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S_{ω}^{ϵ}-CLOSEDNESS IN FUZZY TOPOLOGICAL SPACES

B M Uzzal Afsan[1] and *C K Basu[2]
[1]Department of Mathematics, Sripat Singh College, Jiaganj Murshidabad, PIN-742123, West Bengal
[2]Department of Mathematics, West Bengal State University, Berunanpukuria
P.O. Malikapur, Barasat, North 24-Parganas, Kolkata-700126, India

ABSTRACT

In this paper, a new kind of covering axiom, fuzzy S_{ω}^{ϵ}-closedness, stronger than fuzzy s-closedness due to Sinha and Malakar (1994) is introduced in fuzzy topological spaces (Chang, 1968) in terms of fuzzy semi-ω-ϵ-open sets and fuzzy semi-ω-ϵ-closures. Several characterizations via fuzzy prefilterbases and fuzzy nets (Pu and Liu, 1980) along with various properties of such concept are obtained.

AMS Subject Classification Codes: 54A40, 54D99.

Keywords: Fuzzy θ-semi-ω-ϵ-closure, fuzzy S_{ω}^{ϵ}-closed, fuzzy θ-semi-ω-ϵ-cluster point, fuzzy θ-semi-ω-ϵ-convergence, fuzzy complete θ-semi-ω-ϵ-accumulation point, weakly fuzzy θ-semi-ω-ϵ-closed.

INTRODUCTION

The notion of ω-open sets in general topological spaces introduced by Hdeib (1982) has been studied in recent years by a good number of researchers like Hdeib (1989) Noiri *et al*. (2009), Omari and Noorani (2007) and Zoubi and Nashef (2003).

For a long time, topologists have been interested in investigating properties closely related to compactness in both general topological spaces and fuzzy topological spaces (Chang, 1968) some of those are found in papers of Basu *et al*. (2009). Ganguly and Saha (1990), Joseph and Kwack (1980), Lowen (1979), Malakar *et al*. (1994), Malakar and Mukherjee (1993), Mashhour *et al*. (1987), Mukherjee and Ghosh (1989), Sinha and Malakar (1994), Thompson (1976), and Velicko (1968). Every new invention neighbouring fuzzy compactness, at some stage or other, becomes culminated into a tremendous applications not only within fuzzy topology itself but also in other branches of applied sciences. Keeping this in mind, a new kind of covering property fuzzy S_{ω}^{ϵ}-closedness in fuzzy topological spaces, stronger than the well-known concept s-closedness due to Sinha and Malakar (1994) is introduced in terms of fuzzy semi- ω-ϵ-open sets, fuzzy semi ω-ϵ-closures and allied concepts. We have obtained several characterizations via fuzzy prefilterbases and fuzzy nets along with various properties of such concept.

*Corresponding author email: ckbasu1962@yahoo.com.

PRELIMINARIES

A point x of a general topological space (X,τ) is called a condensation point of a subset A of X if for each open set U containing x, $A \cap U$ is uncountable. A set A is called ω-closed (Joseph and Kwack, 1980) if it contains all of its condensation points and the complement of an ω-closed set is called an ω-open set or equivalently, $A \subset X$ is ω-open if and only if for each $x \in A$, there exists an open set U containing x such that $U - A$ is countable. The set of all ω-open sets of a topological space (X,τ) is denoted by τ_{ω}. It is to be noted that τ_{ω} is a topology on X finer than τ. Throughout this paper spaces (X,δ) and (Y,σ) (or simply X and Y) represent non-empty fuzzy topological spaces due to Chang (1968) and the symbols I and I^X have been used for the unit closed interval $[0,1]$ and the set of all functions with domain X and codomain I respectively. The support of a fuzzy set A is the set $\{x \in X: A(x) > 0\}$ and is denoted by $supp(A)$. A fuzzy set with only non-zero value $p \in (0,1]$ at only one element $x \in X$ is called a fuzzy point and is denoted by x_p and the set of all fuzzy points of a fuzzy topological space is denoted by $Pt(X)$. For any two fuzzy sets A and B of X, $A \leq B$ if and only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy point x_p is said to be in a fuzzy set A (denoted by $x_p \in A$) if $x_p \leq A$, i.e. if $p \leq A(x)$. The set of all fuzzy points having non-zero value ϵ, $0 < \epsilon \leq 1$ and contained in the fuzzy set A is denoted by $Pt(A, \epsilon)$. The constant fuzzy sets of X with values 0 and 1 are denoted by $\underline{0}$ and $\underline{1}$ respectively. A fuzzy set A is said to be quasi-coincident with B (written as $A\hat{q}B$) (Sinha and Malakar, 1994) if $A(x) + B(x) > 1$ for some $x \in X$. A fuzzy set A is said to be not quasi-coincident with B (written as $A\bar{q}B$) (Sinha and Malakar, 1994) if $A(x) + B(x) \leq 1$, for all $x \in X$. A fuzzy open set A of X is called fuzzy quasi-neighbourhood of a fuzzy point x_p if $x_p\hat{q}$ A and the collection of all fuzzy quasi-neighbourhood of a fuzzy point x_p is denoted by $FQN(X, x_p)$. A subset A of a fuzzy topological space X is called fuzzy semi-open (Ahemed and Abo-Khadra, 1989) (resp. regular open (Ahemed and Khadra, 1989)) if $A \leq cl(int(A))$ (resp. $A = int(cl(A))$). The collection of all fuzzy semi-open sets (resp. fuzzy semi-open sets containing the fuzzy point x_p and fuzzy semi-open sets quasi-coincident with the fuzzy point x_p) is denoted by $FSO(X)$ (resp. $FSO(X, x_p)$ and $FSQN(X, x_p)$). The complement of a fuzzy semi-open (resp. regular open) is called fuzzy semi-closed (Ahemed and Abo-Khadra, 1989) (resp. regular closed (Ahemed and Abo-Khadra, 1989). The fuzzy semi-closure of a fuzzy set A of X is the set $scl(A) = \vee\{x_p \in Pt(X) : A\hat{q}U$ for each $U \in FSQN(X, x_p)\}$ (Ghosh, 1990; Lowen, 1979) has defined a fuzzy prefilterbase on X as a non-empty family Ω of non-empty fuzzy sets of X such that for each pair $A, B \in \Omega$, there exists a $C \in \Omega$ such that $C \leq A \wedge B$. A fuzzy point $x_p \in Pt(X)$ is called a fuzzy θ-semi-cluster point of a fuzzy prefilterbase Ω on X if $A\hat{q}\, scl(U)$ for each $A \in \Omega$ and each $U \in FSQN(X, x_p)$ (Sinha and Malakar, 1994). The set of all fuzzy semi-cluster point of a prefilterbase Ω on X is denoted by $s\text{-}ad_\theta\,(\Omega, X)$. A collection Σ of fuzzy sets in X is called a fuzzy cover of $\underline{1}$ if $\vee\{A : A \in \Sigma\} = \underline{1}$, i.e. if $sup\{U(x) : U \in \Sigma\} = 1$ for all $x \in X$. A fuzzy topological space X is fuzzy s-closed (Sinha and Malakar, 1994) (resp. S-closed (Mukherjee and Ghosh, 1989)) if every fuzzy cover of $\underline{1}$ by its fuzzy semi-open sets has a finite subfamily such that the fuzzy semi-closures (resp. closures) of the members of that subfamily is also a cover of $\underline{1}$. In this paper, the cardinalities of an ordinary set K and the set N of all natural numbers are denoted by $|K|$ and $\aleph_0$ respectively.

