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LIGHT D WAVE MESON SPECTRUM IN A NON-RELATIVISTIC QUARK MODEL WITH INSTANTON INDUCED INTERACTION

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ABSTRACT

The mass spectrum of the D wave mesons has been investigated in the frame work of non-relativistic quark model (NRQM). The Hamiltonian used in the investigation has kinetic energy, confinement potential, one-gluon-exchange potential (OGEP) and instanton induced quark-antiquark interaction (III). The calculated D wave meson masses are in agreement with the experimental D wave meson masses. The respective role of III and OGEP in the D wave meson spectrum is discussed.

Keywords: Quark Model, one-gluon-exchange potential, instanton induced interaction, D wave meson spectra.

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INTRODUCTION

The non-relativistic quark models (NRQM) are a class of phenomenological models developed to explain the hadron interactions. They give the most complete description of hadron properties and are the most successful phenomenological models of hadron structure (Chliapnikov, 2000). The Hamiltonian of these quark models usually contains three main ingredients: the kinetic energy, the confinement potential and a hyperfine interaction term, which has often been taken as an effective one-gluon-exchange potential (OGEP) (De Rujula *et al*., 1975). Other type of hyperfine interaction is Instanton-Induced Interaction (III) deduced by a non-relativistic reduction of the ‘t Hooft interaction (Hooft, 1976).

In the constituent quark model, conventional mesons are bound states of a spin ½ quark and spin ½ antiquark bound by a phenomenological potential. Essentially, in all phenomenological QCD based quark models, the Hamiltonian for the quark system consists of the kinetic energy, the two–body confinement potential and OGEP. The short distance behaviour is dominated by one gluon exchange and the behaviour at large distance is accounted by the confinement potential. The confinement potential must come ultimately from a non-perturbative treatment of QCD, whereas the residual interaction OGEP is based on perturbation theory. It is well known that the colour magnetic interaction term in OGEP is responsible to reproduce many of the hadron properties. By adjusting the strength of OGEP (α_s), colour magnetic interaction can reproduce the hyperfine splittings (Bhaduri *et al*., 1981) as well as the short range repulsion of the two-nucleon systems in the relative S-wave (Khadkikar and Vijaya Kumar, 1991). However, the strength of OGEP determined in this empirical way is much greater than 1, which makes it hard to treat it as the perturbative effect. There exists another QCD based candidate for such a residual, effective interaction obtained by 't Hooft from instanton effects (Blask *et al*., 1990; Semay *et al*., 1997, 1999). The main achievement of III in hadron spectroscopy is the resolution of the $U_A(1)$ problem, which leads to a good prediction of the masses of η and η' mesons.

Previously, we had employed the NRQM (Bhavyshri *et al*., 2005, 2008) and the relativistic harmonic model (RHM)(Khadkikar *et al*., 1983) along with III to investigate the ground state masses of S and P wave mesons. In RHM Hamiltonian (Vijaya Kumar, 2004) we had used, a Lorentz scalar plus vector confinement potential along with two body OGEP and III potential. The results obtained for the S and P wave mesons in both NRQM and RHM showed that the inclusion of III diminishes the relative importance of OGEP and also the contribution of III is essential in explaining the mass difference between η and η' mesons and to reproduce S and P wave meson masses. The aim of our earlier investigations was also to test whether quark gluon coupling constant (α_s) can be treated as perturbative effect and to understand the role played by the III in meson spectra. Having met with good success (Bhavyshri *et al*., 2007, 2008), in this work we have extended the NRQM to study the D wave meson spectra. The full Hamiltonian used in the investigation has, in addition to the kinetic energy, two bodies OGEP and III. The III also has antisymmetric spin-orbit term proportional to $\vec{L}\cdot\vec{\Delta}$ where $\vec{\Delta}$ is defined in terms of the Pauli matrices

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as $\frac{1}{2}\left(\vec{\sigma}_1 - \vec{\sigma}_2\right)$. We have examined the role of antisymmetric spin-orbit term of III which couples $^1L_{J=L}$ and $^3L_{J=L}$ in the K-meson sector.

