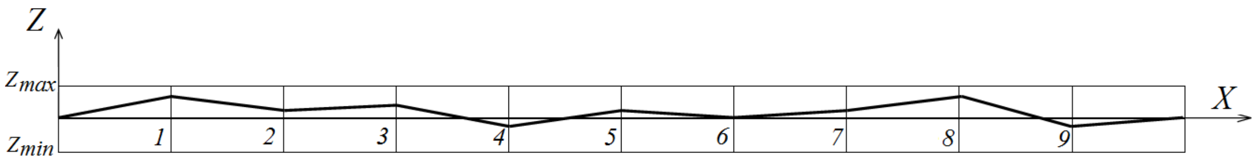

Fig. 2. Bent beam model with nine key points.

Fig. 3. The Optimum design of a bent beam which maximizes the fundamental frequency (exaggerated vertical scale).

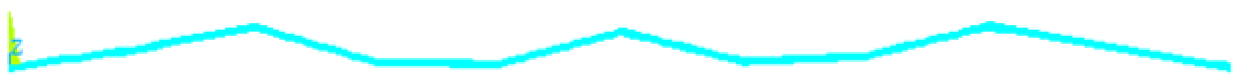

Fig. 4. The Optimum design of a bent beam which maximizes the gap between the fourth and fifth natural frequencies (exaggerated vertical scale).