1. FUZZY SEMI- ω-ϵ-OPEN SETS

Definition 1.1. Let (X, δ) be a fuzzy topological space and $0 < \epsilon \leq 1$. A fuzzy set $A \in I^X$ is called fuzzy ω-ϵ-open if for each fuzzy point $x_p \in Pt(X)$ with $x_p\hat{q}\,A$, there exists an $U \in FQN(x_p)$ such that $|Pt(U, \epsilon) - Pt(A, \epsilon)| \leq \aleph_0$. The complement of a fuzzy ω-ϵ-open set is called a fuzzy ω-ϵ-closed set.

The family of all fuzzy ω-ϵ-open sets of a fuzzy topological space (X, δ) is denoted by δ_ω^ϵ and the family of all fuzzy ω-ϵ-open sets of a fuzzy topological space (X, δ) containing the fuzzy point x_p is denoted by $\delta_\omega^\epsilon(x_p)$.

Theorem 1.2. δ_ω^ϵ is fuzzy topology finer than δ.

Proof: It is trivial to verify that $\underline{0}$, $\underline{1} \in \delta_\omega^\epsilon$. Now consider $A, B \in \delta_\omega^\epsilon$ and $x_p\hat{q}\,(A \wedge B)$. Then $x_p\hat{q}\,A$ and $x_p\hat{q}\,B$ and so there exist $U, V \in FQN(x_p)$ such that
$|Pt(U, \epsilon) - Pt(A, \epsilon)| \leq \aleph_0$ and
$|Pt(V, \epsilon) - Pt(B, \epsilon)| \leq \aleph_0$. The inclusion
$$Pt(U \wedge V, \epsilon) - Pt(A \wedge B, \epsilon) \subset \big(Pt(U, \epsilon) - Pt(A, \epsilon)\big) \cup \big(Pt(V, \epsilon) - Pt(B, \epsilon)\big)$$
implies that
$$|Pt(U \wedge V, \epsilon) - Pt(A \wedge B, \epsilon)| \leq |Pt(U, \epsilon) - Pt(A, \epsilon)| + |Pt(V, \epsilon) - Pt(B, \epsilon)| \leq \aleph_0$$
and so $A \wedge B \in \delta_\omega^\epsilon$. Now let ϵ be an arbitrary index set such that for each $\alpha \in \Delta$, $A_\alpha \in \delta_\omega^\epsilon$ and $x_p\hat{q}\,\vee\{A_\alpha : \alpha \in \Delta\}$. Then there exists an $\alpha_0 \in \Delta$ such that $x_p\hat{q}\,A_{\alpha_0}$ and so there exists $U_0 \in FQN(x_p)$ such that $|Pt(U_0, \epsilon) - Pt(A_{\alpha_0}, \epsilon)| \leq \aleph_0$. The inclusion
$$Pt(U_0, \epsilon) - Pt(\vee\{A_\alpha : \alpha \in \Delta\}, \epsilon) \subset Pt(U_0, \epsilon) - Pt(A_{\alpha_0}, \epsilon)$$
implies that
$$|Pt(U_0, \epsilon) - Pt(\vee\{A_\alpha : \alpha \in \Delta\}, \epsilon)| \leq |Pt(U_0, \epsilon) - Pt(A_{\alpha_0}, \epsilon)|$$
$\leq \aleph_0$ and so $\vee\{A_\alpha : \alpha \in \Delta\} \in \delta_\omega^\epsilon$. So δ_ω^ϵ is a fuzzy topology and is, obviously, finer than δ.

The fuzzy interior and fuzzy closure of a fuzzy set $A \in I^X$ with respect to the fuzzy topology δ_ω^ϵ are denoted by $int_\omega^\epsilon(A)$ and $cl_\omega^\epsilon(A)$.

Definition 1.3. A fuzzy set A of a fuzzy topological space X is called a fuzzy semi-ω-ϵ-open set if $A \leq cl(int_\omega^\epsilon(A))$. The complement of a fuzzy semi-ω-ϵ-open set is a fuzzy semi-ω-ϵ-closed set.

The family of all fuzzy semi-ω-ϵ-open sets of a fuzzy topological space X is denoted by $FS_\omega^\epsilon O(X)$ and the family of all fuzzy semi-ω-ϵ-open sets of a fuzzy topological space X containing the fuzzy point x_p (resp. quasi- coincident with $x_p$is denoted $FS_\omega^\epsilon O(X, x_p)$ (resp. $FS_\omega^\epsilon QN(X, x_p)$).

It is obvious to note that $FSO(X) \subset FS_\omega^\epsilon O(X)$ and $\delta_\omega^\epsilon \subset FS_\omega^\epsilon O(X)$. None of the above inclusions is reversible.

Example 1.4. Consider the space $X = N$ with the fuzzy topology generated by the base $\odot = \{B_n : n \in N\}$, where B_n is the characteristic function of the set $\{1, n\}$. Then the fuzzy topology on X is δ ={ $\underline{0}$ }∪ {$U \in I^N$: U is the characteristic function of a subset of N containing 1} and $FSO(X) = \{\underline{0}\} \cup \{ U \in I^N$: the rang set of U contains 1}. Since N is countable, $\delta_\omega^\epsilon = I^N = FS_\omega^\epsilon O(X)$ for any $\epsilon \in$ (0,1]. This example has also established that $\delta_\omega^\epsilon \neq \delta$ in general.

The following lemma is easy to prove but useful to develop the paper.

Lemma 1.5. Arbitrary join of fuzzy semi-ω-ϵ-open sets of a fuzzy topological space X is a fuzzy semi-ω-ϵ-open.

