SENRA Academic Publishers, Burnaby, British Columbia
Vol. 5, No. 3, pp. 1701-1706, October 2011
Online ISSN: 1920-3853; Print ISSN: 1715-9997

Canadian Journal of pure&applied sciences

PROPAGATION OF LOVE WAVES THROUGH AN IRREGULAR SURFACE LAYER IN PRESENCE OF AN IMPEDING SURFACE

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ABSTRACT

We aimed at studying the propagation of Love waves through the crustal layer of earth in the presence of thin surface impedance. The Wiener-Hopf technique and Fourier transform have been used to find the reflected and scattered Love waves. Numerical computation has been done and analysis of results shows the approximate behavior of reflected and scattered waves. It has been observed that both the material and the thickness of the impeding surface affect the propagation of incident waves. The scattered waves have a logarithmic singularity at the tip of scatterer and behave as a decaying cylindrical wave at distant points, dying out at long distances.

Keywords: Wiener-Hopf technique, scattered waves, seismic waves, surface impedance.

INTRODUCTION

Seismic waves are the waves that cause natural disaster like earthquake, tsunami etc. and are responsible for a lot of destruction. The seismic signals are also applied to investigate the internal structure of earth and they can be used in exploration of valuable materials like mineral, oil, water etc. The mathematical analysis of reflected and scattered waves due to thin uniform distribution of matter in surface layer has been discussed herewith. The discontinuity is present in the half of the surface ($z = -H$, $x < 0$) and the other half of the surface ($z = -H$, $x > 0$) is free surface. The effect of distribution of matter is such that it exerts surface traction proportional to the acceleration in a direction perpendicular to the vertical plane through the direction of propagation.

The model finds its applications in understanding the internal composition of crustal layer of earth. Deshwal and Gogna (1987) have considered the problem of diffraction of compressional waves due to surface impedance. The similar type of problems for Rayleigh waves have also been discussed by Gregory (1966). Tomar and Kaur (2007), and Chattopadhyay *et al*. (2009) have studied the SH-waves in different media. Saito (2010) have studied the excitation of Love waves due to the interaction between propagating ocean wave and sea bottom topography. Here we propose to discuss the propagation of Love waves through a layered structure in the presence of impeding surface. The reflected and scattered waves are obtained to study the internal structure of earth.

MATERIALS AND METHODS

The problem is being analyzed in zx-plane. The z-axis has been taken vertically downwards and x-axis along the interface between the layer of thickness 'H' and the solid half-space $z \geq 0$. The half of the surface ($z = -H$, $x < 0$) contains the thin impeding material and the other half is free surface. A time harmonic Love wave is incident on the impeding surface from the side $x > 0$. The geometry of the problem is shown in figure 1. The displacements for the incident Love wave are given by

$$v_{0,1} = A\cos\theta_{2N} H e^{-(\theta_{1N}z + ik_{1N}x)}, \quad z \geq 0, \tag{1}$$

$$v_{0,2} = A\cos\theta_{2N}(z+H)e^{-ik_{1N}x}, \; -H \leq z \leq 0. \tag{2}$$

where,

$$\theta_{2N} = \sqrt{k_2^2 - k_{1N}^2},$$
$$\theta_{1N} = \sqrt{k_{1N}^2 - k_1^2}, \; |k_1| < |k_2| \tag{3}$$

and k_{1N} is a root of the equation

$$\tan\theta_{2N} H = \gamma \frac{\theta_{1N}}{\theta_{2N}}, \text{ and } \gamma = \frac{\mu_1}{\mu_2}, \; k_{1N} = \frac{\omega}{C_{1N}} \tag{4}$$

μ_1 *and* μ_2 $(\mu_1 > \mu_2)$ being the rigidities of shear waves in the half space and in the crustal layer respectively and C_{1N} represents the phase velocity of Love waves of N[th] mode in a layered structure with a surface layer having a thickness H.