The Model

In NRQM (Bhavyshri *et al*., 2005) the full Hamiltonian is,

$$H = K + V_{OGEP}(\vec{r}_{ij}) + V_{CONF}(\vec{r}_{ij}) + V_{III}(\vec{r}_{ij}) \quad (1)$$

where

$$K = \sum_{i=1}^{2}\left(M_i + \frac{P_i^2}{2M_i}\right) - K_{CM} \quad (2)$$

where M_i and P_i are the mass and momentum of the i^th^ quark. The K is the sum of the kinetic energies including the rest mass minus the kinetic energy of the center of mass motion (CM) of the total system. The potential energy part consists of confinement term V_{CONF}, the residual interaction V_{OGEP} and instanton induced interaction V_{III}.

The confinement term is taken to be linear.

$$V_{CONF}(\vec{r}_{ij}) = -a_c \mathrm{r}_{ij}\left(\boldsymbol{\lambda}_i.\boldsymbol{\lambda}_j\right) \quad (3)$$

where a_c is the confinement strength. The term r_{ij} here and elsewhere in the paper stands for the relative distance between the two quarks Here, $\boldsymbol{\lambda}_i$ and $\boldsymbol{\lambda}_j$ are the generators of the color SU(3) group for the i^{th} and j^{th} quark,

The following central part of two-body potential due to OGEP is usually employed (De Rujula *et al*., 1975),

$$V_{OGEP}^{cent}(\vec{r}_{ij}) = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i.\boldsymbol{\lambda}_j\left[\frac{1}{r_{ij}} - \frac{\pi}{M_i M_j}\left(1 + \frac{2}{3}\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\right)\delta(\vec{r}_{ij})\right] \quad (4)$$

where the first term represents the residual Coulomb energy and the second term the chromo-magnetic interaction leading to the hyperfine splitting. The $\boldsymbol{\sigma}_i$ is the Pauli spin operator and α_s the quark-gluon coupling parameter.

The non-central part of OGEP has two terms, the spin-orbit interaction $V_{OGEP}^{SO}(\vec{r}_{ij})$ and the tensor term $V_{OGEP}^{TEN}(\vec{r}_{ij})$. The spin-orbit interaction of OGEP is given by

$$V_{OGEP}^{SO}(\vec{r}_{ij}) = -\frac{\alpha_s}{4}\boldsymbol{\lambda}_i.\boldsymbol{\lambda}_j\left[\frac{3}{8M_i M_j}\frac{1}{r_{ij}^3}\left(\vec{r}_{ij}\times\vec{P}_{ij}\right)\cdot\left(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j\right)\right] \quad (5)$$

where the relative angular momentum is defined as usual in terms of relative position $\vec{r}_{ij}$ and the relative momentum $\vec{P}_{ij}$.

The tensor part of the OGEP is,

$$V_{OGEP}^{TEN}(\vec{r}_{ij}) = -\frac{\alpha_s}{4}\boldsymbol{\lambda}_i.\boldsymbol{\lambda}_j\left[\frac{1}{4M_i M_j}\frac{1}{r_{ij}^3}\right]\hat{S}_{ij} \quad (6)$$

where,

$$S_{ij} = [3(\vec{\sigma}_i.\hat{r})(\vec{\sigma}_j.\hat{r}) - \vec{\sigma}_i.\vec{\sigma}_j].$$

The tensor potential is a scalar which is obtained by contracting two second rank tensors.

The non-central part of III has contributions from both spin- orbit and tensor terms. The spin-orbit contribution comes from relativistic corrections to the central potential of III. It is given by (Semay *et al*., 1999),

$$V_{III}^{SO}(\vec{r}_{ij}) = V_{LS}(\vec{r}_{ij})\vec{L}\cdot\vec{S} + V_{L\Delta}(\vec{r}_{ij})\vec{L}\cdot\vec{\Delta} \quad (9)$$

