SENRA Academic Publishers, Burnaby, British Columbia
Vol. 5, No. 2, pp. 1567-1571, June 2011
Online ISSN: 1920-3853; Print ISSN: 1715-9997

Canadian Journal of pure&applied sciences

THE SOLUTION OF MATHEMATICAL MODEL FOR TRANSPORT OF OXYGEN IN PERIPHERAL NERVE WITH THE FIRST-ORDER CHEMICAL KINETICS USING FINITE-DIFFERENCE TECHNIQUE DURING PRANAYAMA

*Sarita Singh[1], V K Katiyar[2] and P Pradhan[1]
[1]Gurukul Kangri Univerisity, Haridwar, Uttrakhand
[2]Department of Mathematics, Indian Institute of Technology Roorkee, India

ABSTRACT

We modeled time dependent transport of oxygen in peripheral nerve by the simulation of steady-state oxygen tension, diffusion, chemical reactions and consumption of oxygen in capillaries using Krogh cylinder symmetry. These parameters were assumed to change rapidly to new. To characterize the approach of the oxygen tension to a new value, a monoexponential function is defined. Diffusion of oxygen is radial from capillary to a surrounding cylinder tissue and from arterial distance to veins, diffusion is axial. The time-dependent transport of oxygen is peripheral nerve with forward and backward reactions including first-order chemical kinetics has been considered, which makes this model different from the earlier studied models. We used Finite difference technique for the solution of this model.

Keywords: Nerve oxygen consumption, peripheral nerve, oxygen transport, tissue, forward and backward reaction, first order chemical kinetics.

INTRODUCTION

Through a physical solution in water oxygen is transported by the blood partly with hemoglobin. This transport is affected by forward and backward reactions. Kreutzer (1982) shows a comparison of oxygen consumption following zero-order, first -order or Michaclies-Menten kinetics in a plot of the peripheral tissue oxygen pressure against capillary length shows that kinetics of zero-order provides the lowest values but Michaclies-Menten and first-order kinetics having higher value of oxygen pressure both in capillary as well as in tissue. Krogh (1919) and Reneau *et al*. (1967) formulated mathematical models of the release of oxygen from hemoglobin and diffusion from capillaries into surrounding tissue in various cases. After this Reneau *et al*. (1969) considered time-dependent aspects of transport of oxygen.

Low *et al*. (1986) established that peripheral nerve tissue could resist moderate degrees of ischemia or hypoxemia and continue to conduct impulses for minutes to hours. This is not true for brain nerve. The mechanisms of the relative resistance of a peripheral nerve to ischemia are not completely known. The role of hemoglobin and myoglobin is facilitating oxygen transport to tissue. A totally different approach to the mathematical study of oxygen transport to tissue intended to describe on large scale, convection and diffusion of oxygen occurs over a very large distance within the tissue. Diffusion is the process whereby particles of liquids, gases, and solids blend together as the result of their spontaneous movement caused by thermal campaigning and dissolved substance moves from higher to lower concentration. Similarly, in the human body, transport of oxygen occurs. Lagerlund and Low (1993) studied transport of oxygen and diffusion process in peripheral nerve is steady-state. A non steady state condition which effects a sudden change in arterial oxygen tension blood flow velocity or nerve oxygen consumption rate on the distribution of oxygen tension in endoneurial tissue around a capillary. Sharan *et al*. (1997) examined the transport of oxygen in the blood flowing through the systemic capillaries, the blood has been considered as a homogenous model for transport of oxygen in the capillary and surrounding tissue.

Sharma *et al*. (2004) investigated endoneurial oxygen transport in capillary and a surrounding Krogh cylinder of tissue with forward and reverse chemical reaction. A sudden change in arterial oxygen tension affects blood flow velocity and nerve oxygen consumption rate with forward and backward reactions.

The objective of the present study was to investigate endoneurial transport of oxygen in capillary and a surrounding cylinder of tissue with generation or degeneration in oxygen due to forward and backward chemical reaction in capillary, but a first-order chemical kinetics for tissue.