Definition 1.6. Let A be a fuzzy set of a fuzzy topological space X. The fuzzy semi-ω-ϵ-closure of A is the set $scl_\omega^\epsilon(A) = \vee\{x_p \in Pt(X) : A\hat{q}U$ for each $FS_\omega^\epsilon QN(X, x_p)\}$.

Theorem 1.7. For any two fuzzy sets A and B of a fuzzy topological space X, the following properties hold:

(a) $A \leq scl_\omega^\epsilon(A) \leq scl(A)$.

(b) $scl_\omega^\epsilon(A) = scl_\omega^\epsilon(scl_\omega^\epsilon(A))$.

(c) A is fuzzy semi-ω-ϵ-closed if and only if $A = scl_\omega^\epsilon(A)$.

(d) $A \leq B$ implies that $scl_\omega^\epsilon(A) \leq scl_\omega^\epsilon(B)$.

Proof: The proof is straightforward and is thus omitted. Following example shows that $scl_\omega^\epsilon(A) \neq scl(A)$

Example 1.8. Consider the same fuzzy topological space $X = N$ of Example 1.4. Let $A \in I^X$ defined by:

$$A(x) = \begin{cases} 1, & x = 1 \\ n+1, & x \neq 1 \end{cases}$$

Then $scl_\omega^\epsilon(A)$=A for any $\epsilon \in$ (0,1] and $scl(A) = I^X$.

Definition 1.9. Let A be a fuzzy set of a fuzzy topological space X. The fuzzy θ-semi-ω-ϵ-closure of A is the set $s_\theta cl_\omega^\epsilon(A) = \vee\{x_p \in Pt(X) : A\hat{q}scl_\omega^\epsilon(U)$ for each $FS_\omega^\epsilon QN(X, x_p)\}$. A fuzzy set A is called fuzzy θ-semi-ω-ϵ-closed if $A = s_\theta cl_\omega^\epsilon(A)$ and the complement of a fuzzy θ-semi-ω-ϵ-closed set is called a fuzzy θ-semi-ω-ϵ-open set.

The following theorem can be easily verified:

Theorem 1.10. For any two fuzzy sets A and B of a fuzzy topological space X, the following properties hold:

(a) $scl_\omega^\epsilon(A) \leq s_\theta cl_\omega^\epsilon(A)$.

(b) $A \leq B$ implies that $s_\theta cl_\omega^\epsilon(A) \leq s_\theta cl_\omega^\epsilon(B)$.

2. FUZZY S_ω^ϵ-CLOSED SPACES

Definition 2.1. A fuzzy topological space X is called fuzzy S_ω^ϵ-closed if every fuzzy cover of $\underline{1}$ by its fuzzy semi-ω-ϵ-open sets has a finite subfamily such that the fuzzy semi-ω-ϵ-closures of the members of that subfamily is also a cover of $\underline{1}$.

It is obvious to note that every fuzzy S_ω^ϵ-closed space is s-closed. But the converse is not true in general that is established by the following example.

Example 2.2. The fuzzy topological space $X = N$ of Example 1.4 is fuzzy s-closed, but not S_ω^ϵ-closed because $\odot = \{B_n : n \in N\}$, where B_n is the characteristic function of the set $\{1, n\}$ is a fuzzy cover of $\underline{1}$ by its fuzzy semi-ω-ϵ-open sets without any finite subfamily such that the fuzzy semi-ω-ϵ-closures of the members of that subfamily is also a cover of $\underline{1}$.

Theorem 2.3. A fuzzy topological space X is fuzzy S_ω^ϵ-closed if and only if for each fuzzy prefilterbase Ω in X equipped with fuzzy semi-ω-ϵ-open sets, $\wedge\{scl_\omega^\epsilon(A): A\in\Omega\}\neq \underline{0}$.

Proof: Let X be fuzzy S_ω^ϵ-closed and Ω be a fuzzy prefilterbase in X equipped with fuzzy semi-ω-ϵ-open sets such that $\wedge\{scl_\omega^\epsilon(A): A\in\Omega\} = \underline{0}$. Then $\vee\{\underline{1}-scl_\omega^\epsilon(A): A\in\Omega\} = \underline{1}$ and so the fuzzy S_ω^ϵ-closedness of space X ensures that there is a finite subset Ω_0 of Ω such that $\vee\left\{scl_\omega^\epsilon\left(\underline{1}-scl_\omega^\epsilon(A)\right): A\in\Omega_0\right\} = \underline{1}$, which implies that $\wedge\{\underline{1}-scl_\omega^\epsilon(\underline{1}-scl_\omega^\epsilon(A)): A\in\Omega_0\} = \underline{0}$ i.e. $\wedge\{A: A\in\Omega_0\} = \underline{0}$. Since Ω is a prefilterbase, there exists an $A_0\in\Omega$ such that $A_0\leq\wedge\{A: A\in\Omega_0\}$ and thus $A_0 = \underline{0}$ ---- a contradiction.

Conversely, let X be not fuzzy S_ω^ϵ-closed. Then there exists a fuzzy cover Σ of $\underline{1}$ by fuzzy semi-ω-ϵ-open sets of X such that $\vee\{scl_\omega^\epsilon(A): A\in\Sigma_0\} < \underline{1}$, i.e. $\wedge\{\underline{1}-scl_\omega^\epsilon(A): A\in\Sigma_0\} > \underline{0}$ for any $\Sigma_0\subset\Sigma$ with $|\Sigma_0| < \aleph_0$. Thus $\{\underline{1}-scl_\omega^\epsilon(A)): A\in\Sigma\}$ is a fuzzy prefilterbase in X equipped with fuzzy semi-ω-ϵ-open sets. Since Σ is a fuzzy cover of $\underline{1}$, $\vee\{A: A\in\Sigma\} = \underline{1}$, i. e. $\wedge\{scl_\omega^\epsilon(\underline{1}-scl_\omega^\epsilon(A)): A\in\Sigma\} = \underline{0}$ --- a contradiction.

Theorem 2.4. A fuzzy topological space X is fuzzy S_ω^ϵ-closed if and only if each family Δ of fuzzy semi-ω-ϵ-closed sets in X with the property $\wedge\{\underline{1}-scl_\omega^\epsilon(\underline{1}-V): V\in\Delta_0\}\neq\underline{0}$ for each for any $\Delta_0\subset\Delta$ with $|\Delta_0| < \aleph_0$ has a non-empty meet.

