

PHASE AND GROUP VELOCITIES OF THE SURFACE ELEMENTARY EXCITATIONS ON THE SUPERFLUID HELIUM-II SURFACE AT LOW TEMPERATURES

A A Zakharenko

International Institute of Zakharenko Waves (IIZWs)
660037, Krasnoyarsk-37, 17701, Krasnoyarsk, Russia

ABSTRACT

Both the effective phase velocity and the group velocity of the surface elementary excitations (SEEs), namely quantized capillary waves or ripples are shown in dependence on the wavenumber k . For comparison, the dispersion relation for a free ^4He -atom is also shown. The existence possibilities of three non-dispersive Zakharenko waves are shown. The effective masses of the SEEs are also drawn in dependence on the wavenumber k . Quantum systems in which the non-dispersive Zakharenko waves can be found are also discussed.

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INTRODUCTION

Quantized capillary waves or ripples exist on the liquid helium-II surface at low temperatures similar to ripples, which can exist on a normal liquid surface. Ripples are very interesting to study because they can interact on the surface with electrons, which can form a two-dimensional Wigner crystal on the surface. In addition, ripples can interact with the bulk elementary excitations (phonons, rotons) of liquid helium-II at the liquid surface. It was noted in (Zakharenko, 2001) that energy spectra of surface elementary excitations (SEEs) of the liquid helium-II can qualitatively be described by a polynomial by the same way, by which the energy spectra of the bulk elementary excitations (BEEs) of the liquid helium-II at low temperatures were described. Therefore, the ripplon (SEEs) energy spectra can also consist of three energy zones (the ripplon-phonon, r^- -ripplon-roton and r^+ -ripplon-roton branches, respectively). The name "ripplon-phonon branch" for the first energy zone was introduced because phonons are long wavelength excitations. It is thought that ripples could be experimentally observed even with the same method used to observe the BEEs in the bulk liquid helium-II (for example, by Zn-superconducting bolometers positioned at the liquid helium surface). In many experiments concerning studies of the BEEs and SEEs, a small experimental cell (line sizes ~ 100 mm) with the isotopically pure liquid helium-II (McClintock, 1978) is cooled down to temperatures below 100 mK by the well-known dilution refrigerator techniques described in the excellent and classical books by Betts (1989) and by Lounasmaa (1974).

In this paper, calculations are introduced for both the effective phase velocity c_{ph} and the group velocity c_g of the surface elementary excitations (ripples) using the experimental data by Lauter *et al.* (1991). Dispersion relations for a free ^4He -atom are also given for comparison and the ripplon effective masses are evaluated. It is thought that the studies of this paper can be useful for engineers and physicists working in the field of low temperature physics. Different quantum systems, in which the Cooper pairing phenomenon as a key mechanism of quantum physics can be applied, are also discussed in the section before conclusion.

Existing theory and experimental data

The experimental data in (Lauter *et al.*, 1991; Lauter *et al.*, 1992) for the ripplon energy spectra shows that there is dramatic dependence on the thickness of liquid helium films. In the experimental work by Lauter *et al.* (1991) the measurements were done for several coverages from ~ 7 layers to 3.5 layers. Lurio *et al.* (1992) measured the liquid helium density profile at low temperatures and their results allow one to conclude that ripples are localized at the liquid surface within a slab of $\sim 7 \text{ \AA}$ to 9 \AA .

The ripplon dispersion relation is written by the well-known formula for the case of short wavelengths while neglecting the effects of gravity (Landau and Lifshits, 1987):

$$\omega^2 = \frac{\alpha}{\rho} k^3 \quad (1)$$

where ω and ρ are the angular frequency and density,

Corresponding author email: aazaaz@inbox.ru

The present paper is written in memory of Academician L D Landau.

respectively. In equation (1), α denotes the surface tension of the liquid helium surface at low temperatures. The surface tension is constant at temperatures below 100 mK. The value of α was measured by several research groups (Iino *et al.*, 1985; Nakanishi and Suzuki, 1998; Roche *et al.*, 1997; Zinov'eva and Boldarev, 1969; Eckard *et al.*, 1977; Guo *et al.*, 1971). Therefore there are several different values of α . However, they are relatively close to each other. Relation (1) was obtained by Atkins (1953). Therefore, the ripplon energy $E_r = \omega\hbar$ depends on the wavenumber k , according to equation (1) as $k^{3/2}$.

