

PARAMETRIC ANALYSIS OF BEAM RESTING ON ELASTIC FOUNDATION (ANN)

*Mostafa A M Abdeen and S M Bichir

Department of Engineering Mathematics and Physics
 Faculty of Engineering, Cairo University, Giza 12211, Egypt

ABSTRACT

Mathematical formulation for Timoshenko beam resting on an elastic foundation is presented. Parametric analysis is presented for three types of boundary conditions. In the current paper, artificial intelligence technique is implemented to simulate and then predict the beam's deflection using one row of the results from DQM (Differential Quadrature Method) analysis to study the effect of foundation parameter, beam stiffness and applied load on the Timoshenko beam's deflection for the three types of boundary conditions. The ANN (Artificial Neural Network) results presented in the current study showed that the designed ANN models can simulate and predict very accurately the beam's deflection and the effect of different DQM parameters.

Keywords: Timoshenko beam, elastic foundation, analytic formulation, numerical simulation; artificial neural network.

INTRODUCTION

There are many applications for beam on elastic foundation mainly in mechanical and civil engineering e.g. disc brake pad, shafts supported on ball, roller, building and bridges, submerged floating tunnels, buried pipelines, railroad tracks etc. Two simple models have been used to analyze a beam on an elastic foundation. One is the Winkler (1867) foundation model, which is based on a pure bending beam theory. The second is the Pasternak (1954) shear model, which is based on the assumption of pure shearing of the beam (no bending).

Timoshenko (1921, 1956) solved analytically the problems of beams on elastic foundation with several loading and boundary conditions. DQM is a numerical method used for solving many problems in engineering and mathematics (Bert and Malik, 1997).

Artificial intelligence has been widely used to simulate and predict the behavior of the different physical phenomena in most of the engineering fields. Kheireldin (1998) developed ANN model to study the characteristics of sewer contractions in open channels. Allam (2005) developed artificial intelligence model to predict the maximum and minimum settlement under building near tunnel construction. Mohamed (2006) developed ANN model to reduce the design time of multi-story steel frames. Abdeen (2008) developed several ANN models to simulate the flow discharges of water surface profile in open channels. Simulation and prediction for the internal properties of different materials, using ANN technique, were very important in many researches as in Abdeen and Hodhod (2010) and Gaafar *et al.* (2011). In the present

paper, the parametric analysis of the beam resting on elastic foundation is presented. The ANN technique is used to understand, simulate and predict the beam's deflection for three different end conditions.

Formulation of the Problem

Figure (1) shows the beam model resting on elastic foundation with modulus K_s .

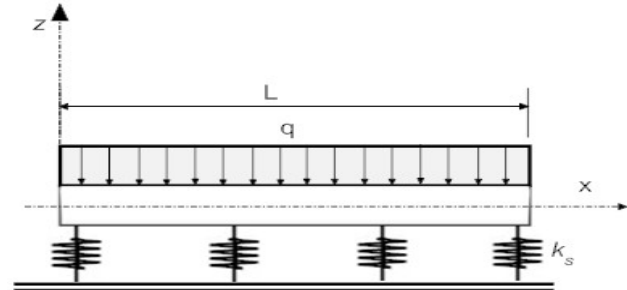


Fig. 1. Proposed Beam Model.

The beam's material is assumed to be homogeneous with elasticity modulus E and shear modulus G . The cross section area and bending stiffness are A and D respectively. The Timoshenko effect constant $C = KGA$ Where: K is the Timoshenko effect.

The equilibrium differential equation for the lateral deflection w can be written as:

$$D \frac{\partial^4(w)}{\partial x^4} - \frac{k_s D}{C} \frac{\partial^2(w)}{\partial x^2} + k_s(w) = q - \frac{D}{C} \frac{\partial^2(q)}{\partial x^2} \quad (1)$$

Where:

$$D = Eh^3/12(1-\nu^2)$$

*Corresponding author email: Mostafa_a_m_abdeen@hotmail.com

Equation (1) is the general equilibrium equation of the beam resting on elastic foundation. By using the DQM the governing differential equation (1) will be in the following form:

$$\sum_{j=1}^N a_{ij}^{(4)} w_j - \frac{k_s}{C} \sum_{j=1}^N a_{ij}^{(2)} w_j + \frac{k_s}{D} w_i = \frac{q_i}{D} - \frac{q_i''}{C} \quad (2)$$

Where:

N is the number of node points and

a_{ij} is weighting coefficients.