MATHEMATICAL MODEL

We used a modified version of the mathematical model of Reneau *et al*. (1969) for the calculation of endoneurial

*Corresponding author email: saritamath@gmail.com

oxygen profiles as a function of time by numerical solution of differential equations which describe the oxygen transport in a capillary and a surrounding cylinder of tissue(the Krogh cylinder). In the capillary region, transport of oxygen takes place both by convection and by diffusion, and oxygen is generated due to its dissociation in the hemoglobin inside the red blood cell and its transport to the blood plasma across the cell membrane, there is only diffusion of oxygen in tissue region by the tissue cells. We assume that P(r, x, t) be the partial pressure of oxygen. r is the radius of capillary v(r, t) the velocity of blood in the fully developed flow in the capillary. To calculate the partial pressure P in the capillary the differential equation is:

$$\frac{\partial P}{\partial t} = D_b\left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial x^2}\right) - vE\left(\frac{\partial P}{\partial x}\right) - k_f P + k_r P \quad (1)$$

Where $$E = 1 + \left(\frac{C_b \eta}{S_b P_{50}}\right)\frac{(P/P_{50})^{\eta-1}}{\left[1+(P/P_{50})^{\eta}\right]^2} \quad (2)$$

Where k_f and k_r are the rate of forward and backward reaction due to erythrocyte boundaries. x is the distance along the capillary from the arterial end i.e. axial coordinate, r is the distance from the capillary centre i.e. radial distance, t is time, D_b is the oxygen diffusivity in blood, v is the velocity of flowing blood, S_b is the solubility of oxygen in blood, C_b is the oxygen content of blood, η is the Hill's coefficient for the hemoglobin dissociation curve, P_{50} is the oxygen tension for 50% hemoglobin saturation.

The model for the partial pressure P'(r, x, t) in the tissue, are given by partial differential equation:

$$\frac{\partial P'}{\partial t} = D_t\left(\frac{\partial^2 P'}{\partial r^2} + \frac{1}{r}\frac{\partial P'}{\partial r} + \frac{\partial^2 P'}{\partial x^2}\right) - K\left(\frac{C}{S_t}\right) \quad (3)$$

The terms including second partial derivative with respect to x and with respect to r, are the axial & radial diffusion of oxygen from capillary into surrounding tissue. D_t the diffusivity of oxygen in tissue [Michael's-Menten], S_t Oxygen solubility in tissue, and C is the consumption rate of tissue oxygen and given as:

$$C = C_{\max}\frac{P}{P + C_{50}} \quad (4)$$

C_{max} is the value of consumption rate at very high tension; C_{50} is a constant representing the oxygen tension at which consumption decreases to 50% of its maximal value. Equation (1) and (3) are solved simultaneously following by the boundary conditions:

$$\frac{\partial P}{\partial x} = 0 \qquad r_c \le r \le r_t \ , \quad x = 0 \quad (5)$$

$$\frac{\partial P}{\partial x} = S(r) \quad 0 \le r \le r_c \qquad x = l \quad (6)$$

$$\frac{\partial P}{\partial r} = 0 \qquad \text{for } r = 0 \quad \text{and all } x \quad (7)$$

$$P = P' \quad \text{for } r = r_c \quad \text{and all } x \quad (8)$$

$$\frac{\partial P'}{\partial x} = S(r) \quad \text{for} \quad r_c \le r \le r_t \ , \ x = l \quad (9)$$

$$\left(D_b S_b \frac{\partial P}{\partial r}\right)_{capillary} = \left(D_t S_t \frac{\partial P'}{\partial r}\right)_{tissue} \quad \text{for } r = r_c$$

and all x (10)

$$\frac{\partial P'}{\partial r} = 0 \qquad \text{for } r = r_t \quad \text{and all } x, \quad (11)$$

$$P = P_0(r,x) \quad \text{for } t = 0 \quad \text{and} \quad \forall r, x \quad (12)$$

$$P = P_a - \left(\frac{C_{\max}}{4D_b S_b}\right)\left(\frac{r_t^2}{r_c^2} - 1\right)\left(r^2 - \frac{r_c^2}{2}\right) \quad 0 \le r \le r_c \quad x = 0 \quad (13)$$

Here P_a is the arterial oxygen tension, r_c capillary radius, r_t radius of Krogh tissue cylinder (one-half of intercapillary distance), l is the capillary length. Equation (5) states that there is no axial diffusion into tissue cylinder at the arterial end, Equation (6) specifies the rate at which oxygen diffuse axially out of the venous end of the capillary as a function S_r. Equation (7) states that there is no radial diffusion at $r = 0$ (from the capillary centre). Equation (8) is the continuity of tensions at the capillary wall. Equation (9) uses a non- zero value for axial diffusion of oxygen out of the venous end of the tissue cylinder because this seems to represent better the actual situation than would a value of zero. Here the axial derivative of P' in tissue is arbitrary taken to be equal to its value at the capillary wall by Reneau *et al*. (1967). Equation (10) guarantees that the rate of diffusion out of the capillary equal to the rate of diffusion into the tissue at the capillary wall. Equation (11) is for the fact that there is no radial diffusion of oxygen out of tissue cylinder. Equation (12) specifies the initial oxygen tension P (at time t=0at each point is equal to the steady-state oxygen tension $P_0(r,x)$, calculated as a solution of equation (1) and (3) at the initial values of parameters ($P_a = P_i, v = v_i$ and $C_{\max} = C_v$) with time derivative set at zero i.e. totally steady-state condition. After time t=0 these parameters are as follows:

$$P_a = P_f + (P_i - P_f)e^{-t/\tau_p}$$

$$v = v_f + (v_i - v_f)e^{-t/\tau_v}$$

$$C_{\max} = C_f + (C_i - C_t)e^{-t/\tau_c}$$

Where P_i, v_i, C_i are initial values; P_f, v_f, C_f are final values; τ_p, τ_v, τ_c are time constant that determine the rate of change of P_a, v and $C_{\max}$. Equation (13) is for the

radial dependence of capillary oxygen tension at the arterial end. Here the radial diffusion outward from the capillary centre. In this study we solve equation (1) and (3) analytically as well as numerically, only numerical solution we considered for the first-order chemical kinetics to calculate the oxygen tension in tissue. There is a good agreement between previous results and ours. For the numerical solution of equation (1) and (3) we use finite difference technique.

RESULTS AND DISCUSSION

Figure 1 shows oxygen tension time profiles at point near the arterial end of the capillary when the arterial oxygen tension changes from normal to 50% of normal with time interval 0.1 second.

Similarly figure 2 shows oxygen tension time profiles at maximum distance from the arterial end, it is clear from the figure 2 that oxygen tension changes sharply at points

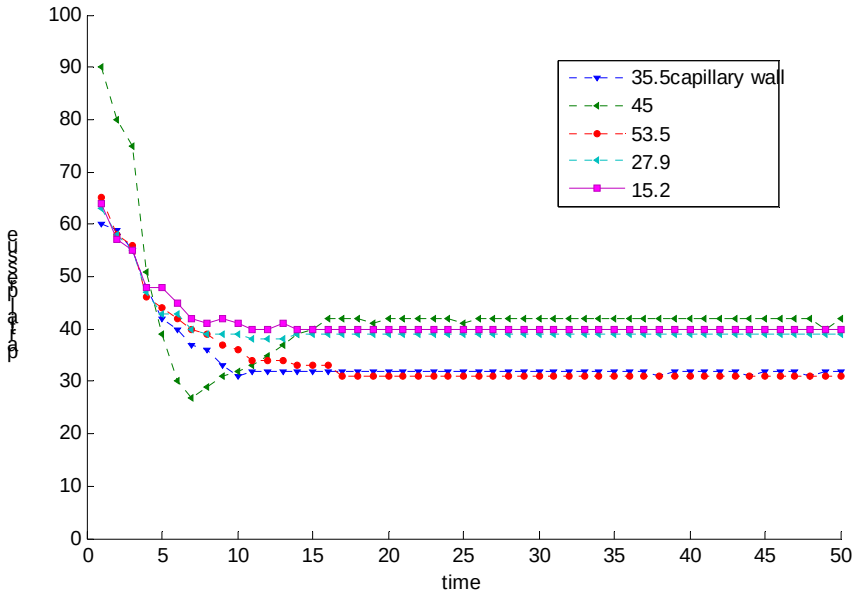

Fig. 1. Time profiles of tissue oxygen tension at arterial end of capillary for various radial distances from capillary centre for 50% of oxygen tension with time constant 0.1 second.