Because the normalized layer thickness kd of ripples can equal to ~ 1 (for example, for the wavenumbers $k < 1\text{Å}^{-1}$) and values of the layer thickness d are smaller than the maximum value of $d_m \sim 9\text{Å}$, it is possible to write the dispersion relation like that for the capillary gravity waves of a normal liquid (Landau and Lifshits, 1987) accounting dependence on the kd :

$$\omega^2 = (g_r k + \alpha k^3 / \rho) \tanh(kd) \quad (2)$$

Indeed, dispersion relation (2) can be used as a first approximation for suitable small values of the kd . In equation (2), g_r is the ripplon gravity factor and has acceleration dimension c_0^2/d , and $g_r\{c_0 \sim 100\text{ms}^{-1}$ (Lauter *et al.*, 1991), $d \sim 10\text{Å}$ (Lounasmaa, 1974)} $\sim 10^{13}\text{ms}^{-2}$. Therefore,

$$c_{ph} = c_g = c_0 = (g_r d)^{1/2} \quad (3)$$

where c_0 is the velocity in the normal, unexcited state for the case of $k \rightarrow 0$ ("pure gravity waves"). Also, if g_r is equal to $g \sim 10\text{ms}^{-2}$, the velocity c_0 reduces to $\sim 10^{-4}\text{ms}^{-1}$. The ripplon gravity factor g_r is also taken in different forms for comparison with experiments, for example, see (Roche *et al.*, 1997; Williams *et al.*, 1998; Roche *et al.*, 1995) incorporating Van der Waals and electrostrictive forces. Formula (2) shows that for normal liquids there is only case (3). On the other hand, for the quantum liquid helium-II several such points can exist at which the group velocity c_g is equal to the effective phase velocity c_{ph} . These points relate to the equality $c_{ph} = c_g$ and will be shown and discussed below.

Also, it is necessary to write that due to the fact that the liquid helium is a quantum liquid consisting of helium atoms, a thin film of the liquid is therefore a quantum-liquid thin film, consisting of helium atoms as well. Therefore, it is natural that the SEE energy spectrum has one ripplon-roton minimum that was initially measured in (Lauter *et al.*, 1991) and recently confirmed in (Lauter *et al.*, 2002) in which it was called "layered roton." The experimental data by Lauter *et al.* (1991) allows one to conclude that there is a possibility to observe the surface second sound and the author completely agrees with the conclusion in (Lauter *et al.*, 1991). The velocity V_2 of the bulk second sound in the bulk liquid helium-II was initially measured by Peshkov (1946) with a precise

thermometer. The velocity V_2 equals to $\sim 140\text{ms}^{-1}$ at low temperatures. Similar measurements of the velocity of the surface second sound today are not carried out possibly due to difficulties in measuring it because the SEEs propagate in thin films of the liquid within a slab of $\sim 10\text{Å}$. Perhaps this can be done with a thermometer-sensor of dimensions $\sim 2-3\text{Å}$ or less. It is thought that this is feasible when the helium atoms or electrons positioned on the liquid surface are used for the sensors. On the other hand, the velocity of the surface second sound could be $(3^{1/2})$ -times smaller than that of the surface first sound, like those for the bulk sounds. Therefore, in both surface and bulk cases, an energy leak can occur between the corresponding first and second sounds that can give a possibility to indirectly observe the surface second sound.