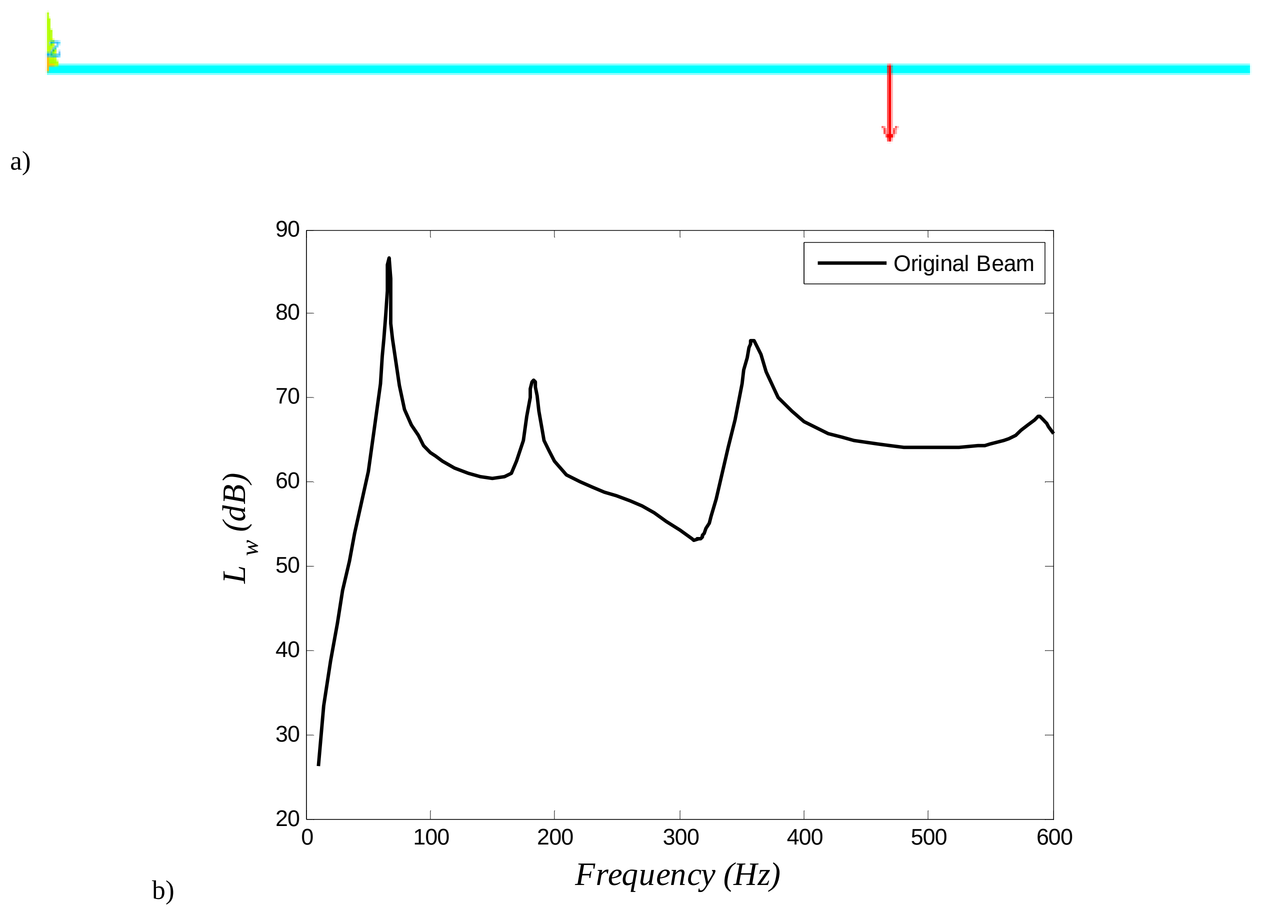

Fig. 5. (a) Beam excited by a point force, and (b) Sound power level spectrum of the beam.

between these adjacent frequencies. In the second example, the distance between the fourth and fifth natural frequencies of a clamped beam is maximized by bending design. The beam has the same dimensions and material properties of the beam in the previous example. The fourth and fifth natural frequencies of this beam before

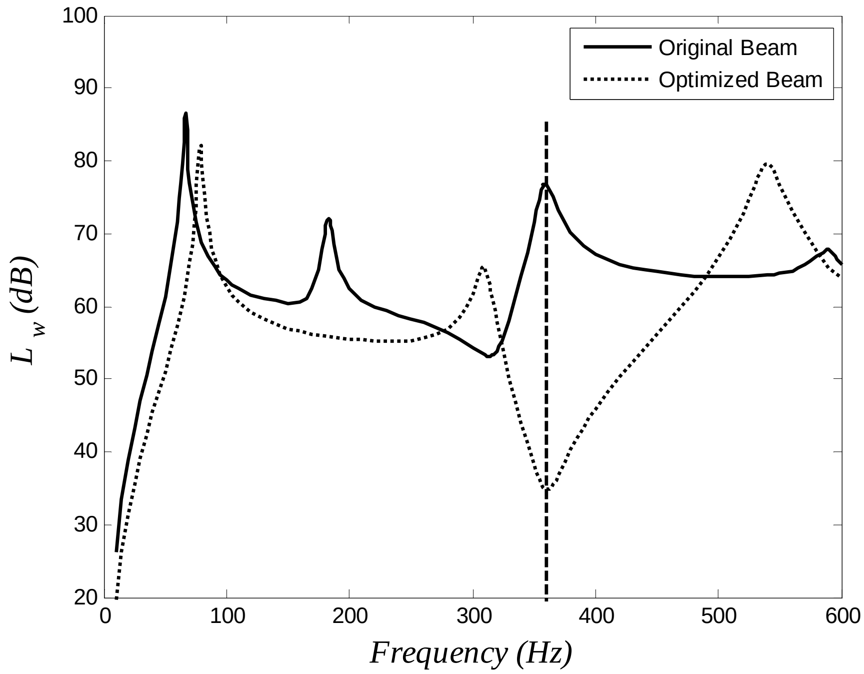

Fig. 6. Sound power level spectrum of the optimized beam at 358 Hz compared with the original beam.

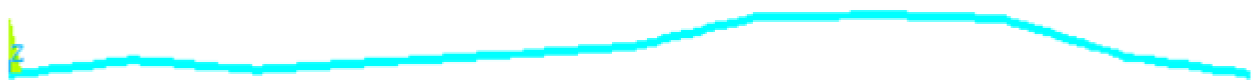

Fig. 7. The optimum design of a bent beam which minimizes the sound power level at 358 Hz (exaggerated vertical scale).

bending are 591.7 Hz and 883.8 Hz, respectively. So the difference between the frequencies is 292.1 Hz. The optimization problem is the same as the problem which is illustrated in Eq. 9, but with maximizing $(f_5 - f_4)$ instead of maximizing f_1.

