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THE ROLE OF ANTI-SYMMETRIC SPIN-ORBIT POTENTIAL OF III IN DWAVE K MESON SPECTRUM IN THE FRAME WORK OF RELATIVISTIC HARMONIC MODEL

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ABSTRACT

The mass spectrum of the D-wave K mesons is considered in the frame work of relativistic harmonic model (RHM). The full Hamiltonian used in the investigation has the Lorentz scalar plus a vector harmonic-oscillator potential, the confined-one-gluon-exchange potential (COGEP) and the instanton-induced quark-antiquark interaction (III). It is shown that the anti-symmetric spin orbit potential of the III contributes substantially to the mass difference between the 1^1D_2 and 1^3D_2 mesons in the K meson sector. A good description of the mass spectrum is obtained. The respective role of III and COGEP in the D-wave meson spectrum is discussed.

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Keywords: Quark model, confined-one-gluon-exchange potential, instanton induced interaction, D-wave meson spectra.

INTRODUCTION

The phenomenological models developed to explain observed properties of hadrons are either non-relativistic quark models (NRQM) with suitably chosen potential or relativistic models where the interaction is treated perturbatively. The NRQM have been proven to be very successful in describing hadronic properties (Gromes, 1977; Bhaduri *et al*., 1981; Godfrey and Isgur, 1985; Blask, *et al*., 1990; Burakovsky and Goldman, 1997; Chliapnikov, 2000). In most of these works, it is assumed that the quark interaction is dominated by a linear or quadratic confinement potential and is supplemented by a short range potential stemming from the one-gluon exchange mechanism. The Hamiltonian of these quark models usually contains three main ingredients: the kinetic energy, the confinement potential and a hyperfine interaction term, which has often been taken as an effective one-gluon-exchange potential (OGEP) (De Rujula *et al*., 1975). Other types of hyperfine interaction have also been introduced in the literature. For ex: the Instanton-Induced Interaction (III), deduced by a non-relativistic reduction of the ‘t Hooft interaction (Hooft, 1976) has already been successfully applied in several studies of the hadron spectra. The main achievement of III in hadron spectroscopy is the resolution of the U_A (1) problem, which leads to a good prediction of the masses of η and η' mesons.

The success of the NRQM in describing the hadron spectrum is somewhat paradoxical, as light quarks should in principle not obey a non-relativistic dynamics. This paradox has been avoided in many works based on the constituent quark models by using for the kinetic energy term of the Hamiltonian a semi-relativistic or relativistic expression (Semay *et al*., 1997; Brau *et al*., 2000). Even in the existing relativistic models though the effect of confinement of quarks has been taken into account, the effect of confinement of gluons has not been taken into account (Vijaya Kumar and Khadkikar, 1998). In our present work, we have investigated the effect of exchange of confinement of gluons on the masses of light D- wave K mesons in the frame work of RHM with III (Vijaya Kumar *et al*., 2004; Khadkikar and Gupta, 1983).

In our present work, for the confinement of quarks we are making use of the RHM which has been successful in explaining the properties of light hadrons. For the confinement of gluons, we have made use of the current confinement model (CCM) which was developed in the spirit of the RHM (Khadkikar and Vijaya Kumar, 1991). The CCM has been quite successful in describing the glue-ball spectra. The confined gluon propagators (CGP) are derived in CCM. Using CGP we have obtained confined one gluon exchange potential (COGEP) (Vinodkumar *et al*., 1992). The full Hamiltonian used in the investigation has Lorentz scalar plus a vector harmonic-oscillator potential, in addition to two-body central and non-central terms of COGEP and III. In addition, III also has anti-symmetric spin-orbit term proportional to $\vec{L}.\vec{\Delta}$, where $\vec{\Delta}$ is defined in terms of the Pauli matrices as $\frac{1}{2}(\overrightarrow{\sigma_1}-\overrightarrow{\sigma_2})$. The role of the anti-symmetric spin- orbit potential of III is discussed exclusively in this paper. The full discussion of the

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Hamiltonian is given in section 2. A brief discussion of the parameters used in our model follows in section 3. The results of the calculation are presented in the section 4. And the conclusions are given in section 5. In our present work, the total mass of the meson is obtained by calculating the energy Eigen values of the Hamiltonian in the harmonic oscillator basis. The mass difference between the $K_2(1770)$ and $K_2(1820)$ is obtained by diagonalising the Hamiltonian matrix. The most important feature of anti-symmetric spin-orbit part that plays crucial role in $K_2(1820)$ meson is explained.