Boundary Conditions

Simply Supported (SS)

Boundary conditions are: $w = 0$ at $x = 0, L$,
 $w'' = 0$ at $x = 0, L$

or, in the DQ discrete

$$\text{domain: } \begin{cases} w_i = 0 & \text{at } i = 1, N \\ \sum_{j=1}^N a_{ij}^{(2)} w_j = 0 & \text{at } i = 1, N \end{cases}$$

Clamped-Clamped (CC)

Boundary conditions are: $w = 0$ at $x = 0, L$,
 $w' = 0$ at $x = 0, L$

or, in the DQ discrete

$$\text{domain: } \begin{cases} w_i = 0 & \text{at } i = 1, N \\ \sum_{j=1}^N a_{ij}^{(1)} w_j = 0 & \text{at } i = 1, N \end{cases}$$

Clamped-Free (CF)

Boundary conditions are: $w = 0$ at $x = 0$,
 $w' = 0$ at $x = 0$,
 $w'' = 0$ at $x = L$,
 $w''' = 0$ at $x = L$

or, in the DQ discrete

$$\text{domain: } \begin{cases} w_1 = 0, & \sum_{j=1}^N a_{1,j}^{(1)} w_j = 0 \\ \sum_{j=1}^N a_{N,j}^{(2)} w_j = 0, & \sum_{j=1}^N a_{N,j}^{(3)} w_j = 0 \end{cases}$$

Numerical Models

Artificial Neural Network (ANN) is a numerical model depends on a certain number of neurons in different layers. Every neuron acts very closely to the real neuron of the human brain. Each layer has a different function than the others. The input layer with its neurons gets the information from the external world (given data), while the hidden layers are working as detectors of these data. The output layer is the final layer of the network and it produces the required results. Neuralyst software, Shin (1994) is used to design the ANN models in the present work.

Simulation Cases

To fully investigate the effect of foundation parameter, bending stiffness and applied load on the beam’s response (deflection) for the three types of boundary conditions, three numerical boundary condition models, using ANN technique, are designed in this study. The developed simulation models used one solution output obtained from DQM to design the ANN models.

Numerical Models Design

To design ANN models to simulate and predict the beam’s deflection, the input and output variables have to be determined. Table 1 shows the three neural network boundary condition models (SS, CC, CF).

Table 1. Key Input and Output Variables for Neural Network Models.

Boundary Condition Model	Input Variables				Output
	Beam’s Span x (m)	K_s (N/m ³)	D (N.m)	q (Pa)	
SS					Deflection w(m)
CC					
CF					

Several ANN models are tested for all numerical models to finally choose the best networks design to simulate, very accurately, the effect of foundation parameter, bending stiffness and applied load on beam’s deflection based on minimizing the Root Mean Square Error (RMS-Error).

The training procedure for the developed ANN models, in the current study, uses one raw of the data from the results of the DQM to let the ANN understands the behavior. After sitting finally the NN models, these models are used to predict the beam’s deflection for different values of K_s , D and q and others.

Table 2 presents the final design of the developed ANN models for the three boundary conditions. The structure of the three models is chosen to be the same but the difference between them will be in RMS-Error and the number of trials to achieve accepted accuracy represented by maximum percentage relative error.

Table 2. The Designed ANN Models.

Boundary Condition Model	No. of layers	No. of Neurons in each layer			
		Input Layer	First Hidden	Second Hidden	Output Layer
SS	4	4	8	6	1
CC					
CF					

The parameters of the designed ANN models are presented in table 3, where:

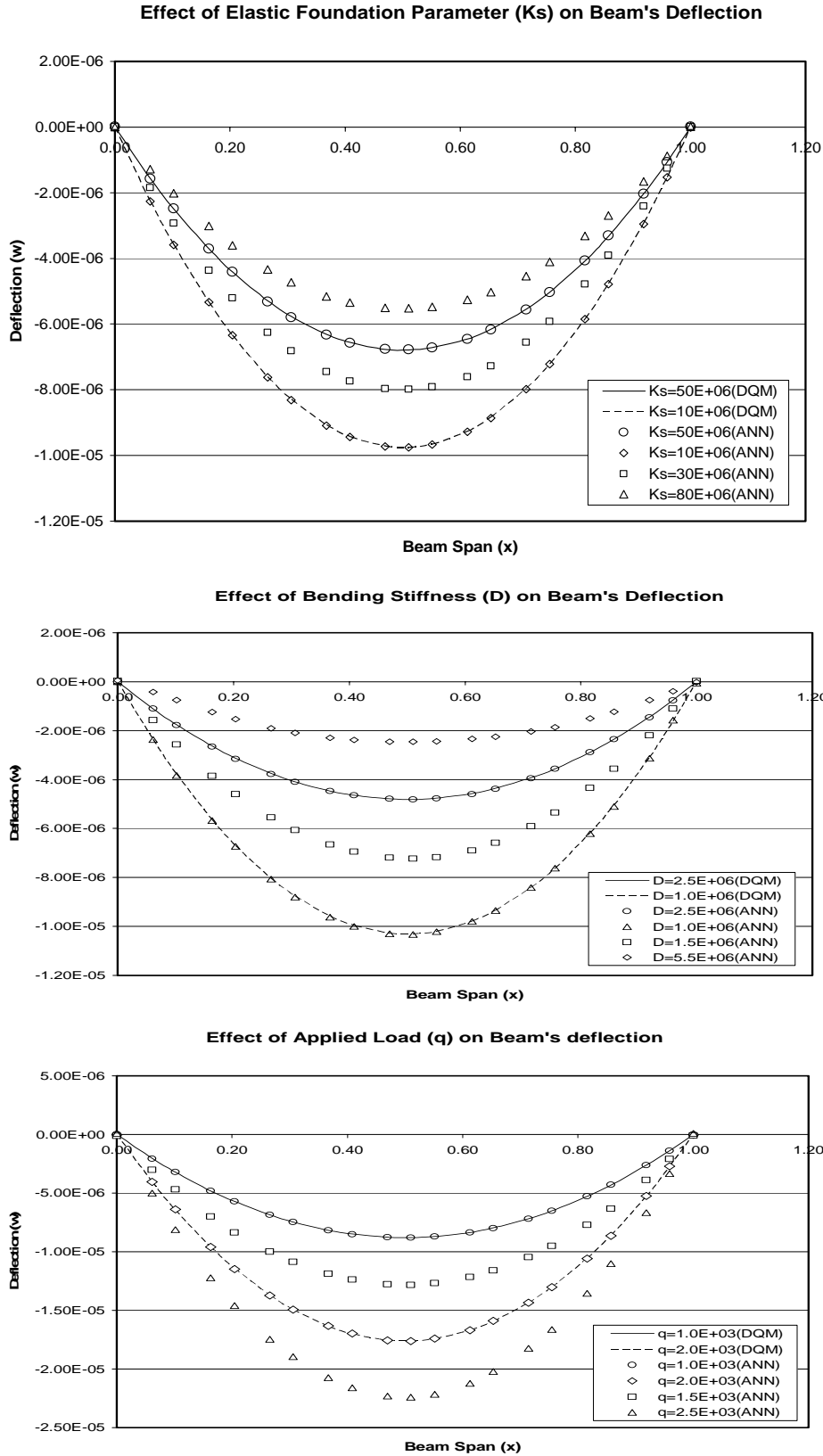


Fig. 2. Simple Supported Edged Beam's Deflection.