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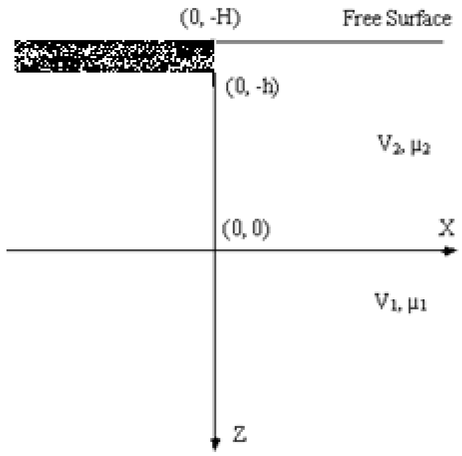

Fig.1. Geometry of the problem.

The wave equation in two dimensions is given as

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}+\frac{\varepsilon}{c^2}\frac{\partial u}{\partial t}, \quad (5)$$

where $\varepsilon > 0$ is the damping constant and c is the velocity of propagation. If the displacement be harmonic in time, then

$$u(x,z,t) = v(x,z)e^{-i\omega t} \quad (6)$$

and equation (5) reduce to

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}+k^2\, v=0. \quad (7)$$

The wave equation in present study is written as

$$(\nabla^2+k_j^2)\ v_j=0, \quad j=1,2 \quad (8)$$

where,

$$k_j=\sqrt{\frac{\omega^2+i\varepsilon\omega}{V_j^2}}=k_j^{'}+ik_j^{''}=\frac{\omega}{V_j} \quad (9)$$

V_1 *and* V_2 $(V_1>V_2)$ are respectively the velocities of shear waves in the half space $z\geq 0$ and in the layer $-H\leq z\leq 0$.

Let the total displacement be given by

$$v=v_{0,1}+v_1, \quad z\geq 0 \ , \ -\infty<x<\infty, \quad (10)$$

$$=v_{0,2}+v_2, -H\leq z\leq 0 \ , \ -\infty<x<\infty. \quad (11)$$

The boundary conditions are

(i) $$\mu_2\frac{\partial v_2}{\partial z}=av, \quad x\leq 0, \quad z=-H. \quad (12)$$

(ii) $$\frac{\partial v_2}{\partial z}=0, \quad x\geq 0, \quad z=-H, \quad (13)$$

(iii) $$v_1=v_2, \ \mu_1\frac{\partial v_1}{\partial z}=\mu_2\frac{\partial v_2}{\partial z},$$

$$z=0, \ -\infty<x<\infty. \quad (14)$$

In condition (12), 'a' is the constant depending upon the nature of material of the impeding surface. The boundary condition (13) represents the physical situation that at each point of the surface there is a resisting force proportional to the velocity along the normal to the vertical plane through the direction of propagation.

Taking Fourier transform of equation (8), we obtain

$$\frac{d^2\overline{v}_j(p,z)}{dz^2}-\theta_j^2\overline{v}_j\,(p,z)=0, \quad (15)$$

where $\theta_j=\pm\sqrt{p^2-k_j^2}$ and $\overline{v}_j\,(p,z)$ represents Fourier transform of $v_j\,(p,z)$ which can be defined as

$$\overline{v}_j\,(p,z)=\int_{-\infty}^{\infty}v_j\,(x,z)e^{ipx}dx, \quad p=\alpha+i\beta$$

$$=\int_{-\infty}^{0}v_j\,(x,z)e^{ipx}dx+\int_{0}^{\infty}v_j\,(x,z)e^{ipx}dx$$

$$=\overline{v}_{j-}(p,z)+\overline{v}_{j+}(p,z). \quad (16)$$

If for a given z, as $|x|\to\infty$ and $M,\tau>0,\ |v_j(x,z)|\sim Me^{-\tau|x|}$, then $\overline{v}_{j+}(p,z)$ is analytic in $\beta>-\tau$ and $\overline{v}_{j-}(p,z)$ is analytic in $\beta<\tau\,(=lm(k_j))$. By analytic continuation, $\overline{v}_j\,(p,z)$ and its derivatives are analytic in the strip $-\tau<\beta<\tau$ in the complex p-plane. Solving equation (15) and choosing the sign of θ_j such that its real part is always positive, we find

$$\overline{v}_1\,(p,z)=A(p)e^{-\theta_1 z}, \quad z\geq 0 \quad (17)$$

and

$$\overline{v}_2\,(p,z)=B(p)e^{-\theta_2 z}+C(p)e^{\theta_2 z}, \quad -H\leq z\leq 0. \quad (18)$$