After solving the optimization problem and creating the optimal design, the fourth and fifth natural frequencies are 569.3 and 1224.0 Hz, respectively. The gap between these frequencies is 654.7 Hz, which is 124.1% greater than the gap between the frequencies of the original beam. The optimal design which gives the maximum gap between the fourth and fifth natural frequencies is shown in figure 4. The optimal design variables are summarized in table 2.

MINIMIZING THE RADIATED SOUND POWER OF VIBRATING BEAMS BY BENDING TECHNIQUE

To demonstrate the efficiency of bending technique to minimize the radiated sound power of vibrating clamped beams, two examples are presented in this section. In the first example, we seek the design of a bent clamped beam that yields a minimum sound radiation at a specific frequency. In the second example, we seek the design of a bent clamped beam that yields a minimum sound radiation at the broad frequency band. In both examples, the GA is used to find the heights of the key points which give the minimum sound radiation.

The beam under consideration has the same dimensions and material properties of the beam in previous examples. It has loss factor η = 0.02. It is excited by a harmonic force at point (x = 0.7*L*, see figure 5(a)), having uniform amplitude of 0.5 N in the frequency range of 10-600 Hz. The location of the excitation point is selected such that this force will excite all the mode shapes within the frequency range of interest. The plate is assumed to be in an infinite baffle. The acoustic medium is air having acoustical impedance $\rho c = 415$ rayle. The sound power is calculated numerically based on Eq. 7. The sound power spectrum is shown in figure 5(b). The peaks in the plot represent the sound radiation at beam first four natural frequencies. Among these, there are two odd mode shapes that radiate high levels of sound power. The first one is associated with the first mode shape ($f_1 \approx 66.2$ Hz), the second one is associated with the third mode shape ($f_3 \approx 358.0$ Hz). It is noted that there are small sound power peaks in the second and fourth mode shapes, and this result is expected due to the volume velocity cancellation effect at these even modes.

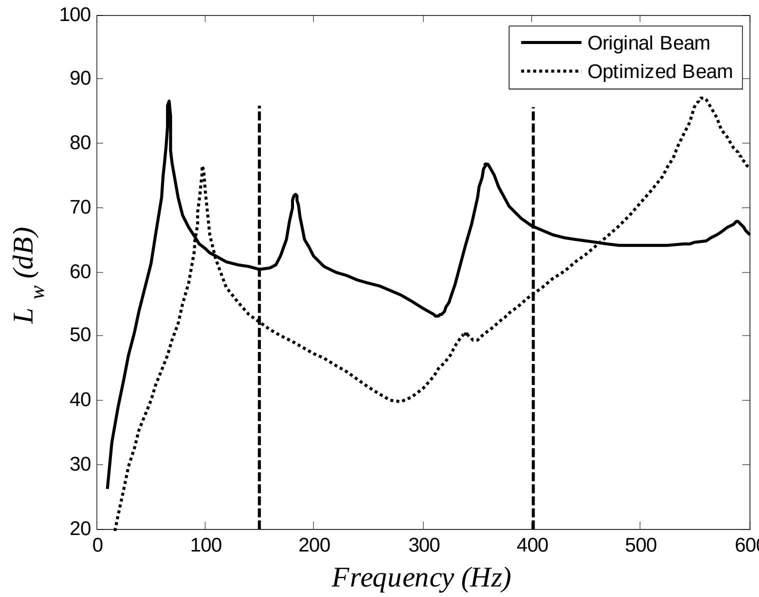

Fig. 8. Sound power level spectrum of the optimized beam over broad frequency band compared with the original beam.

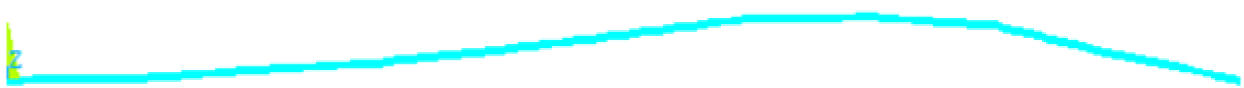

Fig. 9. The optimum design of a bent beam which minimizes the sound power level at over broad frequency band (exaggerated vertical scale).