The Model

In RHM (Khadkikar and Gupta, 1983; Vijaya Kumar *et al.*, 2004) quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic-oscillator potential

$$V_{conf}(\vec{r}) = \frac{1}{2}\left(1+\gamma_0\right)A^2r^2 + M \qquad (1)$$

where γ_0 is the Dirac matrix:

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (2)$$

M is the quark mass and A^2 is the confinement strength. They have a different value for each quark flavour. In RHM, the confined single quark wave function (ψ) is given by:

$$\psi = N\begin{pmatrix} \phi \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M}\phi \end{pmatrix} \qquad (3)$$

with the normalization

$$N = \sqrt{\frac{2(E+M)}{3E+M}}; \qquad (4)$$

where E is an eigenvalue of the single particle Dirac equation with the interaction potential given in (1). The lower component is eliminated by performing the similarity transformation,

$$U\psi = \phi \qquad (5)$$

Where U is given by,

$$\frac{1}{N\left[1+\dfrac{\mathbf{P}^2}{(E+M)^2}\right]}\begin{pmatrix} \mathbf{1} & \dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M} \\ -\dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M} & \mathbf{1} \end{pmatrix} \qquad (6)$$

Here, U is a momentum and state (E) dependent transformation operator. With this transformation, the upper component ϕ satisfies the harmonic oscillator wave equation.

$$\left[\frac{\mathbf{P}^2}{E+M} + A^2r^2\right]\phi = (E-M)\phi\ , \qquad (7)$$

which is like the three dimensional harmonic oscillator equation with an energy-dependent parameter Ω_n^2 :

$$\Omega_n = A\left(E_n + M\right)^{1/2} \qquad (8)$$

The eigenvalue of (7) is given by,

$$E_n^2 = M^2 + (2n+1)\Omega_n^2\,. \qquad (9)$$

Note that eqn. (7) can also be derived by eliminating the lower component of the wave function using the Foldy-Wouthuysen transformation as it has been done in (Khadkikar and Gupta, 1983).

Adding the individual contributions of the quarks we obtain the total mass of the hadron. The spurious centre of mass (CM) is corrected by using intrinsic operators for the $\sum_i r_i^2$ and $\sum_i \nabla_i^2$ terms appearing in the Hamiltonian. This amounts to just subtracting the CM motion zero point contribution from the E^2 expression. It should be noted that this method is exact for the 0S-state quarks as the CM motion is also in the 0S state.

The two body quark-antiquark potential is the sum of COGEP and III potential.

$$V_q(\vec{r}_{ij}) = V_{COGEP}(\vec{r}_{ij}) \ + \ V_{III}(\vec{r}_{ij}) \qquad (10)$$

COGEP is obtained from the scattering amplitude (Khadkikar and Vijaya Kumar,1991)

$$M_{fi} = \frac{g_s^2}{4\pi}\bar{\psi}_i\gamma^\mu\frac{\lambda_i^a}{2}\psi_i D_{\mu\nu}^{ab}(q)\bar{\psi}_j\gamma^\nu\frac{\lambda_j^b}{2}\psi_j\,, \qquad (11)$$

where, $\bar{\psi} = \psi^+\gamma_0$, $\psi_{i/j}$ are the wave functions of the quarks in the RHM, $D_{\mu\upsilon}^{ab} = \partial_{ab}D_{\mu\upsilon}$ are the CCM gluon propagators in momentum representation, $g_s^2/4\pi\ (=\alpha_s)$ is the quark-gluon coupling constant and λ_i is the color $SU(3)_c$ generator of the i^{th} quark. The details can be found in reference (Khadkikar and Vijaya Kumar, 1991; Vinodkumar *et al.*, 1992). Below, we list the expressions for the central, tensor and spin-orbit part of the COGEP. The central part of COGEP is (Vinodkumar *et al.*, 1992)

$$V_{COGEP}^{cent}(\vec{r}_{ij}) = \frac{\alpha_s N^4}{4} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \left[D_0(\vec{r}_{ij}) + \frac{1}{(E+M)^2} \left[4\pi\delta^3(\vec{r}_{ij}) - c^4 r^2 D_1(\vec{r}_{ij}) \right] \left[1 - 2/3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \right] \quad (12)$$