Solving equations (17) and (18) by using boundary condition (14), we get

$$\overline{v}_2\,(p,z)=A(p)\frac{[\theta_2\cosh\theta_2 z-\gamma\,\theta_1\sinh\theta_2 z]}{\theta_2} \quad (19)$$

$$\bar{v}_1(p,z) = \bar{v}_1(p,0)e^{-\theta_1 z} \tag{20}$$

Differentiating equations (19) and (20) with respect to z and putting $z=-H$ and denoting $\bar{v}_j'(p,-H)$ by $\bar{v}_j'(p)$ etc., eliminating A(p) using conditions (12) and (13), we obtain

$$\bar{v}_{2+}'(p) = 0 \tag{21}$$

and

$$\bar{v}_{2-}'(p) = \frac{a\bar{v}_{2-}}{\mu_2} + \frac{aA}{i\mu_2(p-k_{1N})} \tag{22}$$

Adding equations (21) and (22), we find

$$\bar{v}_2'(p) = \bar{v}_{2+}'(p) + \bar{v}_{2-}'(p) = \frac{a\bar{v}_{2-}}{\mu_2} + \frac{aA}{i\mu_2(p-k_{1N})} \tag{23}$$

Now, from equation (19), we have

$$\bar{v}_2(p,z) = \frac{\theta_2\cosh\theta_2 z - \gamma\theta_1\sinh\theta_2 z}{\theta_2\cosh\theta_2 H + \gamma\theta_1\sinh\theta_2 H}\bar{v}_2(p) \tag{24}$$

Differentiating equation (24) with respect to z and putting $z=-H$ and denoting $\bar{v}_2'(p,-H)$ by $\bar{v}_2'(p)$, we obtain

$$\bar{v}_2'(p) = -\frac{\theta_2(\theta_2\sinh\theta_2 H + \gamma\theta_1\cosh\theta_2 H)}{\theta_2\cosh\theta_2 H + \gamma\theta_1\sinh\theta_2 H}\bar{v}_2(p) \tag{25}$$

Using equations (23) and (25), we find

$$\frac{a\bar{v}_{2-}}{\mu_2} + \frac{aA}{i\mu_2(p-k_{1N})} = -\frac{\theta_2(\theta_2\sinh\theta_2 H + \gamma\theta_1\cosh\theta_2 H)}{\theta_2\cosh\theta_2 H + \gamma\theta_1\sinh\theta_2 H}\bar{v}_2(p) \tag{26}$$

The equation (26) is Wiener-Hopf type differential equation whose solution will give the reflected and scattered waves. Now, we write

$$L(p) = \frac{f_1(p)}{f_2(p)} = \frac{\theta_2\cosh\theta_2 H + \gamma\theta_1\sinh\theta_2 H}{\theta_2\sinh\theta_2 H + \gamma\theta_1\cosh\theta_2 H} \tag{27}$$

where L(p) tends to 1 as $|p|$ tends to infinity. So, by infinite product theorem (Noble, 1958), L(p) can be factorized. If $p=\pm p_{1n}$ and $p=\pm p_{2n}$ are the zeros of $f_1(p)$ and $f_2(p)$ respectively, then we write (Sato, 1961)

$$\frac{f_1(p)}{f_2(p)} = \left[\prod_{n=1}^{\infty}\frac{p^2-p_{1n}^2}{p^2-p_{2n}^2}\right]\frac{G_1(p)}{G_2(p)} \tag{28}$$

where,

$$G_1(p) = \frac{f_1(p)}{\prod_{n=1}^{\infty}(p^2-p_{1n}^2)} \quad \text{and}$$

$$G_2(p) = \frac{f_2(p)}{\prod_{n=1}^{\infty}(p^2-p_{2n}^2)}. \tag{29}$$

Further, we write

$$P(p) = \frac{G_1(p)}{G_2(p)} = G_+(p)G_-(p) \tag{30}$$

where,

$$\log G_+(p) = \frac{1}{\pi}\int_0^{\infty}\frac{N_1-N_2}{t-ip}dt - \frac{1}{\pi}\int_0^{k_1}\frac{M_1-M_2}{t+p}dt - \frac{1}{\pi}\int_{k_1}^{k_2}\frac{1}{t+p}dt, \tag{31}$$