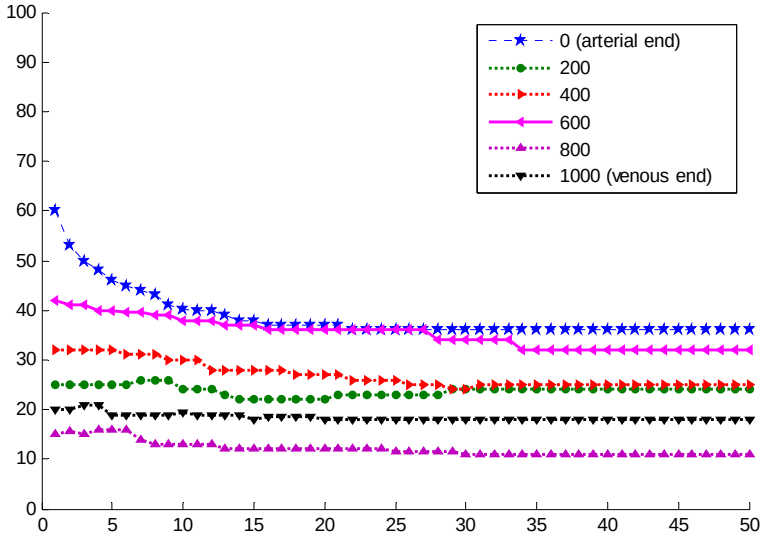

Fig. 2. Time profile at the maximum distance from capillary for various axial distances for 50% of oxygen tension with time constant 0.1 second.

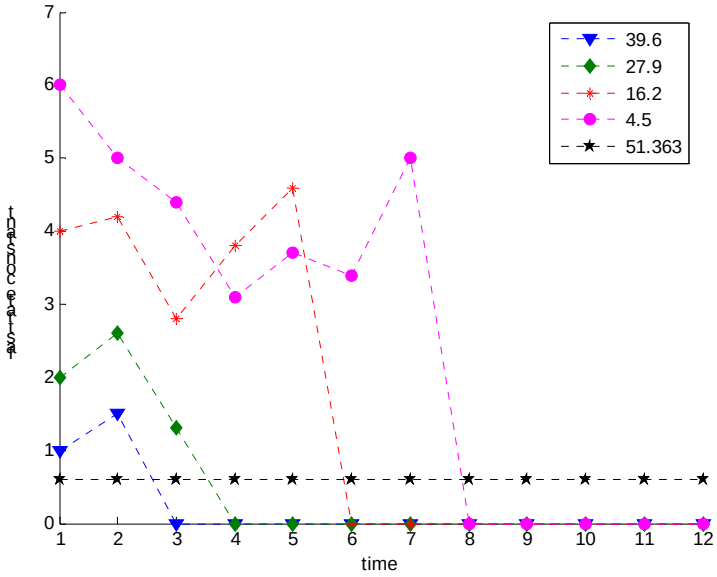

Fig. 3. Axial profiles of fast rate constant for various capillary centre when arterial oxygen tension changes from normal to 50% with time constant 0.1 second.

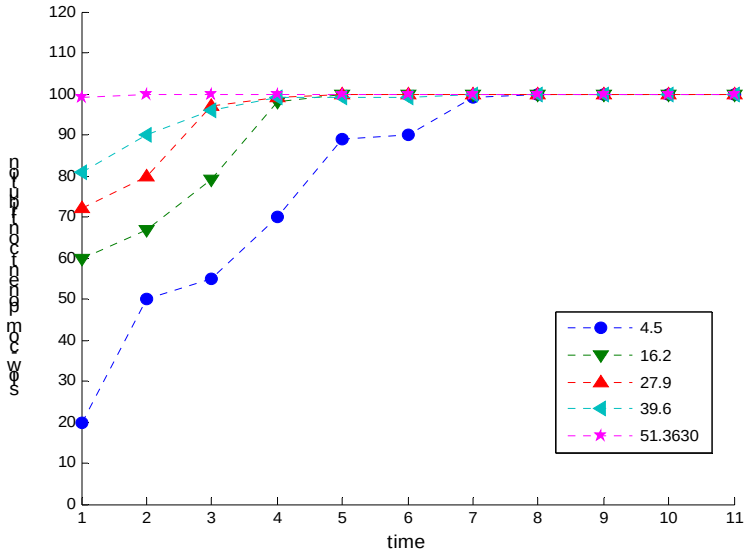

Fig. 4. Axial profile of slow-component contribution when arterial oxygen tension changes from normal to 50% with time constant 0.1 second.

near the capillary and near arterial end. A single exponential function is used to express the time course of changing the oxygen at all locations excepting points near arterial end and near the capillary end, for these excluding points, a biexponential function is used.

There is a small variation in figure 1 and figure 2, due to the first order chemical kinetics. If we take k=1 then pervious results occurs smoothly. This changed reaction gives good results numerically. Figure 3 shows an axial profile of fast rate constant for various capillary centres when arterial oxygen tension changes from normal to 50% with time constant 0.1 second. Here fast rate constant decreases with increasing radial and axial distance up to an abrupt transition point at which the oxygen-tension profiles become monoexponential.