It is possible to note that expression (1) is a first approximation to describe ripplon dispersion relation that is true for shorter wavelengths in the first energy zone. Expression (2) is a second approximation for this purpose because dependence on the layer thickness is taken into account. However, it is also indispensable to account the following facts:

- the liquid helium is a quantum liquid with the roton minimum;
- there is an influence of different "clean-dirty" substrates, which can shift position of both the maxon maximum and the roton minimum in dependence on slab's thickness of the liquid helium.

Because the ripplon energy spectra are qualitatively similar to the energy spectra of the BEEs (there are both the ripplon-maxon maximum and the ripplon-roton minimum) the effective phase velocity c_{ph} of the SEEs is treated instead of the SEE phase velocity $v_{ph} = \omega/k$. This was initially done in (Zakharenko, 2005a) for the energy spectra of the BEEs. It is thought that to treat the phase velocity of the BEEs in that case is "not relevant," according to the results by Stirling (1985). This can be true and also applicable for the SEEs.

Dispersion relations

The experimental data used for this treatment relates to 3.85 helium atom layers in (Lauter *et al.*, 1991). The ripplon energy dependence on the wavenumber k is shown in figure 1. As it is shown in the figure, the ripplon energy spectra consist of three energy zones (the three branches, which correspond to three types of the SEEs similar to the BEEs). Unfortunately, the experimental data by Lauter *et al.* (1991) did not provide us with enough points for the third energy zone (the r^+ - ripplon-roton branch) and therefore, it is only possible to discuss what occurs in this energy zone. In each energy zone there exist dispersive waves ("dispersive SEEs") for which the effective phase velocity c_{ph} is unequal to the group velocity c_g of the dispersive waves ($c_{ph} \neq c_g$). This

dispersion relation, namely the coupling of the velocities c_{ph} and c_g can be written as dependence of the c_g on the velocity c_{ph} , wavenumber k , and first derivative dc_{ph}/dk with respect to the k in the following form (k_Δ is a constant):

$$c_g = c_{ph} + (k - k_\Delta) \frac{dc_{ph}}{d(k - k_\Delta)} \quad (4)$$

where the c_{ph} can be defined as follows:

$$c_{ph} = \text{Abs} \left[\frac{\eta}{2\mu_r^*} (k - k_\Delta) \right] \quad (5)$$

and $\hbar = h/2\pi = 1.054571628(53) \times 10^{-34}$ [J×s] is the reduced Planck constant, and $h = 6.62606896(33) \times 10^{-34}$ [J×s] is the Planck constant. The reduced Planck constant, also known as the Dirac constant, differs from the Planck constant by a factor of 2π .

In equation (5), μ_r^* is the effective mass of the SEEs. Values of k_Δ are different for each energy zone, for example, $k_\Delta = 0$ for $k \rightarrow 0$ and $k_\Delta = k_{\Delta m} \approx 1.325 \text{ \AA}^{-1}$ for the ripplon-maxon region with energy $E_{\Delta m} \approx 8 \text{ K}$, but $k_\Delta = k_{\Delta r} \approx 1.725 \text{ \AA}^{-1}$ around the ripplon-roton minimum with energy $E_{\Delta r} \approx 6.3 \text{ K}$. It is clearly seen in equations (4) and (5) that both the c_g and the c_{ph} approach zero at $k = k_\Delta$. That occurs at both the ripplon-maxon maximum $k_\Delta = k_{\Delta m}$ and the ripplon-roton minimum $k_\Delta = k_{\Delta r}$. Also, it was shown in (Zakharenko, 2005b) that if the c_{ph} decreases, the values of the c_g must be smaller than the values of the c_{ph} . On the other hand, if the c_{ph} increases, the values of the c_g must be larger than those of the c_{ph} . Therefore, there are points in the ripplon energy zones, at which the non-dispersive Zakharenko waves ($c_{ph} = c_g \neq 0$) can exist. This condition of $c_{ph} = c_g \neq 0$ occurs at extreme points of the c_{ph} that is seen in equation (4) and was also shown in (Zakharenko, 2005a) for the BEEs. Indeed, the c_{ph} and the c_g are coupled, and the first derivative dc_{ph}/dk with respect to the wavenumber k becomes zero.