and

$$\tan N_1 = \frac{\alpha'\cos\alpha' H}{\gamma(t^2+k_1^2)^{1/2}\sin\alpha' H},$$

$$\tan N_2 = \frac{\gamma(t^2+k_1^2)^{1/2}\cos\alpha' H}{\alpha'\sin\alpha' H},$$

$$\tan M_1 = \frac{\alpha''\cos\alpha'' H}{\gamma(k_1^2-t^2)^{1/2}\sin\alpha'' H},$$

$$\tan M_2 = \frac{\gamma(k_1^2-t^2)^{1/2}\cos\alpha'' H}{\alpha''\sin\alpha'' H},$$

$$\alpha' = \sqrt{t^2+k_2^2} \text{ and } \alpha'' = \sqrt{k_2^2-t^2}.$$

Hence, we write

$$L(p) = \prod_{n=1}^{\infty}\frac{p^2-p_{1n}^2}{p^2-p_{2n}^2}G_+(p)G_-(p) \tag{32}$$

where,

$$L_{\pm}(p) = \prod_{n=1}^{\infty}\frac{p \pm p_{1n}}{p \pm p_{2n}}G_{\pm}(p) \tag{33}$$

Decomposition of equation (26) results into the equation

$$\frac{1}{L_+(p)}\left[\bar{v}_{2-}\sqrt{p+k_2} - \bar{v}_{2-}(-p_{1n})\sqrt{k_2-p_{1n}}\right] + \frac{L_-(p)}{\sqrt{p-k_2}}\left[\bar{v}_{2-} + \frac{A}{i(p-k_{1n})}\right]\frac{a}{\mu_2}$$

$$= -\frac{\bar{v}_{2+}\sqrt{p+k_2}}{L_+(p)} - \frac{\bar{v}_{2-}(-p_{1n})\sqrt{k_2-p_{1n}}}{L_+(p)} \tag{34}$$

There is a pole at $p=k_{1n}$ and branch point at $p=\pm k_2$. Hence the left hand member of equation (34) is analytic in $\beta < \tau (= \mathrm{Im}(k_1))$ and the right hand member is analytic in the region $\beta > -\tau$ (The contour of integration is shown in figure 2). Therefore, by analytic continuation they represent an entire function in the strip

$-\tau < \beta < \tau$. Hence, by Liouville's theorem, each member in (34) has a constant value $-$ c.

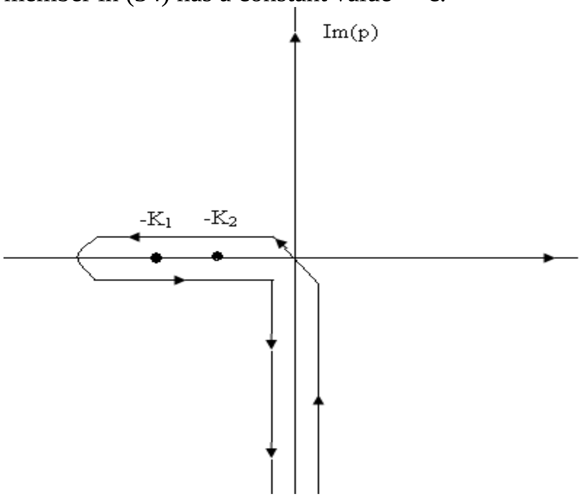

Fig. 2. Contour of integration in complex p- plane

Hence, we write

$$\bar{v}_2(p) = -\frac{a}{\mu_2\theta_2}\left[\bar{v}_{2-} + \frac{A}{i(p-k_{1N})}\right]L(p) \qquad (35)$$

where,

$$\bar{v}_{2-} = \frac{\mu_2\sqrt{p-k_2}}{\mu_2\theta_2 + aL(p)}\left[c - cL_+(p) - \frac{AaL(p)}{i\mu_2(p-k_{1N})\sqrt{p-k_2}}\right] \qquad (36)$$