Proof: Let X is fuzzy S_ω^ϵ-closed and Δ is a family of fuzzy semi-ω-ϵ-closed sets in X with the property $\wedge\{\underline{1}-scl_\omega^\epsilon(\underline{1}-V): V\in\Delta_0\}\neq\underline{0}$ for each for any $\Delta_0\subset\Delta$ with $|\Delta_0| < \aleph_0$. If possible let $\wedge\{V : V\in\Delta\} = \underline{0}$. Then $\{\underline{1}-V: V\in\Delta\}$ is a fuzzy cover of $\underline{1}$ by its fuzzy semi-ω-ϵ-open sets and so fuzzy S_ω^ϵ-closedness of X ensures the existence of a $\Delta_0\subset\Delta$ with $|\Delta_0| < \aleph_0$ such that $\vee\{scl_\omega^\epsilon(\underline{1}-V): V\in\Delta_0\} = \underline{1}$, i.e. $\wedge\{\underline{1}-scl_\omega^\epsilon(\underline{1}-V): V\in\Delta_0\} = \underline{0}$ --- a contradiction.

Conversely, let X be not fuzzy S_ω^ϵ-closed. Then by Theorem 2.3, there exists a fuzzy prefilterbase Ω in X equipped with semi-ω-ϵ-open sets of X such that $\wedge\{scl_\omega^\epsilon(A): A\in\Omega\} = \underline{0}$.

Therefore $\{\underline{1}-scl_\omega^\epsilon(A): A\in\Omega\}$ is a fuzzy cover of $\underline{1}$ by its fuzzy semi-ω-ϵ-open sets and so fuzzy non- S_ω^ϵ-closedness of X ensures that $\vee\{scl_\omega^\epsilon(\underline{1}-scl_\omega^\epsilon(A)): A\in\Omega_0\} < \underline{1}$, i.e. $\wedge\left\{\underline{1}-scl_\omega^\epsilon\left(\underline{1}-scl_\omega^\epsilon(A)\right): A\in\Omega_0\right\} > \underline{0}$ for each $\Omega_0\subset\Omega$ with $|\Omega_0| < \aleph_0$. Thus $\{scl_\omega^\epsilon(A): A\in\Omega\}$ is a family of fuzzy semi-ω-ϵ-closed sets in X with the property stated in the theorem having empty meet.

Definition 2.5. A fuzzy point $x_p\in Pt(X)$ is called a fuzzy θ-semi-ω-ϵ-cluster point of a fuzzy prefilterbase Ω on X if $A\hat{q}\, scl_\omega^\epsilon(U)$ for each $A\in\Omega$ and each $U\in FS_\omega^\epsilon QN(X, x_p)$. The set of all fuzzy semi-ω-ϵ-cluster point of a prefilterbase Ω on X is denoted by S_ω^ϵ-$ad_\theta\ (\Omega, X)$.

Lemma 2.6. A fuzzy point $x_p\in Pt(X)$ is a fuzzy θ-semi-ω-ϵ-cluster point of a fuzzy prefilterbase Ω on X if and only if $x_p\in s_\theta cl_\omega^\epsilon(A)$, for all $A\in\Omega$.

Proof: Let $x_p\in Pt(X)$ be a fuzzy θ-semi-ω-ϵ-cluster point of fuzzy prefilterbase Ω on X, $A\in\Omega$ and $U\in FS_\omega^\epsilon QN(X, x_p)$. Then by Definition 2.5, $A\hat{q}\, scl_\omega^\epsilon(U)$ and so $x_p\in s_\theta cl_\omega^\epsilon(A)$

To establish the converse, let $A\in\Omega$ and $U\in FS_\omega^\epsilon QN(X, x_p)$. Then $x_p\in s_\theta cl_\omega^\epsilon(A)$ and so by Definition 1.9, $A\hat{q}\, scl_\omega^\epsilon(U)$ and hence $x_p$is a fuzzy θ-semi-ω-ϵ-cluster point of fuzzy prefilterbase Ω.

Theorem 2.7. A fuzzy topological space X is fuzzy S_ω^ϵ-closed if and only if for every fuzzy prefilterbase Ω on X, S_ω^ϵ-$ad_\theta\ (\Omega, X)\neq\underline{0}$.

Proof: Let X be a fuzzy S_ω^ϵ-closed space and Ω be a fuzzy prefilterbase on X such that S_ω^ϵ-$ad_\theta\ (\Omega, X) = \underline{0}$. Then for each $x\in X$ and each positive integer n, there exist an $U_n^x\in FS_\omega^\epsilon QN(X, x_{\frac{1}{n}})$ and an $A_n^x\in\Omega$ such that $A_n^x\bar{q}\, scl_\omega^\epsilon(U_n^x)$. Since $U_n^x + \frac{1}{n} > 1$,

$\{U_n^x : x \in X, n \in N\}$ is a cover of $\underline{1}$. Then the fuzzy S_ω^ϵ-closedness of X ensures the existence of subsets N_0 and X_0 of N and X respectively such that $\vee\{scl_\omega^\epsilon(U_n^x) : x \in X_0, n \in N_0\} = \underline{1}$, $|X_0| < \aleph_0$ and $|N_0| < \aleph_0$. Again $A_n^x \bar{q}\, scl_\omega^\epsilon(U_n^x)$ for each $x \in X_0$ and $n \in N_0$ implies that $\wedge\{A_n^x : x \in X_0, n \in N_0\}\bar{q}\underline{1}$. Since Ω is a fuzzy prefilterbase on X, there exists an $A \in \Omega$ such that $A \leq \wedge\{A_n^x : x \in X_0, n \in N_0\}$ and so, that $A\bar{q}\underline{1}$ i.e. $A = \underline{0}$ --- a contradiction.

Conversely, let X be not fuzzy S_ω^ϵ-closed. Then by Theorem 2.3, there exists a fuzzy prefilterbase Ω in X equipped with semi-ω-ϵ-open sets of X such that $\wedge\{scl_\omega^\epsilon(A) : A \in \Omega\} = \underline{0}$. Then $\{\underline{1} - scl_\omega^\epsilon(A) : A \in \Omega\}$ is a cover of $\underline{1}$ and so for each $x_p \in Pt(X)$, there exists an $A \in \Omega$ such that $U = \underline{1} - scl_\omega^\epsilon(A) \in FS_\omega^\epsilon QN(X, x_p)$ and $\bar{q}\, scl_\omega^\epsilon(U)$. Hence $x_p \notin s_\theta cl_\omega^\epsilon(A)$ and thus by Lemma 2.6, $S_\omega^\epsilon\text{-}ad_\theta\ (\Omega, X) = \underline{0}$.