From figure 4 and figure 5, it is clear that an increase in the arterial oxygen tension rate of change increases the fast rate constant but has little effect on either the slow

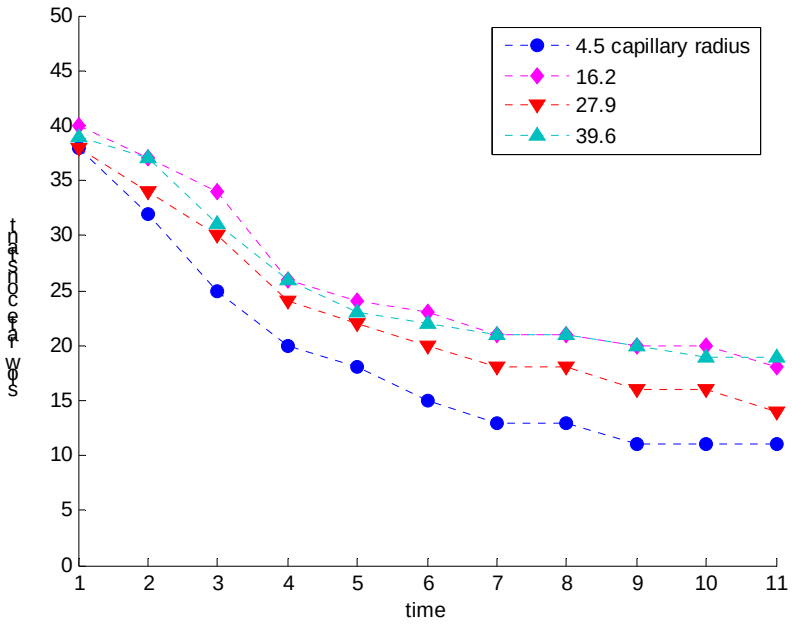

Fig. 5. Axial profile of slow rate contribution when arterial oxygen tension changes from normal to 50% with time constant 0.1 second.

rate constant or the slow-component contribution. As soon as flow velocity increases slow rate and fast rate constant decreases.

CONCLUSION

A Solution of Mathematical Model for Transport of Oxygen in Peripheral nerve with first-order chemical kinetics using finite-difference technique during Pranayama is obtained in this study. The transport of oxygen in peripheral nerve with forward and backward chemical reactions is time-dependent, analyzed using finite difference technique. The chemical reactions as well as new factor (K) (in first order chemical kinetics) were observed to affect the time profiles of tissue oxygen tension in the body with a time step 0.1 second.

REFERENCES

Kreuzer, F. 1982. Oxygen supply to tissue: The Krogh model and its assumptions. Experientia. Birkhauser verlag, CH-4010 Basel/Switzerland. 38:1415-1426.

Krogh, A. 1919. The number and Distribution of capillaries in muscles with calculations of the oxygen pressure head necessary for supplying the tissue. Journal of Physiology. 52(6):409-415.

Low, PA., Schemlzer, JD. and Ward, KK. 1986. The effect of age on energy metabolism and Resistance to ischaemic conduction failure in rat peripheral nerve. Journal of Physiology. 374:263-271.

Low, PA., Ward, K., Schmerzer JD. and Brimijon, S. 1986. Ischemic conduction failure and energy metabolism in experimental diabetic neuropathy. American Journal of Physiology. 248:E457-E462.

Lagerlund, TD. and Low, PA. 1993. Mathematical Modeling of time-dependent oxygen transport in rat peripheral nerve. Journal of Computational Biology Medicine. 23:29-47.

Reneau, DD., Bruley, DF. and Knisely, MH. 1967. A mathematical simulation of oxygen release, diffusion and consumption in the capillaries and tissue of the human brain, proceedings of the 33rd Annual chemical Engg. Symposium. Plenum press, Cincinnati. 135-241.

Reneau, DD., Bruley, DF. and Knisely, MH. 1969. A digital Simulation of transient oxygen transport in capillary tissue systems: development of a numerical method for solution of transport equations describing coupled convection-diffusion systems. AICHE Journal. 15:916-925.

Sharan, M. Singh, B. and Kumar, P. 1997. A two layers model for studying the effect of plasma layer on the delivery of oxygen to tissue using a finite element method. Journal of Applied Mathematical Modeling. 21:419-426.

Sharma, GC. and Jain, M. 2004. A Computational solution of Mathematical model for oxygen transport in peripheral nerve. Journal of Computers in Biology and Medicine. 34:633-645.

Received: Feb 26, 2010; Revised: Dec 21, 2010; Accepted: Jan 25, 2011