Therefore, each energy zone consists of two modes of different dispersive waves ("dispersive SEEs") beginning with the corresponding non-dispersive Zakharenko wave and becoming zero at both the ripplon-maxon maximum and the ripplon-roton minimum (see Fig. 1). The first mode in the energy zone 1 begins at $k = 0$ with the ripplon velocity $c_0 \sim 95 \text{ ms}^{-1}$ that corresponds to the normal, unexcited state of the helium atomic layers. The surface phase velocity c_0 depends on the layer thickness d in (3) and runs from $\sim 90 \text{ ms}^{-1}$ to $\sim 110 \text{ ms}^{-1}$, according to the experimental data in (Lauter *et al.*, 1991). For the treated 3.85 layers, the first non-dispersive Zakharenko wave must be in the first energy zone due to the possible backflow in the ripplon-phonon region, similar to the phonon-backflow of the BEEs in the bulk liquid helium-II at low temperatures that was measured by Stirling (1985).

Note that the theoretical results can be improved as soon as improved high-resolution neutron-scattering experimental data is available.

For comparison, both the group velocity V_g^{at} and the phase velocity V_{ph}^{at} of a free ^4He -atom are shown in dependence on the wavenumber k by two straight lines in figure 1. From this point of view, a free ^4He -atom as a quasi-particle represents dispersive wave with the constant relationship $V_g^{at} = 2V_{ph}^{at}$:

$$V_g^{at} = \frac{\eta k}{m_4} \quad \text{and} \quad V_{ph}^{at} = \frac{\eta k}{2m_4} \quad (6)$$

The atom phase velocity V_{ph}^{at} crosses the effective phase velocity c_{ph} of the SEEs, probably only in the first energy zone (see Fig. 1). This can mean that only such SEEs can be experimentally excited at the liquid-helium surface by the helium-atom beams because an energy leak can occur at the crossing point of the phase velocities between two different types of oscillations (see the first energy zone in Fig. 1). This is like the situation, which occurs in solid layered systems (Lardat *et al.*, 1971) where two different types of oscillations can exist and an energy leak, for example, between Love type waves with polarization perpendicular to the sagittal plane and dispersive Rayleigh type waves with polarization in the sagittal plane can be at crossing points of the phase velocities of the dispersive waves.

It is possible to discuss the possibilities for the other cases, when electrons or the other atoms listed in table 1 (which can form cryocrystals on the liquid surface) are treated instead of the helium atoms. It is obvious that if a free quasi-particle has its own mass, being larger than the helium atom mass, the linear behavior of the phase velocity of such free quasi-particle on the wavenumber k will show a possibility to excite either one SEE or two in each energy zone. Simply, for a quasi-particle being heavier than ^4He -atom, a linear dependence of the phase velocity will lie below that of a ^4He -atom shown in figure 1. Therefore, in the second and third energy zones, an energy leak can occur at possible crossing points between a heavy quasi-particle in table 1 and the corresponding SEEs. From this point of view, for a free electron with mass being several orders smaller than the helium atom mass, there is the possibility to excite the SEEs only in the first energy zone with $c_{ph} \approx c_0$. This can mean that a free electron coming to the liquid surface can excite SEEs with the velocity $\sim c_0$ (the velocity of the normal, unexcited state of the liquid helium layers). Such elementary excitations will negligibly change the phase velocity c_0 of the SEEs on the liquid surface. However, electrons can form a two-dimensional Wigner crystal on the liquid surface that makes possible to change the dispersion relations in the second and third energy zones of the SEE energy spectra. Also, the Wigner

crystallization of electrons is extensively studied in solids (Iordanskii and Muzykantskii, 1991; Piel *et al.*, 2002; Rontani *et al.*, 2005; Lenac, 2005) and it is thought that the electron crystallization study can be applied to cuprates.