Definition 2.8. A fuzzy point $x_p \in Pt(X)$ is called a fuzzy θ-semi-ω-ϵ-cluster point of a fuzzy net $\{A_\lambda : \lambda \in (\Pi, \leq)\}$ if for each $U \in FS_\omega^\epsilon QN(X, x_p)$ and for each $\lambda \in \Pi$ there exists a $\lambda_0 \in \Pi$ with $\lambda_0 \geq \lambda$ such that $A_{\lambda_0}\hat{q}\, scl_\omega^\epsilon(U)$. The set of all fuzzy θ-semi-ω-ϵ-cluster points of the fuzzy net $\{A_\lambda : \lambda \in (\Pi, \leq)\}$ is denoted by $S_\omega^\epsilon\text{-}ad_\theta\ (\Pi, X)$.

A fuzzy net $\{A_\lambda : \lambda \in (\Pi, \leq)\}$ is said to θ-semi-ω-ϵ-converge to a fuzzy point $x_p \in Pt(X)$ if for each $U \in FS_\omega^\epsilon QN(X, x_p)$, there exists a $\lambda_0 \in \Pi$ such that $A_\lambda \hat{q}\, scl_\omega^\epsilon(U)$ for all $\lambda \geq \lambda_0$.

Theorem 2.9. A fuzzy topological space X is fuzzy S_ω^ϵ-closed if and only if for every fuzzy net $\{A_\lambda : \lambda \in (\Pi, \leq)\}$ on X, $S_\omega^\epsilon\text{-}ad_\theta\ (\Pi, X) \neq \underline{0}$.

Proof: Let X be a fuzzy S_ω^ϵ-closed space and $\{A_\lambda : \lambda \in (\Pi, \leq)\}$ be a fuzzy net on X such that $S_\omega^\epsilon\text{-}ad_\theta\ (\Pi, X) = \underline{0}$. Then for each $x_p \in Pt(X)$, there exist an $U(x_p) \in FS_\omega^\epsilon QN(X, x_p)$ and a $\lambda(x_p) \in \Pi$ such that $A_\mu \hat{q}\, scl_\omega^\epsilon(U(x_p))$ for all $\mu \geq \lambda(x_p)$. We claim that the family $\Delta = \{\underline{1} - U(x_p) : x_p \in Pt(X)\}$ of fuzzy semi-ω-ϵ-closed sets satisfies the property given in Theorem 2.4. Consider a subfamily $\Delta = \{\underline{1} - U(x_{p_i}^i) : i = 1, 2, \ldots, n\}$, where n is a finite positive integer. Then there exists a $\nu \in \Pi$ with $\nu \geq \lambda(x_{p_i}^i)$ for each $i = 1, 2, \ldots, n$ such that $A_\mu \bar{q}\, scl_\omega^\epsilon(U(x_{p_i}^i))$ for all $\mu \geq \nu$ and so $A_\mu \leq \underline{1} - \vee\{scl_\omega^\epsilon(U(x_{p_i}^i)), i = 1, 2, \ldots, n\}$ $= \wedge\{\underline{1} - scl_\omega^\epsilon(U(x_{p_i}^i)), i = 1, 2, \ldots, n\}$ for all $\mu \geq \nu$. Thus $\wedge\{\underline{1} - scl_\omega^\epsilon(U(x_{p_i}^i)), i = 1, 2, \ldots, n\} \neq \underline{0}$. Then Theorem 2.4 ensures that $\wedge\{\underline{1} - U(x_p) : x_p \in Pt(X)\} \neq \underline{0}$. Let $y_r \in \wedge\{\underline{1} - U(x_p) : x_p \in Pt(X)\}$. Then $y_r \leq \underline{1} - U(x_p)$ for each $x_p \in Pt(X)$. Then for the particular case, $y_r \leq \underline{1} - U(y_r)$ and so, $y_r \bar{q} U(y_r)$ --- a contradiction.

Conversely, let X be not fuzzy S_ω^ϵ-closed. Then Theorem 2.3 ensures the existence of a fuzzy prefilterbase Ω in X equipped with fuzzy semi-ω-ϵ-open sets such that $\wedge\{scl_\omega^\epsilon(A) : A \in \Omega\} = \underline{0}$, i.e. that $\vee\{\underline{1} - scl_\omega^\epsilon(A) : A \in \Omega\} = \underline{1}$. It is obvious to note that $\Sigma = \{x_\lambda \in \lambda : \lambda \in \Omega\}$ is a fuzzy net where Ω is the directed set under the relation "$\ll$" defined by $\lambda_1 \ll \lambda_2$ if and only if $\lambda_2 \leq \lambda_1$ for all $\lambda_1, \lambda_2 \in \Omega$. Then Σ has a fuzzy θ-semi-ω-ϵ-cluster point $x_p \in Pt(X)$. Since $\{\underline{1} - scl_\omega^\epsilon(A) : A \in \Omega\}$ is a cover of $\underline{1}$, $x_p \hat{q}(\underline{1} - scl_\omega^\epsilon(A))$ for some $A \in \Omega$. Then $x_B \bar{q}\, scl_\omega^\epsilon(\underline{1} - scl_\omega^\epsilon(A))$ for all $B \in \Omega$ (directed set) with $A \ll B$. Thus x_p can not be fuzzy θ-semi-ω-ϵ-cluster point of the fuzzy net Σ --- a contradiction.

Theorem 2.10. A fuzzy topological space X is fuzzy S_ω^ϵ-closed if and only if every fuzzy net has a fuzzy subnet θ-semi-ω-ϵ-converging to a fuzzy point of X.

Proof: Let fuzzy topological space X be fuzzy S_ω^ϵ-closed and $\Sigma = \{A_\lambda : \lambda \in (\Pi, \leq)\}$ be a fuzzy net on X. Then by Theorem 2.9, there exists an $x_p \in Pt(X)$ at which Σ θ-semi-ω-ϵ-clusters. So for each $U \in FS_\omega^\epsilon QN(X, x_p)$ there exists a $\lambda \in \Pi$ such that $A_\lambda \hat{q}\, scl_\omega^\epsilon(U)$. Then the set $\otimes = \{(\lambda, U) : U \in FS_\omega^\epsilon QN(X, x_p), \lambda \in \Pi,$ $A_\lambda \hat{q}\, scl_\omega^\epsilon(U)\}$ is a directed set under the relation "$\ll$" defined by

$(\lambda, U) \ll (\nu, V)$ if and only if $\lambda \leq \nu$ and $V \leq U$ and so the mapping $\phi: (\otimes, \ll) \to \Sigma$ defined by $\phi((\lambda, U)) = A_\lambda$ is a fuzzy subnet of Σ. We claim ϕ θ-semi-ω-ϵ-converges to x_p. To establish this, suppose $G \in FS_\omega^\epsilon QN(X, x_p)$. Then there exists a $\lambda_0 \in \Pi$ (and so $(\lambda_0, G) \in \otimes$) such that $A_{\lambda_0} \hat{q}\, scl_\omega^\epsilon(G)$. Therefore for all $(\lambda, U) \in \otimes$ with $(\lambda_0, G) \ll (\lambda, U)$, i.e. for all $U \in FS_\omega^\epsilon QN(X, x_p)$ and for all $\lambda \in \Pi$ satisfying $A_\lambda \hat{q}\, scl_\omega^\epsilon(U)$ with $\lambda_0 \leq \lambda$ and $U \leq G$ implies that $\phi((\lambda, U))\hat{q}\, scl_\omega^\epsilon(G)$. Thus ϕ θ-semi-ω-ϵ-converges to x_p. The converse part is clear.