To calculate the matrix elements (MEs) of COGEP, we have fitted the exact expressions of $D_0(\vec{r})$ and $D_1(\vec{r})$ by Gaussian functions. It is to be noted that the $D_0(\vec{r})$ and $D_1(\vec{r})$ are different from the usual Coulombic propagators. However, in the asymptotic limit ($\vec{r} \to 0$) they are similar to Columbic propagators and in the infra-red limit ($\vec{r} \to \infty$) they fall like Gaussian. In the above expression the c (fm^{-1}) gives the range of propagation of gluons and is fitted in the CCM to obtain the glue ball spectra. The $D_0(\vec{r})$ and $D_1(\vec{r})$ are given by,

$$D_0(\vec{r}) = \left(\frac{\alpha_1}{r} + \alpha_2 \right) \exp\left[\frac{-r^2 c_0^{\,2}}{2} \right];$$

$$D_1(\vec{r}) = \frac{\gamma}{r} \exp\left[\frac{-r^2 c_2^{\,2}}{2} \right]$$

Where α_1 = 1.035994, α_2 = 2.016150 fm^{-1}, c_0 = $(3.001453)^{1/2}$ fm^{-1}, γ = 0.8639336

And $c_2 = (4.367436)^{1/2}$ fm^{-1}.

Tensor part of COGEP is,

$$V_{COGEP}^{TEN}(\vec{r}_{ij}) = -\frac{\alpha_s}{4} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \frac{N^4}{(E+M)^2} \left(\frac{D_1''(\vec{r}_{ij})}{3} - \frac{D_1'(\vec{r}_{ij})}{3r} \right) \hat{S}_{ij} \quad (13)$$

Where $\hat{S}_{ij} = [3(\boldsymbol{\sigma}_i.\hat{r})(\boldsymbol{\sigma}_j.\hat{r}) - \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j]$

Where $\hat{\mathbf{r}} = \hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j$ is the unit vector in the direction of $\vec{r}$. In the above expression primes and double primes corresponds to first and second derivatives of $D_1(\vec{r})$. The derivatives of $D_1(\vec{r})$ were fitted to Gaussian functions.

$$D_1'(\vec{r}_{ij}) = \frac{1}{r} \varepsilon \exp\left[\frac{-r^2 c_3^{\,2}}{2} \right] - \frac{1}{r^2} \gamma \exp\left[\frac{-r^2 c_2^{\,2}}{2} \right]$$

$$D_1''(\vec{r}_{ij}) = \frac{2}{r^3} \gamma \exp\left[\frac{-r^2 c_2^{\,2}}{2} \right] - \frac{2}{r^2} \varepsilon \exp\left[\frac{-r^2 c_3^{\,2}}{2} \right] + \frac{1}{r} \kappa r^2 \exp\left[-\frac{r^2 c_4^{\,2}}{2} \right]$$

ε = -1.176029 fm^{-1}, κ =5.118019 fm^{-4} $c_2 = (4.367436)^{1/2}$ fm^{-1} $c_3 = (2.117112)^{1/2}$ fm^{-1}, $c_4 = (3.255009)^{1/2} fm^{-1}$

The spin-orbit part of COGEP is

$$V_{COGEP}^{LS}(\vec{r}_{ij}) = -\frac{\alpha_s}{4} \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \frac{N^4}{(E+M)^2} \frac{1}{2r} \left[\left(\left[\vec{r}_{ij} \times (\vec{p}_i - \vec{p}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \right] \left[D_0'(\vec{r}_{ij}) + 2D_1'(\vec{r}_{ij}) \right] \right) \right. \quad (14)$$

Where $D_0'(\vec{r}_{ij}) =$

$$\frac{1}{r}\left[\beta_1 + r\ \beta_2\right] \exp\left[\frac{-r^2 c_1^{\,2}}{2}\right] - \frac{1}{r^2}\left[\alpha_1 + r\ \alpha_2\right] \exp\left[\frac{-r^2 c_0^{\,2}}{2}\right]$$

$$2 D_1'(\vec{r}_{ij}) = 2\left(\frac{1}{r} \varepsilon \exp\left[\frac{-r^2 c_3^{\,2}}{2} \right] - \frac{1}{r^2} \gamma \exp\left[\frac{-r^2 c_2^{\,2}}{2} \right] \right)$$

Where β_1 =2.680358 fm^{-1}, β_2 =-7.598860 fm^{-2} and $c_1 = (2.373588)^{1/2}$.