Table 1. The masses of some free quasi-particles. The mass of a free electron is 0.911×10^{-30} [kg].

Element	a.e.m.	Atom mass, 10^{-27} [kg]
He ²	4.0026	6.667
Kr ³⁶	83.80	139.583
Xe ⁵⁴	131.30	218.702

The effective masses μ_r^* of the SEEs in dependence on the wavenumber k are shown in figure 2. The ripplon effective mass μ_r^* increases to its maximum value at the ripplon-maxon maximum, from which it starts to decrease to its minimum value at the ripplon-roton minimum. It is seen in figure 2 that the maximum ripplon effective mass is smaller than the atom mass of a free ⁴He-atom, $m_4 = 6.667 \times 10^{-27}$ kg, and that the minimum mass is approximately one order smaller than the maximum mass. Probably, the maximum ripplon effective mass approaches the mass m_4 , or even is equal to the m_4 . It is thought that improved high-resolution neutron-scattering experimental data is required for that purpose. The crossing point between two phase velocities in the first energy zone can represent the binding energy $E_B = 7.15$ K by Wilks (1967).

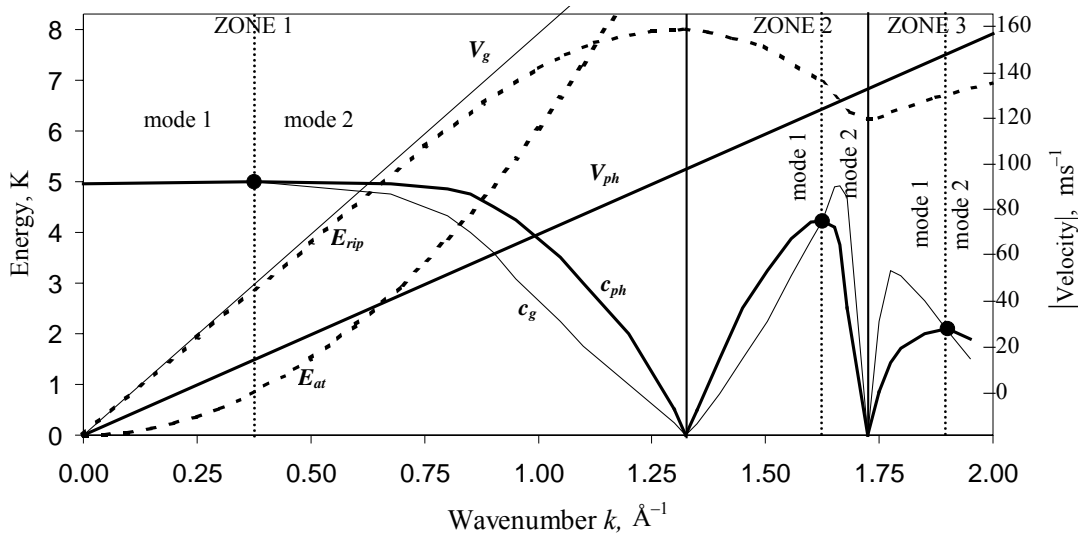


Fig. 1. Both the ripplon energy E_{rip} and the energy E_{at} of a free ⁴He-atom shown by the corresponding point lines. The phase velocity V_{ph} and the group velocity V_g of the helium atom are shown by two straight lines. Absolute values of both the effective phase velocity c_{ph} and the group velocity c_g of ripples are shown by bold and simple solid lines, respectively. Three energy zones with corresponding two modes of the dispersive waves are introduced. Three big points show the corresponding non-dispersive Zakharenko waves.