The first term in Eqn. (9) is the traditional symmetric spin-orbit term proportional to the operator $\vec{L}\cdot\vec{S}$. The other term is the anti-symmetric spin-orbit term proportional to $\vec{L}\cdot\vec{\Delta}$ where $\vec{\Delta} = \frac{1}{2}\left(\vec{\sigma}_1 - \vec{\sigma}_2\right)$. The radial functions of Eqn. (9) are expressed as (Semay *et al*., 1999),

$$V_{LS}(\vec{r}_{ij}) = \left(\frac{1}{M_i^2} + \frac{1}{M_j^2}\right)\sum_{k=1}^{2}\kappa_k\frac{\exp(-\mathrm{r}_{ij}^2/\eta_k^2)}{(\eta_k\sqrt{\pi})^3} + \left(\frac{1}{M_i M_j}\right)\sum_{k=3}^{4}\kappa_k\frac{\exp(-\mathrm{r}_{ij}^2/\eta_{k-2}^2)}{(\eta_{k\text{-}2}\sqrt{\pi})^3} \quad (10)$$

and

$$V_{L\Delta}(\vec{r}_{ij}) = \left(\frac{1}{M_i^2} - \frac{1}{M_j^2}\right)\sum_{k=5}^{6}\kappa_k\frac{\exp(-\mathrm{r}_{ij}^2/\eta_{k\text{-}4}^2)}{(\eta_{k\text{-}4}\sqrt{\pi})^3} \quad (11)$$

The term $V_{LS}(\vec{r})$ is responsible for the splitting of the ^3L_J states with $J = L-1,\ \mathrm{L},\ \mathrm{L}+1$. With such a term L is still good quantum numbers but S is not. The term $V_{L\Delta}(\vec{r})$ couples states $^1L_{J=L}$ and $^3L_{J=L}$. Due to the mass dependence in Eqn. (11), it is clear that this term is inoperative when the quarks are identical. In practice the antisymmetric spin orbit term is important only in the K-sector. The κ_i and η_i are free parameters in the theory (Semay *et al*., 1999; Bhavyshri *et al*., 2008). The M_i corresponds to the mass of the strange quark (s) and M_j corresponds to mass of (u/d) quark. This term accounts for the splitting between 1^1D_2 and 1^3D_2 states in the K sector.

The tensor interaction of III is,

$$V_{III}^{TEN}(\vec{r}_{ij}) = \frac{\hat{S}_{ij}}{M_i M_j} \sum_{k=7}^{8} \kappa_k \frac{\exp(-r_{ij}^2/\eta_{k-4}^2)}{(\eta_{k-4}\sqrt{\pi})^3} \quad (12)$$

With the tensor interaction, L is no longer a good quantum number since this term couples the states $^3L_{J=L+1}$ and $^3(L+2)_{J=L+1}$. It is to be noted that III and OGEP have the same spin dependence except for $V_{L\Delta}(\vec{r})$ term.

RESULTS AND DISCUSSIONS

In our calculation we have expressed the product of quark-antiquark oscillator wave functions in terms of oscillator wave functions corresponding to the relative and centre-of-mass coordinates (CM).

We are able to calculate the light D wave meson masses with all η's and κ·s held fixed and by varying only the κ_7 and κ_8 parameters. The parameters used in our model are listed in tables 1 and 2. The oscillator quantum number for the CM wave functions is restricted to N_{cm}= 0. The Hilbert space of relative wave functions is truncated at radial quantum number n_{max} = 4. The Hamiltonian matrix is constructed for each meson separately in the basis states of $\left| N_{CM} = 0, L_{CM} = 0;\ ^{2S+1}L_J \right\rangle$ and is diagonalised.

Table 1. Values of parameters for D wave parameters.

b	0.62 fm
$M_{u,d}$	380 MeV
M_s	560 MeV
a_c	10 MeV fm^{-1}
α_s	0.2
η_1	0.2 fm
η_2	0.29 fm
η_3	1.4 fm
η_4	1.3 fm
κ_1	1.8
κ_2	1.7
κ_3	1.9
κ_4	2.1
κ_5	-22.0
κ_6	-24.5

The masses of the singlet and triplet D wave mesons after diagonalisation in harmonic oscillator basis with n_{max}= 4 are listed in tables 3 and 4, respectively. The results show that III along with OGEP interaction is necessary to obtain the meson mass spectra. If OGEP is taken as the only source of hyperfine interaction, the value of α_s necessary to reproduce the hadrons spectrum is generally much larger than one; this leads to a large spin-orbit interaction, which destroys the overall fit to the spectrum (Bhayshri *et al*., 2005). The important role played by III in obtaining the masses of these mesons can be understood by examining table 5. In table 5, we have listed the calculated masses of triplet D wave mesons without the inclusion of III potential. The role of III is crucial in explaining the mass differences of D wave K mesons. It is interesting to note that the calculated masses without III contribution of triplet D wave mesons involving only u/d quarks are higher than the experimental masses. The inclusion of III lowers the masses of D mesons in u/d sector. In case of other triplet D wave mesons III has attractive contribution.