It should be noted that in the limit c $\to$ 0, the central, tensor and spin-orbit part of the COGEP goes over to the corresponding potentials of the OGEP.

The central part of III potential is given by (Blask *et al*., 1990; Semay and Silvestre- Brac, 1997),

$$V_{III} = \begin{cases} -8g\delta(r_{ij})\,\delta_{S,0}\,\delta_{L,0}, \ for\ I = 1, \\ -8g'\delta(r_{ij})\,\delta_{S,0}\,\delta_{L,0}, \ for\ I = 1/2, \\ 8\begin{pmatrix} g & \sqrt{2}g' \\ \sqrt{2}g' & 0 \end{pmatrix} \delta(r_{ij})\,\delta_{S,0}\delta_{L,0}, \ for\ I = 0 \end{cases} \quad (15)$$

The symbols S, L and I are respectively the spin, the relative angular momentum and the iso-spin of the system. The g and g' are the coupling constants of the interaction. The Dirac delta-function appearing has been regularized and replaced by a Gaussian- like function:

$$\delta_{ij} \to \frac{1}{(\Lambda\sqrt{\pi})^3} \exp\left[\frac{r_{ij}^2}{\Lambda^2} \right] \quad (16)$$

where Λ is the size parameter.

The non-central part of III has contributions from both spin- orbit and tensor terms. The spin-orbit contribution comes from relativistic corrections to the central potential of III. It is given by,

$$V_{III}^{SO}(\vec{r}_{ij}) = V_{LS}(\vec{r}_{ij}) \vec{L} \cdot \vec{S} + V_{L\Delta}(\vec{r}_{ij}) \vec{L} \cdot \vec{\Delta} \quad (17)$$

The first term in Eqn. (17) is the traditional symmetric spin-orbit term proportional to the operator $\vec{L} \cdot \vec{S}$. The other term is the anti-symmetric spin-orbit term proportional to $\vec{L} \cdot \vec{\Delta}$ where $\vec{\Delta} = \frac{1}{2}\left(\vec{\sigma}_1 - \vec{\sigma}_2\right)$. The radial functions of Eqn. (17) are expressed as (Semay and Silvestre-Brac, 1999),

$$V_{LS}(\vec{r}_{ij}) = \left(\frac{1}{M_i^2} + \frac{1}{M_j^2}\right)\sum_{k=1}^{2}\kappa_k \frac{\exp(-r_{ij}^2/\eta_k^2)}{(\eta_k\sqrt{\pi})^3} + \left(\frac{1}{M_i M_j}\right)\sum_{k=3}^{4}\kappa_k \frac{\exp(-r_{ij}^2/\eta_{k-2}^2)}{(\eta_{k-2}\sqrt{\pi})^3} \quad (18)$$

and

$$V_{L\Delta}(\vec{r}_{ij}) = \left(\frac{1}{M_i^2} - \frac{1}{M_j^2}\right)\sum_{k=5}^{6}\kappa_k \frac{\exp(-r_{ij}^2/\eta_{k-4}^2)}{(\eta_{k-4}\sqrt{\pi})^3} \quad (19)$$

The term $V_{LS}(\vec{r})$ is responsible for the spitting of the ${}^3L_J$ states with $J = L-1$, L, L+1. With such a term L is still good quantum numbers but S is not. The term $V_{L\Delta}(\vec{r})$ which couples states ${}^1L_{J=L}$ and ${}^3L_{J=L}$. Due to the mass dependence in Eqn. (19), it is clear that this term is inoperative when the quarks are identical. In practice the antisymmetric spin obit term is important only in the K-sector. The terms κ_i and η_i are free parameters in the theory (Semay and Silvestre-Brac, 1999). M_i corresponds to the mass of the strange quark (s) and M_j corresponds to mass of u/d quark. This term accounts for the splitting between 1^1D_2 and 1^3D_2 states in the K sector.

The tensor interaction of III is

$$V_{III}^{TEN}(\vec{r}_{ij}) = \frac{\hat{S}_{ij}}{M_i M_j}\sum_{k=7}^{8}\kappa_k \frac{\exp(-r_{ij}^2/\eta_{k-4}^2)}{(\eta_{k-4}\sqrt{\pi})^3} \quad (20)$$

With the tensor interaction, L is no longer a good quantum number since this term couples the states ${}^3L_{J=L+1}$ and ${}^3(L+2)_{J=L+1}$.