The displacement $v_2(x,z)$ is obtained by inversion of Fourier transform as given below

$$v_2(x,z) = \frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta} \bar{v}_2(p,z)e^{-ipx}dp$$

$$= \frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta}\left[\frac{\theta_2\cosh\theta_2 z - \gamma\theta_1\sinh\theta_2 z}{\theta_2\sinh\theta_2 H + \gamma\theta_1\cosh\theta_2 H}\right]$$

$$\times\left[\bar{v}_{2^+}(p) + \bar{v}_{2^-}(p)\right]e^{-ipx}dp, \qquad (37)$$

where $\bar{v}_2(p)$ is given in equation (35).

RESULTS AND DISCUSSION

The incident Love waves are scattered when these waves encounter with surface irregularities like impeding surface in the layer. For finding the scattered component of the incident Love waves, we evaluate the integral in equation (37). There is a branch point $p = -k_2$ in the lower half-plane. For contribution around this point we put $p = -k_2 - it$, t being small. The branch cut is obtained by taking $\mathrm{Re}(\theta_2) = 0$. $Now\ \theta_2^2 = p^2 - k_2^2$ should be negative, so $\theta_2 = \pm i\bar{\theta}_2$, $\bar{\theta}_2 = \sqrt{t^2 + 2k_2^{''}t}$, $\theta_1 = \pm i\bar{\theta}_1$ and $\bar{\theta}_1 = \sqrt{(k_2+it)^2 - k_1^2}$. The imaginary part of θ_2 has different signs on two sides of the branch cut. Integrating equation (37) along two sides of branch cut, we get

$$v_{2,1}(x,z) = \frac{1}{2\pi}\int_0^\infty\left[\{\bar{v}_2(p,z)\}_{\theta_2 = i\bar{\theta}_2} - \{\bar{v}_2(p,z)\}_{\theta_2 = -i\bar{\theta}_2}\right]e^{-k_2^{''}x}e^{-tx}dt$$

$$= -\frac{ac}{\pi}e^{-k_2^{''}x}\int_0^\infty\left[\frac{\xi(t)\ \cos\sqrt{t^2+2k_2^{''}t}\,(z+H)}{\sqrt{t^2+2k_2^{''}t}}\right.$$

$$\left. + \frac{\psi(t)\ \sin\sqrt{t^2+2k_2^{''}t}\,(z+H)}{\sqrt{t^2+2k_2^{''}t}}\right]e^{-tx}dt \qquad (38)$$

where,

$$\xi(t) = \frac{(\bar{\theta}_2\cos\bar{\theta}_2 H + \gamma\bar{\theta}_1\sin\bar{\theta}_2 H)\theta_2^{'}}{\mu_2\bar{\theta}_2 + a\eta(t)}, \qquad (39)$$

$$\psi(t) = \frac{\bar{\theta}_2(\bar{\theta}_2\sin\bar{\theta}_2 H - \gamma\bar{\theta}_1\cos\bar{\theta}_2 H)\theta_2^{'}}{\mu_2\bar{\theta}_2 + a\eta(t)} \qquad (40)$$

and

$$\eta(t) = \frac{\bar{\theta}_2\cos\bar{\theta}_2 H + \gamma\bar{\theta}_1\sin\bar{\theta}_2 H}{-\bar{\theta}_2\sin\bar{\theta}_2 H + \gamma\bar{\theta}_1\cos\bar{\theta}_2 H}, \qquad (41)$$

where $\theta_2^{'} = \sqrt{-i(2k_2^{''}+t)}$.

For evaluation of integral in equation (38), we retain $\xi(0)$ and $\psi(0)$ only as 't' is small. Now, we write

$$K_0(k_2^{''}r) = e^{-k_2^{''}x}\int_0^\infty\frac{\cos\sqrt{t^2+2k_2^{''}t}\,(z+H)}{\sqrt{t^2+2k_2^{''}t}}e^{-tx}dt \qquad (42)$$

where K_0 is the modified Hankel function of zero order and $r = \sqrt{x^2+(z+H)^2}$ is the distance from the scatterer. Hence, equation (38) is written as