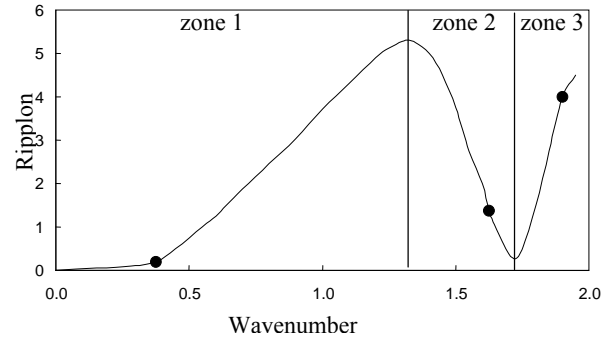


Fig. 2. The ripplon effective mass $\mu_r^* = |(k - k_\Delta)\hbar/c_g|$, 10^{-27} [kg] in dependence on the wavenumber k . Three energy zones are shown. The ripplon effective masses shown by points correspond to the non-dispersive Zakharenko waves.

Discussions about different quantum systems

In addition to the superfluidity phenomenon in the liquid helium-II possessing a nucleus, consisting of two protons and two neutrons, the same phenomenon can be found in liquid ³He, the rarer isotope, which has a nucleus of two protons and one neutron. It is noted that due to measurement difficulties a superfluid ³He was finally observed only in 1972 at a temperature below 3mK. Note that there is difference between ³He and ⁴He masses and between the nuclear spins. The ³He spin is + 1/2 instead of zero, making ³He a fermion. It is thought that the mass difference makes the ³He-boiling point 3.2K instead of 4.2K for ⁴He due to the larger zero-point energy for the

lighter atom. The ^3He atoms associated in the Cooper pairs with their nuclear spins aligned in order to give spin + 1 and the resulting bosons could associate into a superfluid. It is also thought that the superfluidity of liquid ^3He is caused by spin fluctuations. Also it is interesting that there is an estimate based on the assumption of a Van der Waals attraction mechanism for ^3He leading to a much lower estimate for the critical temperature ($\sim 10^{-6}$ K instead of $\sim 10^{-3}$ K).

The superfluidity phenomenon is also akin to the superconductivity phenomenon in solids, according to the results by Landau (1941), where electrons can be also treated as an electron liquid. At the present time it is well-established that superconductivity is a consequence of formation of the Cooper pairs. The latter takes place, if the attractive forces between electrons exceed the Coulomb repulsion. In spite of many other suggestions, only two mechanisms of such attraction are definitely established: namely the exchange of phonons and of spin fluctuations. The latter mechanism is the cause of superconductivity in some heavy-fermion metals, e.g. UPt_3 . It is noted that the BCS formula describing superconductivity represents only an isotropic model. There is a recent suggestion that there is no theory for (high temperature) superconductivity, except the Cooper pairs, in order to predict existence of room temperature superconductivity (RTSC). As a result, the multi-promising RTSC has yet to be discovered and within the last two decades the high temperature superconductivity (HTSC) was discovered. It is noted that there is still a great interest in the HTSC in cuprates. Recently, in the discussion about the HTSC mechanism in cuprates, a lot of passion was involved, which led to suppression of references to “unpleasant” experimental data, referring to all fashionable, even mutually contradicting theories and other violations of scientific ethics. This resulted in the appearance of a legend about an “unsolvable mystery of the HTSC.” However, Abrikosov believes that there is a suitable theory (Abrikosov, 1999; Abrikosov, 2000) based not on abstract models, but on the peculiar properties of the electron energy spectrum in real substances. First of all, a theory must include the quasi-two-dimensionality of electrons in the CuO_2 planes and the “extended saddle point singularities in electron energy spectra, weak screening of the Coulomb interaction and phonon-mediated interaction between electrons plus a spin-fluctuation” near the Brillouin zone boundaries.