Definition 2.11. A fuzzy point $x_p \in Pt(X)$ is called a fuzzy complete θ-semi-ω-ϵ-accumulation point of a fuzzy set A in X if for each $U \in FS_\omega^\epsilon QN(X, x_p)$, $|suppA| = |\{y \in X: A(y) + (scl_\omega^\epsilon(U))(y) < 1\}|$.

Theorem 2.12. Let X be a fuzzy S_ω^ϵ-closed fuzzy topological space. Then every fuzzy set A of X with $|suppA| \geq \aleph_0$ has a fuzzy complete θ-semi-ω-ϵ-accumulation point.

Proof: Let a fuzzy set A of X with $|suppA| \geq \aleph_0$ has no fuzzy complete θ-semi -ω-ϵ-accumulation point. Then for each $x_{\frac{1}{n}} \in Pt(X)$, where $n \in N$, there exists an $U_n^x \in FS_\omega^\epsilon QN(X, x_{\frac{1}{n}})$, such that $|suppA| > |\{y \in X: A(y) + (scl_\omega^\epsilon(U))(y) > 1\}|$.

Clearly, $\{U_n^x : x \in X, n \in N\}$ is a cover of $\underline{1}$ by its semi-ω-ϵ-open sets. Then the fuzzy S_ω^ϵ-closedness of X ensures the existence of subsets N_0 and X_0 of N and X respectively such that $\vee\{scl_\omega^\epsilon(U_n^x): x \in X_0, n \in N_0\} = \underline{1}$, $|X_0| < \aleph_0$ and $|N_0| < \aleph_0$. Let $a \in suppA$. Then there exist an $r \in N_0$ and a $z \in X_0$ such that $(scl_\omega^\epsilon(U_r^z))(a) = 1$ and so $(scl_\omega^\epsilon(U_r^z))(a) + A(a) > 1$. Thus $a \in \{y \in X: A(y) + (scl_\omega^\epsilon(U_r^z))(y) > 1\}$ and so $suppA \subseteq \cup\{A_{U_r^z}: r \in N_0\ z \in X_0\}$, i.e. $|suppA| \leq |\cup\{A_{U_r^z}: r \in N_0\ z \in X_0\}|$, where $A_{U_r^z} = \{y \in X: A(y) + (scl_\omega^\epsilon(U_r^z)(y) > 1\}$. Again $|A_{U_r^z}| \leq suppA$ for each $z \in X_0$, $r \in N_0$. Therefore $|\cup\{A_{U_r^z}: r \in N_0\ z \in X_0\}| = max\{|A_{U_r^z}|: r \in N_0\ z \in X_0\} < |suppA|$ --- a contradiction.

Since it has already been shown that the space $X = N$ of the Example 1.4 is not fuzzy S_ω^ϵ-closed though every fuzzy set A of X with $|suppA| = \aleph_0$ has a fuzzy complete θ-semi-ω-ϵ-accumulation point, the converse statement of Theorem 2.12 need not be true.

3. FUZZY S_ω^ϵ-CLOSED SETS RELATIVE TO A FUZZY TOPOLOGICAL SPACE

Definition 3.1. A fuzzy set S of a topological space X is called fuzzy S_ω^ϵ-closed relative to X if for every family Π of fuzzy semi-ω-ϵ-open sets of X with $sup\{U(x): U \in \Pi\} = 1$ for each $x \in supp(S)$ has a finite subfamily Π_0 of Π such that $\vee\{scl_\omega^\epsilon(U): U \in \Pi_0\} \geq S$.

Theorem 3.2. A fuzzy set S is fuzzy S_ω^ϵ-closed relative to X if and only if for each fuzzy prefilterbase Ω in X equipped with fuzzy semi-ω-ϵ-open sets and $(\wedge\{scl_\omega^\epsilon(A): A \in \Omega\}) \wedge S = \underline{0}$, there exists a subset Ω_0 of Ω such that $|\Omega_0| < \aleph_0$ and $(\wedge\{A: A \in \Omega_0\})\bar{q}\, S$.

Proof: Let S be fuzzy S_ω^ϵ-closed relative to X and Ω be a fuzzy prefilterbase in X equipped with fuzzy semi-ω-ϵ-open sets and $(\wedge\{scl_\omega^\epsilon(A): A \in \Omega\}) \wedge S = \underline{0}$. We claim that $\Pi = \{\underline{1} - scl_\omega^\epsilon(A): A \in \Omega\}$ is the family of fuzzy semi-ω-ϵ-open sets of X satisfying the property given in Definition 3.1. In fact for each $x \in supp(S)$, $\wedge\{scl_\omega^\epsilon(A): A \in \Omega\})(x) = 0$ and so $(\vee\{\underline{1} - scl_\omega^\epsilon(A): A \in \Omega\})(x) = 1$, i.e. $sup\{(\underline{1} - scl_\omega^\epsilon(A))(x): A \in \Omega\} = 1$.

Therefore by fuzzy S_ω^ϵ-closedness of S relative to X, there exists a subset Ω_0 of Ω with $|\Omega_0| < \aleph_0$ such that $\vee\{scl_\omega^\epsilon(\underline{1} - scl_\omega^\epsilon(A)): A \in \Omega_0\} \geq S$, i.e. $\wedge\{A: A \in \Omega_0\} \leq \underline{1} - S$. Thus $\wedge\{A: A \in \Omega_0\}\bar{q}\, S$.