In the first example, the goal is to minimize the sound radiation from the vibrating clamped beam at a frequency of 358 Hz by bending it at 9 key points. The frequency 358 Hz is selected because the beam radiates high sound power at this frequency (L_W = 76.7 dB). Again, the GA is used in the minimization process. It is used to find the heights of the key points which give the minimum sound radiation at 358 Hz. The population size in each generation is assumed 150. The stopping criterion is chosen as the maximum number of generations is 150 generations or the objective function tolerance is less than 10^{-3}. The optimization problem is the same as the problem which illustrated in Eq. 9, but with minimizing the sound power level at 358 Hz instead of maximizing f_1. Figure 6 shows the sound power spectrum plot of the optimized beam compared with the original one. By creating the optimal beam, the sound power level at 358 Hz can be decreased from 76.7 dB to 34.8 Hz (41.9 dB decrease). Figure 7 shows the optimal design of the bent beam and Table 3 summarizes the optimal design variables. The sound minimization is achieved by shifting the natural frequencies far away from frequency of 358 Hz and by redistributing the volume velocity to achieve the maximum volume velocity cancelation. In other words, the reduction of the overall volume velocity results in a reduction of the sound power. Physically, volume velocity cancelation means sound pressure cancelation at the surface of the plate, and this causes little sound energy to be radiated.

In the second example, the sound radiation is minimized over a broad frequency band by bending design of the vibrating clamped beam. The objective function in this example is selected as the average radiated sound power level over a frequency range of interest,

$$L_{W,avg} = \frac{1}{N_{freq}} \sum_{i=1}^{N_{freq}} L_{W,i}, \qquad (10)$$

where N_{freq} is the number of frequency increments between f_{min} and f_{max}, and $L_{W,i}$ is the sound power level at the i^{th} frequency. The beam under consideration and the optimization constraints are similar to those in the last example. However, the objective function is to minimize the sound radiation within f_{min} = 150 Hz and f_{max} = 400 Hz. The frequency range of interest is divided into 50 increments. Again, the minimization is carried out using the genetic algorithm. The results of sound minimization using bending technique are summarized in table 4. Figure 8 shows the sound power spectrum plot of the optimized beam compared with the original beam. By creating the optimal beam as shown in figure 9, the average sound power level decreases from 63.8 dB to 48.5 dB (15.3 dB). Figure 9 shows that the radiated sound power is decreased by increasing the stiffness near the excited area. Similar results were observed by Lamancusa (1993) and Marburg *et al.* (2006).

Table 1. Optimum design variables which maximize the beam fundamental frequency.

z_1(mm)	z_2(mm)	z_3(mm)	z_4(mm)	z_5(mm)	z_6(mm)	z_7(mm)	z_8(mm)	z_9(mm)
1.30	1.75	2.61	3.32	3.81	3.33	2.62	1.74	1.31

Table 2. Optimum design variables which maximize the gap between the beam fourth and fifth natural frequencies.

z_1(mm)	z_2(mm)	z_3(mm)	z_4(mm)	z_5(mm)	z_6(mm)	z_7(mm)	z_8(mm)	z_9(mm)
2.06	4.96	0.61	0.41	4.37	0.73	1.37	5.00	2.50

Table 3. Optimum design variables which minimize the sound power level at frequency of 358 Hz.

z_1(mm)	z_2(mm)	z_3(mm)	z_4(mm)	z_5(mm)	z_6(mm)	z_7(mm)	z_8(mm)	z_9(mm)
1.14	0.37	0.99	1.54	2.19	4.52	4.74	4.43	1.39

Table 4. Optimum design variables which minimize the average sound power level over a broad frequency band.

z_1(mm)	z_2(mm)	z_3(mm)	z_4(mm)	z_5(mm)	z_6(mm)	z_7(mm)	z_8(mm)	z_9(mm)
0.11	0.78	1.38	2.41	3.55	4.86	5.00	4.30	1.95