$$v_{2,1}(x,z) = -\frac{ac}{\pi}\left[\xi(0)K_0(k_2^{''}r) + \psi(0)\int_{-H}^{z}K_0(k_2^{''}s)\,du\right] \qquad (43)$$

where,

$$\xi(0) = \frac{\sqrt{2k_2^{''}}(1+\gamma\theta_1^{'}H)\gamma\theta_1^{'}e^{-i\frac{\pi}{4}}}{a(1+\gamma\theta_1^{'}H) + \mu_2\gamma\theta_1^{'}} \qquad (44)$$

$$\psi(0) = -\frac{\sqrt{2k_2^{''}}(\gamma^2\theta_1^{'2})e^{-i\frac{\pi}{4}}}{a(1+\gamma\theta_1^{'}H) + \mu_2\gamma\theta_1^{'}} \qquad (45)$$

and $\theta_1^{'} = \sqrt{k_2^2 - k_1^2}$, $s = \sqrt{x^2+(t+H)^2}$. (46)

Equation (43) gives the scattered wave due to surface impedance in layer $-H \le z \le 0$. It is clear from the result that if there is no impedance on the surface i.e. a=0, the scattered waves are absent and we have only the incident Love waves.

For finding the reflected component, we evaluate the integral in equation (37) in upper part $\beta > -\tau$ of the complex plane. In order that the integral along the contour at infinity vanishes in the region $x < 0$, the contribution due to the pole at $p = k_{1N}$, is given by

$$v_{2,2} = -A\cos\theta_{2N}(z+H)e^{-ik_{1N}x} \qquad (47)$$

which cancels the incident wave. Now we find the reflected component of Love wave of N[th] mode. So consider the equation

$$L(p) = -\frac{\mu_2\theta_2}{a}$$

$$\frac{\theta_2\cosh\theta_2 H + \gamma\theta_1\sinh\theta_2 H}{\theta_2\sinh\theta_2 H + \gamma\theta_1\cosh\theta_2 H} = -\frac{\mu_2\theta_2}{a} \qquad (48)$$

Let $k_{2N}(N = 1,2,3,.........)$ be the roots of equation (48) which may also be written as

$$\tan\theta'_{2N}\delta = \gamma\frac{\theta'_{1N}}{\theta'_{2N}}, \text{ where } \delta = H - h \qquad (49)$$

and $\theta'_{2N} = \sqrt{k_2^2 - k_{2N}^2}$ and $\theta'_{1N} = \sqrt{k_{2N}^2 - k_1^2}$ (50)

The impeding surface behave as a surface layer and the poles at $p = k_{2N}(N = 1,2,3,.....)$ contributes to

$$v_{2,3}(x,z) = -\frac{a\theta'_{2N}}{\cos\theta'_{2N}\delta}\left[\frac{c(L_+(k_{2N})-1)}{\sqrt{k_2+k_{2N}}} - \frac{A}{i(k_{2N}-k_{1N})}\right] \times \frac{\cos\theta'_{2N}(z+\delta)\,e^{-ik_{2N}x}}{G(k_{2N})} \qquad (51)$$

Equation (51) gives the reflected wave of N[th] mode due to surface impedance in layer $-H < z < 0$. It is also clear from the result that if there is no impedance on the surface i.e. a=0, the reflected waves are absent and we have only the incident Love waves.The numerical computation has been done by considering the value of $k_2\delta$ very small i.e. the width of impeding surface is very small as compared to the wavelength of the wave. For calculation purpose, we have taken $z = -H$, $k_{1N} = k_2 V_1 = 4.6\,km./s$, $V_2 = 3.9\,km./s$, $\mu_1 = 7.98\times10^{10}\,Pa$, $k_2\delta < 0.01$ $\mu_2 = 4.11\times10^{10}\,Pa$. Also for small r, $K_0(k_2''r) \cong \log z - \log r - k$, and for large r, $K_0(k_2''r) \cong \frac{e^{-k_2''r}}{\sqrt{r}}$, which gives the idea of the behavior of the scattered waves at different distances from the scatterer. Figure 3 shows the phase and group velocities of the Love waves in the layered structure while the variation of amplitude versus phase velocity of reflected waves is shown in figure 4.