Conversely, let fuzzy set S be not fuzzy S_ω^ϵ-closed relative to X. Then there exists a family Π of fuzzy semi-ω-ϵ-open sets of X with $sup\{U(x): U \in \Pi\} = 1$ for each $x \in supp(S)$ such that $\vee\{scl_\omega^\epsilon(U): U \in \Pi_0\} < S$, i.e. $\wedge\{\underline{1} - scl^{\epsilon}_{\omega}(U): U \in \Pi_0\} > \underline{1} - S \geq \underline{0}$ for any $\Pi_0 \subseteq \Pi$ with $|\Pi_0| < \aleph_0$. Thus $\{\underline{1} - scl^{\epsilon}_{\omega}(U): U \in \Pi\}$ is a fuzzy prefilterbase in X equipped with fuzzy semi-ω-ϵ-open sets. We claim that $(\wedge\{scl^{\epsilon}_{\omega}(\underline{1} - scl^{\epsilon}_{\omega}(U)): U \in \Pi\})\wedge S = \underline{0}$. For, if $y \in X$ with $S(y) > 0$, then $y \in supp(S)$ and so $sup\{U(y): U \in \Pi\} = 1$. Again $(\wedge\{scl^{\epsilon}_{\omega}(\underline{1} - scl^{\epsilon}_{\omega}(U)): U \in \Pi\})(y) > 0$ implies that $(\wedge\{\underline{1} - U: U \in \Pi\})(y) > 0$, i.e. $sup\{U(y): U \in \Pi\} = (\vee\{U: U \in \Pi\})(y) < 1$, which is a contradiction. Therefore $(\wedge\{scl^{\epsilon}_{\omega}(\underline{1} - scl^{\epsilon}_{\omega}(U)): U \in \Pi\})\wedge S = \underline{0}$. So by the hypothesis, there is a finite subset Π_0 of Π such that $(\wedge\{\underline{1} - scl^{\epsilon}_{\omega}(U): U \in \Pi_0\})\bar{q}S$ and thus $S \leq \wedge\{scl^{\epsilon}_{\omega}(U): U \in \Pi_0\}$ --- a contradiction.

Theorem 3.3. For a fuzzy set S of X, the following statements are equivalent:

(a) S is fuzzy S^{ϵ}_{ω}-closed relative to X.

(b) For every family Σ of fuzzy semi-ω-ϵ-closed sets of X with $\wedge\{V : V \in \Sigma\}\wedge S = \underline{0}$, there exists a finite subfamily Σ_0 of Σ such that $\wedge(\{\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V) : V \in \Sigma_0\})\bar{q}S$.

(c) For each fuzzy prefilterbase Ω of fuzzy semi-ω-ϵ-closed sets in X such that $(\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V))\hat{q}S$ for each $V \in \Omega$, $\wedge\{V : V \in \Omega\}\wedge S \neq \underline{0}$.

Proof: (a) implies (b): Let S be a fuzzy S^{ϵ}_{ω}-closed relative to X and Σ be a family of fuzzy semi-ω-ϵ-closed sets of X with $\{V : V \in \Sigma\}\wedge S = \underline{0}$. Then it is clear that $\{\underline{1} - V : V \in \Sigma\}$ is a family of fuzzy semi-ω-ϵ-open sets of X satisfying the condition given in Definition 3.1 and so fuzzy S^{ϵ}_{ω}-closedness of S relative to X ensures the existence of a finite subfamily Σ_0 of Σ such that $\vee(\{scl^{\epsilon}_{\omega}(\underline{1} - V) : V \in \Sigma_0\}) \geq S$, i.e. $\wedge(\{\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V) : V \in \Sigma_0\})\bar{q}S$.

(b) implies (c): Let Ω be a fuzzy prefilterbase of fuzzy semi-ω-ϵ-closed sets in X such that $(\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V))\hat{q}S$ for each $V \in \Omega$ and $\wedge(\{V : V \in \Omega\}\wedge S = \underline{0}$. Then by (b), there exists a finite subfamily Ω_0 of Ω such that $\wedge(\{\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V) : V \in \Omega_0\})\bar{q}S$. Since Ω is a fuzzy prefilterbase, there exists a $V_0 \in \Omega$ such that $V_0 \leq \wedge\{V : V \in \Omega_0\}$. Then $\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V_0) \leq \underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - \wedge\{V : V \in \Omega_0\} \leq \wedge(\{\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V) : V \in \Omega_0\})$ and so $(\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V_0))\hat{q}S$ --- a contradiction.

(c) implies (a): Let S be not fuzzy S^{ϵ}_{ω}-closed relative to X. Then there exists a family Π of fuzzy semi-ω-ϵ-open sets of X with $sup\{U(x): U \in \Pi\} = 1$ for each $x \in supp(S)$ such that $\vee\{scl^{\epsilon}_{\omega}(U): U \in \Pi_0\} < S$ for each $\Pi_0 \subset \Pi$ with $|\Pi_0| < \aleph_0$. Then $\Omega = \{V = \wedge\{scl^{\epsilon}_{\omega}(\underline{1} - scl^{\epsilon}_{\omega}(U)), U \in \Pi_0\}, \Pi_0 \subset \Pi$ with $|\Pi_0| < \aleph_0\}$ is a fuzzy prefilterbase of fuzzy semi-ω-ϵ-closed sets in X. We claim that $(\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V))\hat{q}S$ for each $V \in \Omega$. In fact, $(\underline{1} - scl^{\epsilon}_{\omega}(\underline{1} - V))\bar{q}\, S$ for some $V \in \Omega$ implies the existence of $\Pi_0 \subset \Pi$ with $|\Pi_0| < \aleph_0$ such that $S \leq scl^{\epsilon}_{\omega}(\underline{1} - V) = scl^{\epsilon}_{\omega}(\underline{1} - \wedge\{scl^{\epsilon}_{\omega}(\underline{1} - scl^{\epsilon}_{\omega}(U)): U \in \Pi_0\})$
$\leq$
$scl^{\epsilon}_{\omega}(\underline{1} - \wedge\{\underline{1} - scl^{\epsilon}_{\omega}(U): U \in \Pi_0\}) \leq \vee\{scl^{\epsilon}_{\omega}(U): U \in \Pi_0\}$
, which is a contradiction of the hypothesis. Hence by (c), $(\wedge(\{V : V \in \Omega\}\wedge S \neq 0$. Then there exists an $x \in supp(S)$ such that $0 < inf\{V(x): V \in \Omega\} = inf\{(scl^{\epsilon}_{\omega}(\underline{1} - scl^{\epsilon}_{\omega}(U)))(x), U \in \Pi\} \leq inf\{\left(scl^{\epsilon}_{\omega}(\underline{1} - U)\right)(x), U \in \Pi\} = inf\{(\underline{1} - U)(x), U \in \Pi\}$.
Hence $1 > sup\{U(x), U \in \Pi\}$ --- a contradiction.