CONCLUSION

The bending method for optimizing the natural frequencies and for minimizing the sound radiation from vibrating beams was demonstrated. The design approach couples the finite element method for modal and harmonic analysis, the Lumped Parameter Model for acoustic analysis, and the genetic algorithm for optimization. The beams under consideration were bent at some key points. The z-coordinates of these key points were considered as design variables. The optimization examples show the efficiency of bending technique to increase the fundamental frequency and the gap between the fourth and fifth natural frequencies of clamped beams. Moreover, the bending technique is efficient in minimizing the sound radiation at a single frequency and over broad frequency band.

REFERENCES

Alshabatat, N. and Naghshineh, K. 2012. Optimization of the Natural Frequencies of Beams via Dimpling Technique. Journal of Modeling and Simulation. 32(2):134-144.

Cheng, WN., Cheng, CC. and Koopmann, GH. 2011. A New Design Strategy for Minimizing Sound Radiation of Vibrating Beam Using Dimples. Journal of Vibration and Acoustics. 133(5): 76-82.

Ece, MC., Aydogdu, M. and Taskin, V. 2007. Vibration of a Variable Cross-Section Beam. Mechanics Research Communications. 34:78-84.

Ewins, DJ. 2000. Modal Testing: Theory and Practice. RSP Ltd.

Fahnline, JB. and Koopmann, GH. 1996. A Lumped Parameter Model for the Acoustic Power Output from a

Vibrating Structure. Journal of Acoustical Society of America. 100(6):3539-3547.

Fritze, D., Marburg, S. and Hardtke, HJ. 2009. Estimation of Radiated Sound Power: A Case Study on Common Approximation Methods. Acta Acustica United with Acustica. 833-842.

Gupta, VK. and Murthy, PN. 1978. Optimal Design of Uniform Non-Homogeneous Vibrating Beams. Journal of Sound and Vibration. 59(4):521-531.

Junger, MC. and Feit, D. 1993. Sound, Structures, and their Interaction. Acoustical Society of America.

Karihaloo, BL. and Niordson, FI. 1973. Optimum Design of Vibrating Cantilevers. Journal of Optimization Theory and Applications. 11(6):638-654.

Kelly, JM., Sackman, JL. and Javid, A. 1991. The influence of perform on the modes of vibrating beams. Earthquake Engineering Structural Dynamics. 20:1145-1157.

Koopmann, GH. and Fahline, JB. 1997. Designing quiet structures: a sound power minimization approach. Academic Press.

Lamancusa, JS. 1993. Numerical optimization techniques for structural-acoustic design of rectangular panels. Computers & Structures. 48(4):661-675.

Marburg, S., Dienerowitz, F., Fritze, D. and Hardtke, HJ. 2006. Case studies on structural-acoustic optimization of a finite beam. Acta Austica United with Acustica. 92(3):427-439.

Nagshineh, K. and Koopmann, G. 1992. Material tailoring of materials to achieve a minimum radiation condition. Journal of Acoustical Society of America. 92(2):841-856.

Szelag, D. and Mroz, Z. 1979. Optimal Design of Vibrating Beams with Unspecified Support Reactions. Computer Methods in Applied Mechanics and Engineering. 19(3):333-349.

Wang, PW. and Cheng, CC. 2005. Natural frequency Tuning Using Structural Patches. Journal of Vibration and Acoustics. 127(1):28-35.

Wu, JS. and Chou, HM. 1998. Free Vibration Analysis of a Cantilever Beam Carrying any Number of Elastically Mounted Point Masses with the Analytical-and-Numerical-Combined Method. Journal of Sound and Vibration. 213(2):317-332.

Wu, JS. and Lin, TL. 1990. Free Vibration Analysis of a Uniform Cantilever Beam with Point Masses by an Analytical-and-Numerical-Combined Method. Journal of Sound and Vibration. 136(2):201-213.

Zhou, D. and Ji, T. 2006. Dynamic Characteristics of a Beam and Distributed Spring-Mass System. International Journal of Solids and Structures. 43:5555-5569.

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