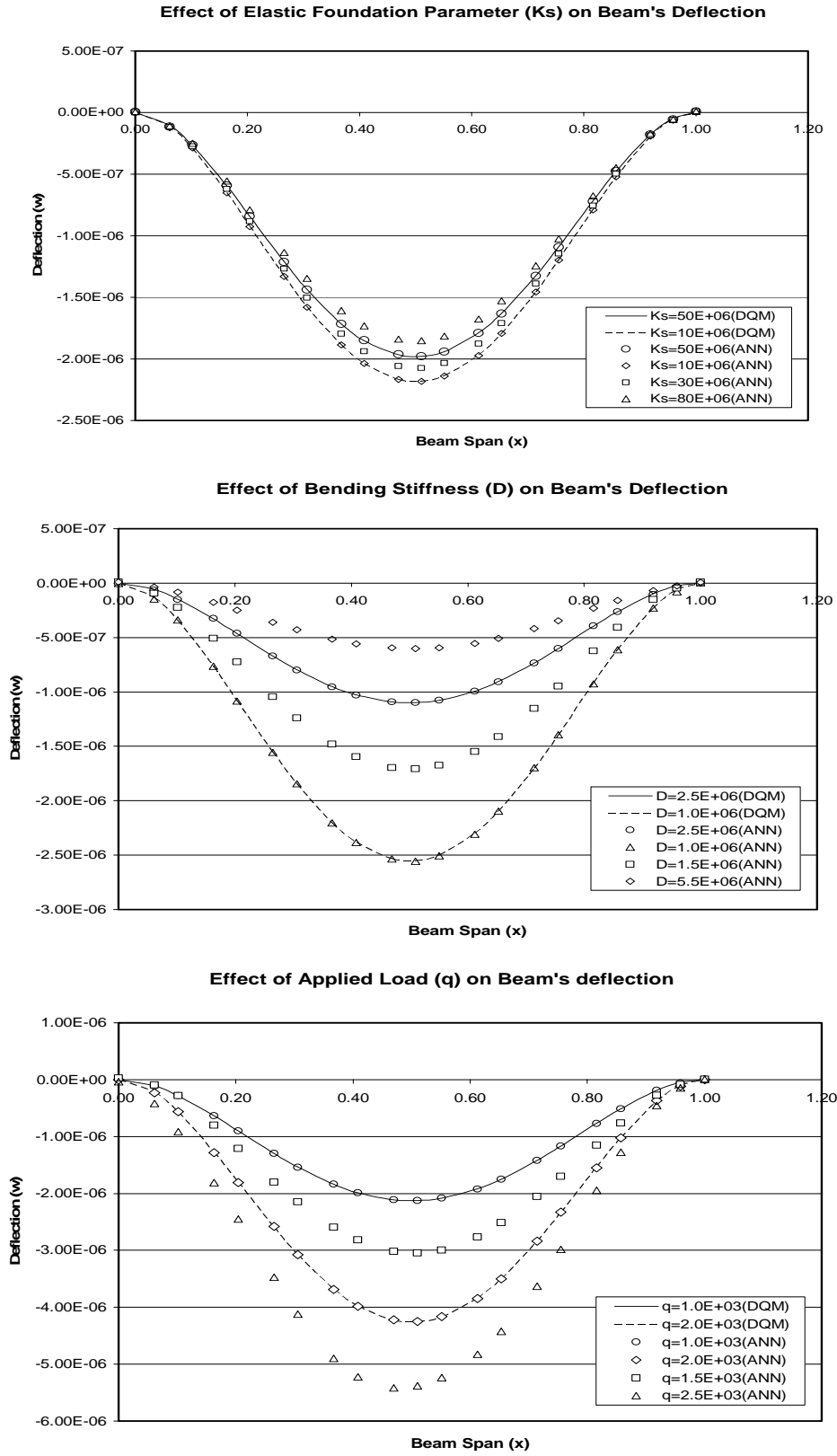


Fig. 3. Clamped-Clamped Edge Beam's Deflection.