Theorem 3.4. A fuzzy set S of a fuzzy topological space X is fuzzy S^{ϵ}_{ω}-closed relative to X if and only if every fuzzy prefilterbase Ω on X with the property that $S \leq scl^{\epsilon}_{\omega}(U)$ and $V\hat{q}\, scl^{\epsilon}_{\omega}(U)$ for each $V \in \Omega$ and for each $U \in FS^{\epsilon}_{\omega}O(X)$ fuzzy θ-semi-ω-ϵ-adheres at some point in S.

Proof: Let fuzzy set S of a fuzzy topological space X be fuzzy S^{ϵ}_{ω}-closed relative to X and a fuzzy prefilterbase Ω on X with the property given in the theorem does not fuzzy θ-semi-ω-ϵ-adhere at any point of S. Since for each $x \in supp(S)$, there always exists an $n_x \in N$ such that

$\frac{1}{n_x} < S(x)$ and $n \geq n_x$ implies $x_{\frac{1}{n}} \leq S$, then for each $n \in N$ with $n \geq n_x$, there exist an $U_n^x \in FS_\omega^\epsilon QN(X, x_{\frac{1}{n}})$ and a $V_n^x \in \Omega$ such that $V_n^x \overline{q}\, scl_\omega^\epsilon(U_n^x)$. Clearly, $sup\{U_n^x(x): n \in N$ with $n \geq n_x\} = 1$. Therefore the family $\left\{U_n^x \in FS_\omega^\epsilon QN\left(X, x_{\frac{1}{n}}\right): x \in supp(S), n(\in N) \geq n_x\right\}$ satisfies the property given in Definition 3.1 and so the fuzzy S_ω^ϵ-closedness of S relative to X ensures the existence of $\Psi \subseteq supp(S)$ and $N_0 \subset N$ with $|\Psi| < \aleph_0$ and $|N_0| < \aleph_0$ such that $S \leq \vee\{scl_\omega^\epsilon(U_n^x): x \in \Psi, n(\in N_0) \geq n_x\}$. Consider $U = \vee\{U_n^x: x \in \Psi, n(\in N_0) \geq n_x\}$ $FS_\omega^\epsilon O(X)$ and a $V \in \Omega$ with $V \leq \vee\{V_n^x \in \Omega: x \in \Psi, n(\in N_0) \geq n_x\}$. Then $S \leq scl_\omega^\epsilon(U)$ and $V \overline{q}\, scl_\omega^\epsilon(U)$ --- a contradiction.

Conversely, let fuzzy set S of a fuzzy topological space X be not fuzzy S_ω^ϵ-closed relative to X. Then Theorem 3.2 ensures the existence of a fuzzy prefilterbase Ω in X equipped with fuzzy semi-ω-ϵ-open sets and $(\wedge\{scl_\omega^\epsilon(A): A \in \Omega\}) \wedge S = \underline{0}$ such that $(\wedge\{A: A \in \Omega\}) \hat{q}\, S$ for each $\Omega_0 \subset \Omega$ with $|\Omega_0| < \aleph_0$. Then $\otimes = \{V = \wedge\{A: A \in \Omega_0\}: \Omega_0 \subset \Omega$ with $|\Omega_0| < \aleph_0\}$ is a fuzzy prefilterbase such that $S \leq scl_\omega^\epsilon(U)$ implies $V \hat{q}\, scl_\omega^\epsilon(U)$ for every $V \in \otimes$ and for every $U \in FS_\omega^\epsilon O(X)$. Then the hypothesis ensures the existence of an $x_p \in Pt(X)$ with $x_p \leq S$ such that $x_p \in S_\omega^\epsilon\text{-}ad_\theta(\Omega, X)$. Again $x_p \leq S$ and $(\wedge\{scl_\omega^\epsilon(A): A \in \Omega\}) \wedge S = \underline{0}$ imply that $x_p \hat{q}(\underline{1} - scl_\omega^\epsilon(A))$ for some $A \in \Omega$ and so the fact $scl_\omega^\epsilon(\underline{1} - scl_\omega^\epsilon(A)) \overline{q}$A establishes $x_p \notin S_\omega^\epsilon$-$ad_\theta(\Omega, X)$--- a contradiction.

Definition 3.5. A fuzzy set A of a topological space X is called fuzzy weakly θ-semi-ω-ϵ-closed if for every fuzzy point $x_p \notin A$, there exists a $V \in FS_\omega^\epsilon QN(X, x_p)$ such that $scl_\omega^\epsilon(V) \wedge A = \underline{0}$. Clearly, every fuzzy weakly θ-semi-ω-ϵ-closed set is fuzzy θ-semi-ω-ϵ-closed.

Theorem 3.6. Let A, B be fuzzy subsets of a space X. If A is fuzzy weakly θ-semi-ω-ϵ-closed and B is fuzzy S_ω^ϵ-closed relative to X, then $A \wedge B$ is fuzzy S_ω^ϵ-closed relative to X.

Proof: Let $\Pi = \{U_\alpha: \alpha \in \Delta\}$ be a family of fuzzy semi ω-ϵ-open sets of X with $sup\{U(x): U \in \Pi\} = 1$ for each $x \in supp(S)$. Since A is fuzzy weakly θ-semi-ω-ϵ-closed, then for each $x \in supp(B) - supp(A)$ and each $n \in N$, there exists $V_n^x \in FS_\omega^\epsilon QN(X, x_{\frac{1}{n}})$ such that $scl_\omega^\epsilon(V_n^x) \wedge A = \underline{0}$. Then it is obvious to note that the family $\Pi' = \{U_\alpha: \alpha \in \Delta\} \wedge \{V_n^x: x \in supp(B) - supp(A), n \in N\}$ satisfies the property $sup\{G(x): G \in \Pi'\} = 1$ for each $x \in supp(B)$. Since B is fuzzy S_ω^ϵ-closed relative to X, there exist finite number of points $x_1, x_2, \ldots, x_k \in supp(B) - supp(A)$, finite number of natural numbers $n_1, n_2, \ldots, n_k \in N$ and finite number of indices $\alpha_1, \alpha_2, .\alpha_3 \ldots \ldots \ldots, \alpha_m \in \Delta$ such that $B \leq (\vee_{i=1}^k scl_\omega^\epsilon(V_{n_i}^{x_i})) \vee (\vee_{j=1}^m scl_\omega^\epsilon(U_{\alpha_j}))$ and so $A \wedge B \leq \vee_{j=1}^m scl_\omega^\epsilon(U_{\alpha_j})$. Thus $A \wedge B$ is S_ω^ϵ-closed relative to X.

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