Fitting Procedure

The parameters of the RHM are the masses of the quarks, M_u=M_d and Ms, the respective confinement strengths, A_u^2=A_d^2, A_s^2, and the oscillator size parameter b_n (=1/Ω_n). They have been chosen to reproduce various nucleons' properties: the root mean square charge radius, the magnetic moment and the ratio of the axial coupling to the vector coupling (Khadkikar and Gupta, 1983). The confinement strength $A_{u,d}$ is fixed by the stability condition for the nucleon mass against the variation of the size parameter b_n

$$\frac{\partial}{\partial b_n}\langle N | H | N \rangle = 0 \quad (21)$$

The parameters associated with the strange quark M_s and A_s^2 have been fitted in order to reproduce the magnetic moments of the strange baryons. The α_s of COGEP is fixed from S wave meson spectroscopy (Vijaya Kumar *et al*., 2004). The value of α_s turns out to be 0.2 for D wave mesons, which is compatible with the perturbative treatment. Among the non central parts of the potential, the hyperfine terms of III has 12 additional strength and size parameters κ's and η's (in Eqns. 18-20) respectively. We note that the κ values can have both positive and negative values (Semay and Silvestre-Brac, 1997, 1999). The values of the III parameters κ's and η's are fixed from S and P wave meson spectroscopy (Bhavyashri *et al*., 2005, 2008) and are listed in table 1.

RESULTS AND DISCUSSION

In the present study, the product of quark-antiquark oscillator wave functions is expressed in terms of oscillator wave functions corresponding to the relative and CM coordinates. The oscillator quantum number for the CM wave functions are restricted to N_{CM} = 0. The Hilbert space of relative wave functions is truncated at radial quantum number n = 4. The Hamiltonian matrix is constructed for each meson separately in the basis states of $\left| N_{CM} = 0, L_{CM} = 0; {}^{2S+1}L_J \right\rangle$ and diagonalised**.** The oscillator size parameter b is fixed by minimizing the expectation value of the Hamiltonian for the vector mesons. We constructed 5x5 Hamiltonian matrix for both $K_2(1770)$ and $K_2(1820)$ in the harmonic oscillator basis. It should be noted that $K_2(1770)$ receives contribution only from the central part of COGEP, where as $K_2(1820)$ receives additional contribution from the tensor and spin-orbit part of COGEP and III. The contributions of the various terms to the masses of the K mesons are listed in table 2. The contributions from the central part of the colour electric and colour magnetic terms of COGEP are found to be negligible. But, there is a substantial repulsive contribution from the tensor part of III. Table 3 gives the masses of the $K_2(1770)$ and $K_2(1820)$ without III after diagonalsing the 5x5 hamiltonian matrix. Though, the mass of the $K_2(1770)$ is in agreement with the experimental results, the theoretically calculated mass of the $K_2(1820)$ is in very poor agreement with the observed masses. Table 4 gives the masses of the mesons after diagonalisation including III but without $V_{III}^{L\Delta}$. Having diagonalised the Hamiltonian matrix, the anti-symmetric term of III was added by diagonalising eqn. 19. The eqn.19 mixes $K_2(1770)$ and $K_2(1820)$ mesons. The $V_{III}^{L\Delta}$ lowers the masses of both $K_2(1820)$ and $K_2(1770)$ meson states. For both K2(1770) and $K_2(1820)$, the contribution of III is attractive. Table 5 gives the masses of the $K_2(1770)$ and $K_2(1820)$ by carrying out the diagonalisation by including the $V_{III}^{L\Delta}$. From table 5, the calculated meson masses are in good agreement with the experimental masses.

Table 1. values of the parameters used in the model.

b	0.62 fm
M_{ud}	380.0 MeV
M_s	560.0 MeV
α_s	0.2
η_1	0.2 fm
η_2	0.29 fm
η_3	1.4 fm
η_4	1.3 fm
κ_1	1.8
κ_2	1.7
κ_3	1.9
κ_4	2.1
κ_5	-22.0
κ_6	-24.5
k_7	36.0
k_8	45.0

CONCLUSIONS

In this work, we have investigated the effect of anti-symmetric spin-orbit term of III on the masses of the $K_2(1770)$ and $K_2(1820)$ mesons in the frame work of RHM. To obtain the masses of $K_2(1720)$ and $K_2(1820)$ mesons, 5x5 matrix was constructed and was diagonalised. The contribution from the tensor and spin-orbit part of the III is found to be significant $K_2(1820)$. To obtain the physical masses of the mesons in the K sector it is necessary to include the anti-symmetric part of III. To conclude, we have investigated the effect of the III on the masses of the D wave K mesons in the frame work of RHM. We have shown that the computation of the masses using only COGEP is inadequate. The contribution of the III is found to be significant. It is shown that the diagonalisation of the interaction matrix using the anti-symmetric III spin orbit potential leads to the lowering of the masses of the tensor and pseudovector meson so as to agree with the observed masses.