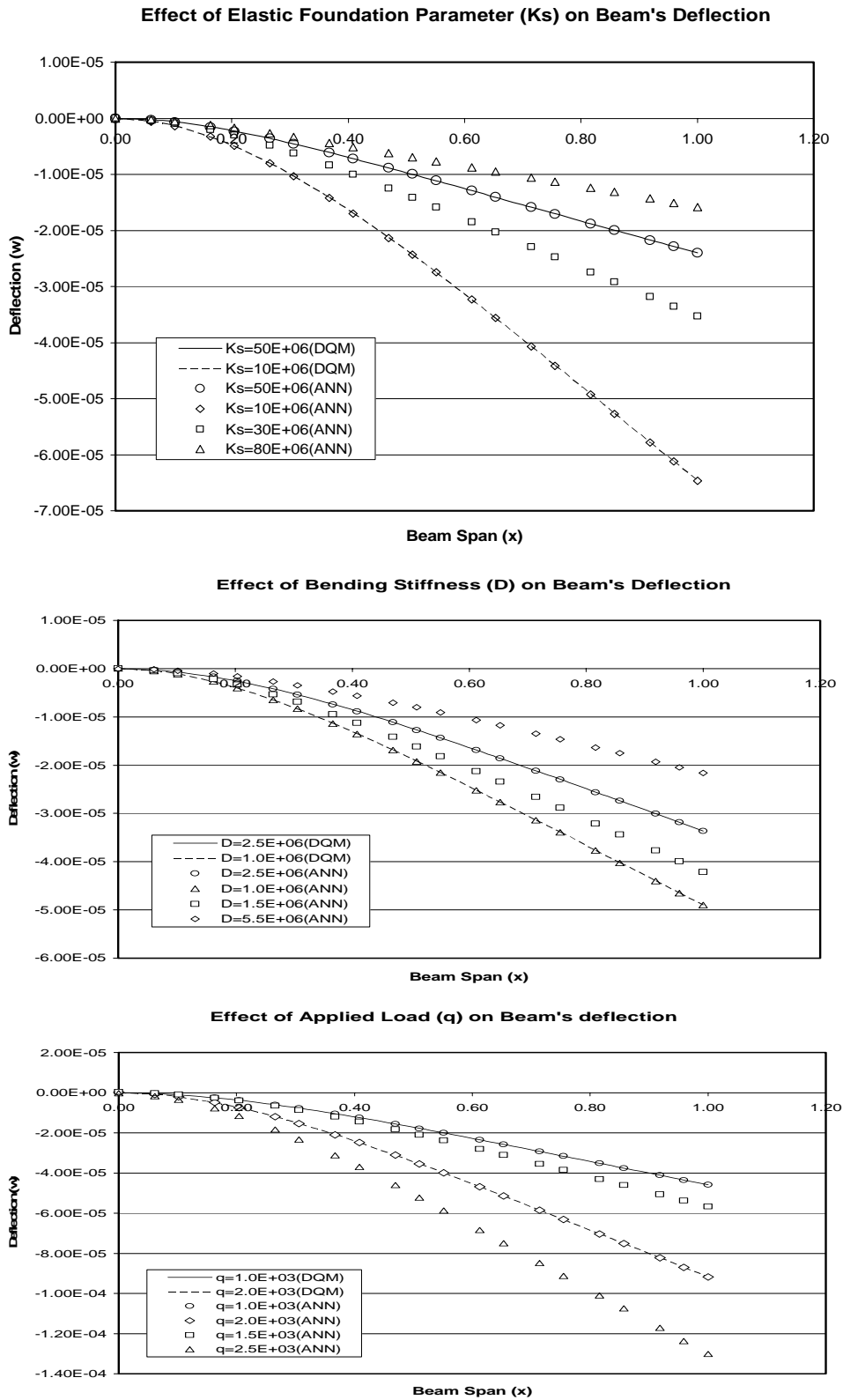


Fig. 4. Clamped-Free Edged Beam's Deflection.

Training Epochs: Number of trails to achieve the present accuracy.

Percentage Relative Error (PRR): Percentage relative error between the numerical results and actual measured value for and is computed according to equation (6) as follows:

$$PRE = (\text{Absolute Value (ANN_PR - AMV)}/\text{AMV}) * 100$$

Where :

ANN_PR : Predicted results using the developed ANN model

AMV : Actual Measured Value

MPRE: Maximum percentage relative error during the model results for the training step (%)

Table 3. Parameters used in the Artificial Neural Network Models.

Simulation Parameter	SS	CC	CF
Training Epochs	558516	685412	473258
MPRE	0.65	0.88	0.45
RMS-Error	0.0007	0.0008	0.0004

(SS-Simply Supported, CC- Clamped-Clamped, CF-Clamped-Free)

RESULTS AND DISCUSSION

Numerical results using ANN technique will be presented in this section for the three neural network boundary condition models (SS, CC and CF) to show the simulation and prediction powers of ANN technique studying the effect of elastic foundation parameter, bending stiffness and applied load on beam's deflection. Figures 2-4 shows the ANN results (symbols) and DQ results (line and dash) for the three boundary condition models. It is very clear, from these figures, that the developed neural network models can simulate and predict the beam's deflection for any variation of foundation parameter, bending stiffness and applied load very accurately.

For all of the case studies, the default values used before varying each parameter are:

$$k_s = 20e6 \text{ N/m}^3, C = 26.667e6 \text{ N/m}, D = 1.2e6 \text{ N.m}, N = 13, q = 1e3 \text{ Pa (uniformly distributed load)}.$$

From the parametric studies conducted in the present work, it could be noticed that, by increasing the elastic foundation parameter and bending stiffness the deflection decreases, while by increasing the applied load the deflection increases.

CONCLUSION

Based on the output results of the developed ANN models in this study, the following can be concluded:

1. The developed ANN boundary condition models are very smarting in understanding the effect of elastic foundation parameter, bending stiffness and applied load on the beam's deflection.
2. The designed ANN models can successfully capable of direct predicting the response behavior of beam resting on an elastic foundation for different parameters and boundary conditions.
3. Using single set of output results from DQM, The ANN models succeeded to understand the behavior of beam on elastic foundation and became ready to give the beam's response for different parameters without solving such kind of problem again.

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