Indeed, the common feature for all quantum systems is that they possess Brillouin energy zones (energy branches) and at boundaries of each zone there is the condition for both the phase and group velocities such as $C_g \sim C_{ph} \rightarrow 0$, except the possible case at the wavenumber $k = 0$ with $C_g \sim C_{ph} \sim C_0$ representing naturally unexcited normal state. This is true for electron energy branches in solids and the energy branches of BEEs and SEEs in the

liquid helium-II, as well as for superfluid ^3He . Note that there are also the well-known acoustic and optic branches in solids. It is thought that there is one corresponding non-dispersive Zakharenko wave in each energy branch (in each Brillouin zone) like that shown in this paper for the SEE energy branches and initially in (Zakharenko, 2005a) for the BEE energy branches. Also, the Cooper pairing phenomenon for the suitable BEEs (positive rotons) in the liquid helium-II is readily verified by the quantum evaporation and condensation phenomena (Zakharenko, 2007a, b). However, it is thought that the same verification for the suitable SEEs can be done using cryocrystals at the liquid surface. Clements *et al.* (1994a) and Clements *et al.* (1994b) studied structure and growth of quantum films and calculated the elementary excitation spectrum of mono- and double-layer ^4He -films. They also included time-dependent short-range correlations of the liquid helium that can be essential to some quantitative predictions. Also, Szybisz (1996) theoretically investigated elementary excitations in rather thick symmetric films of liquid ^4He at $T=0$ K. Elementary excitations in (Szybisz, 1996) can be described by continuum states obtained by solving a Bogoliubov-type equation formulated within the framework of the paired-phonon analysis and the hypernetted-chain approximation. The lowest-lying branch of the states in (Szybisz, 1996) is always bound and describes surface ripplon excitations for wavenumbers smaller than the wavenumber q_r at the roton minimum, $q_r \sim 1.9 \text{ \AA}^{-1}$. It is also worth noting that several postulates concerning existence of corresponding non-dispersive Zakharenko wave in a quantum energy branch (mode of dispersive wave) were recently introduced in (Zakharenko, 2010).

The other well-known examples of the Cooper pairing in quantum systems are as follows:

- polariton-exciton quantum systems (Saba *et al.*, 2003; Eastham and Littlewood, 2001; Keldysh, 1995; Saba *et al.*, 2001) where electrons and holes are treated as free particles, but above a certain threshold density undergo a “BCS-like” phase transition, as the Cooper pairing of one electron and one hole is energetically favored;
- Josephson-junction devices consisting of ferromagnetic (or insulator) and superconducting layers in sandwiched structures (Zheng and Xing, 2009; Cakir and Kulik, 2003; Ota *et al.*, 2009) where Cooper pairs can tunnel through an energetic barrier in a non-dissipative manner.

CONCLUSIONS

The experimental data of the SEE energy spectra shows that there is dependence on the thickness of helium atom layers and the SEE energy spectra are qualitatively similar to the energy spectra of the BEEs in the bulk liquid

helium-II, because there are both the ripplon-maxon maximum and the ripplon-roton minimum. Therefore, the SEE energy spectra consist of several energy zones, and within each energy zone there are two modes of dispersive waves (“dispersive SEEs”) with different dispersions ($c_g < c_{ph}$ or $c_g > c_{ph}$). It is natural that three energy zones were shown in this paper. The aforementioned two modes are confined between the effective phase velocity for the corresponding non-dispersive Zakharenko waves ($c_g = c_{ph} \neq 0$) and the effective phase velocity ($c_g = c_{ph} = 0$) at the corresponding boundaries (the ripplon-maxon maximum and ripplon-roton minimum) between two neighbor energy zones, which represent the boundaries of the Brillouin zones. In the first energy zone at $k = 0$, the phase velocity c_0 is equal to $\sim 95 \text{ ms}^{-1}$ for the treated liquid thin film. The ripplon effective mass μ_r^* is also evaluated, which increases from zero at $k = 0$ to its maximum value at the ripplon-maxon maximum approaching the mass of a free ^4He -atom, $m_4 = 6.667 \times 10^{-27} \text{ kg}$. After that, the mass μ_r^* decreases to its minimum value at the ripplon-roton minimum. Also, the previous section addressed the discussions about the Cooper pairing phenomenon in different quantum systems.

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