Table 2. Values of κ_7 and κ_8 parameters.

Meson	κ_7	κ_8
$\omega(1650)$	32.0	46.0
K*(1680)	32.0	46.0
$K_2(1820)$	38.0	46.0
ω_3 (1670)	-13.5	-16.0
K*(1780)	1.5	3.2
ϕ_3 (1850)	29.2	35.0

We have also investigated two singlet light D wave mesons and six triplet light D wave mesons namely π_2 (1670) (1^1D_2) , K_2(1770) (1^1D_2), ω(1650) (1^3D_1), K*(1680) (1^3D_1), K_2(1820) (1^3D_2), ω_3 (1670) (1^3D_3), K*(1780) (1^3D_3), ϕ_3 (1850) (1^3D_3) (Amsler et.al., 2008). In case of singlet mesons, both colour electric (CE) and colour magnetic (CM) parts of OGEP are attractive. However, the dominant contribution to the masses comes from the kinetic energy and linear confinement potential. In case of triplet D wave mesons the contribution of III potential is very significant. We have noted that in case of 1^3D_1 mesons along with the tensor contribution of III, the spin orbit contribution of III is also significant and attractive. The contribution of tensor term of III in case of 1^3D_2 is repulsive. It is to be noted that the anti-symmetric spin orbit potential of the III contributes substantially to the mass difference between the 1^1D_2 and 1^3D_2 mesons in the K meson sector. The mass difference between K*(1680) and K_2(1820) (Burakovsky *et al*., 1997) mesons is due to the large difference in tensor part of III potential which in case of the former is attractive and in the latter case is repulsive. In case of ω_3 (1670) and K*(1780) mesons III contribution is repulsive, whereas in case of ϕ_3 (1850) it is attractive. From tables 3 and 4, we conclude that the calculated meson masses are in good agreement with the experimental masses.

Table 3. Masses of the singlet mesons (in MeV).

$N^{2S+1}L_J$	Meson	Experimental Mass	Calculated Mass
1^1D_2	π_2 (1670)	1670±20	1696.6
	K_2 (1770)	1773±8	1727.4

Table 4. Masses of the triplet mesons (in MeV).

$N^{2S+1}L_J$	Meson	Experimental Mass	Calculated mass
1^3D_1	ω(1650)	1649 ± 24	1649.8
	K*(1680)	1717 ± 27	1720.2
1^3D_2	K_2(1820)	1816 ±13	1817.2
1^3D_3	ω_3 (1670)	1667 ± 4	1667.1
	K*(1780)	1776 ± 7	1778.9
	ϕ_3 (1850)	1854 ± 7	1855.3

Table 5. Masses of triplet mesons (in MeV) without III.

Meson	Experimental Mass	Calculated Mass
ω(1650)	1649 ± 24	1679.9
K*(1680)	1717 ± 27	1716.1
K_2(1820)	1816 ±13	1730.1
ω_3 (1670)	1667 ± 4	1737.4
K*(1780)	1776 ± 7	1755.9
ϕ_3 (1850)	1854 ± 7	1777.7

CONCLUSIONS

We have investigated the effect of the III on the masses of the D wave mesons in the frame work of NRQM. We have shown that the computation of the mesonic masses using only OGEP is inadequate. The contribution of the III is found to be significant. To obtain the masses of D wave mesons, 5x5 Hamiltonian matrix was constructed and was diagonalised. The contribution from the tensor and spin-orbit part of the III is found to be significant in case of triplet D wave mesons. To obtain the physical masses of the mesons in the K sector it is necessary to include the anti-symmetric part of III. There is a good agreement between the calculated and experimental masses of D wave mesons with the inclusion of III.

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