CONCLUSIONS

The analysis of results shows that the impeding surface affect the propagation of Love waves through the layered structure. The discussion specifies that, it is not only the type of material that affects the propagation but thickness of impeding surface also plays an important part. We have derived the approximate solution for the case that the thickness of impeding surface is small compared with wave-length, leaving the solution for larger thickness in future. The scattered waves have a logarithmic singularity at the tip of scatterer and behave as decaying cylindrical waves at large distances from the scatterer, dying out at very large distances. This fact may be used in predicting the internal structure of earth to some extent by measuring the form of scattered waves at a particular place. The plot of amplitude versus phase velocity of the reflected waves shows that the amplitude decreases as the phase velocity increases but it reduces to zero after a very long time, which explain the reason why Love waves cases large scale destruction during earthquake.

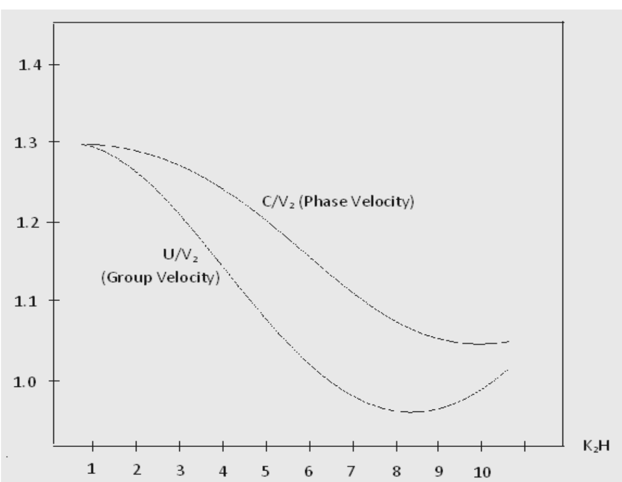

Fig. 3. Group and Phase velocities of Love waves.

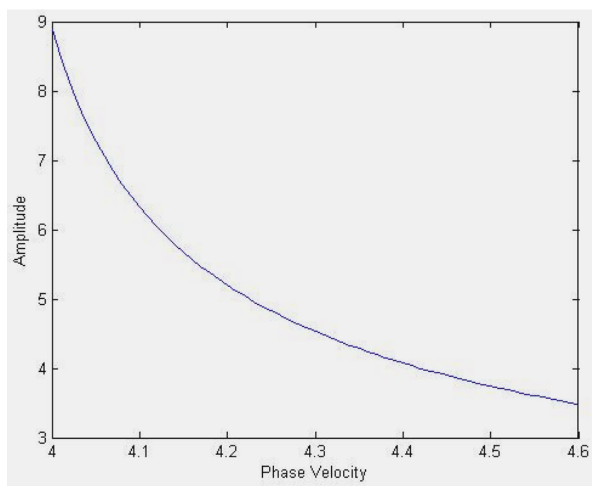

Fig. 4. Amplitude versus Phase velocity.

REFERENCES

Chattopadhyay, A., Gupta, S., Sharma, VK. and Kumari, P. 2009. Propagation of Shear waves in visco- elastic medium at irregular boundaries. Acta Geophysica. 58:195- 214.

Deshwal, PS. and Gogna, ML. 1987. Diffraction of compressional wave in a solid half space by a surface impedance. Acta Geophysica Polonica. 35(2):167-182.

Gregory, RD. 1966. The attenuation of a Rayleigh wave in a half space by surface impedance, Proc. Camb. Phil. Soc. 62:811-827.

Noble, B. 1958. Methods based on the Wiener-Hopf Technique, Pergamon Press.

Saito, T. 2010. Love wave excitation due to the interaction between a propagating Ocean wave and sea bottom topography. Geophy. J. Int. 182:1515-1523.

Sato, R. 1961. Love waves in case the surface layer is variable in thickness, J. Phys. Earth. 9:19-36.

Tomar, SK. and Kaur, J. 2007. SH-waves at a corrugated interface between a dry sandy half-space and an anisotropic elastic half space. Acta Mechanica. 190:1-28.

Received: Jan 3, 2011; Revised: April 15, 2011;
Accepted: April 20, 2011