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Table 3. The masses of K- mesons without III contribution (in MeV).

Meson	Experimental Mass	Calculated Mass without III
$K_2(1770)$	1773±8	1768.8
$K_2(1820)$	1816±13	1768.2

Table 4. The masses of K- mesons without anti-symmetric spin-orbit III contribution (in MeV).

Meson	Experimental Mass	Calculated Mass without $V_{III}^{L\Delta}$
$K_2(1770)$	1773±8	1768.8
$K_2(1820)$	1816±13	1976.52

Table 5. Masses of K-mesons (in MeV).

Meson	Experimental Mass	Calculated Mass
$K_2(1770)$	1773±8	1761.18
$K_2(1820)$	1816±13	1818.6

Appendix: Matrix elements of the anti-symmetric spin-orbit potential ($\vec{L}\cdot\vec{\Delta}$)

In Eqn. (17) $\vec{L}\cdot\vec{\Delta}$ is the anti-symmetric spin-orbit potential $V_{L\Delta}(\vec{r})$, where $\vec{\Delta}=\frac{1}{2}\left(\vec{\sigma}_i-\vec{\sigma}_j\right)$. $V_{L\Delta}(\vec{r})$ couples states ${}^1L_{J=L}$ and ${}^3L_{J=L}$. We have to evaluate the matrix element,

Table 2. The contributions to the masses of K-mesons by the colour-electric (CE), colour-magnetic (CM), spin orbit and tensor terms of COGEP and spin orbit, tensor terms of III (in MeV).

Meson	V_{conf}	V_{COGEP}^{CE}	V_{COGEP}^{CM}	V_{COGEP}^{LS}	V_{COGEP}^{TEN}	V_{III}^{LS}	V_{III}^{TEN}
$K_2(1770)$	1770.786	-3.131	1.220	-	-	-	-
$K_2(1820)$	1770.786	-3.131	-0.407	0.6027	0.3776	-9.913	220.595

D = $\langle l`, S=0; j,m | f(r)\, \mathbf{L}_{ij}^{1} \cdot \frac{(\vec{\sigma}_i - \vec{\sigma}_j)}{2} | l,\ S=1; j,m \rangle$ (A.1)

Where f(r) are radial integrals. $\vec{\sigma}_i$ acts on the first quark and $\vec{\sigma}_j$ acts on the second quark. Since $\mathbf{L}_{ij}^{1}\cdot$ and $\left(\vec{\sigma}_i - \vec{\sigma}_j\right)$ commute,

Using Eq. (7.1.6) of ref [R.Edmonds, 1974],

D= $(-1)^{l+S'+J}\begin{Bmatrix} J & S' & l' \\ 1 & l & S \end{Bmatrix}\langle l' \| f(r) L_{ij}^{1} \| l \rangle \langle S \| (\vec{\sigma}_i - \vec{\sigma}_j)/2 \| S \rangle$ (A.2)

= $f(r)\,(-1)^{l+S'+J}\begin{Bmatrix} J & S' & l' \\ 1 & l & S \end{Bmatrix}\sqrt{l(l+1)(2l+1)}\delta_{ll'}\langle S \| (\vec{\sigma}_i - \vec{\sigma}_j)/2 \| S \rangle$ (A.3)

Evaluation of the reduced matrix element

$\langle S \| (\vec{\sigma}_i - \vec{\sigma}_j)/2 \| S \rangle$

$\langle S \| (\vec{\sigma}_i - \vec{\sigma}_j)/2 \| S \rangle$ =

$\frac{1}{2}\left[\langle 1/2, 1/2, S' \| (\vec{\sigma}_i) \| 1/2, 1/2, S \rangle - \langle 1/2, 1/2, S' \| (\vec{\sigma}_j) \| 1/2, 1/2, S \rangle\right]$

Using Eq. (7.1.7) of ref. [R.Edmonds, 1974]

$\langle 1/2, 1/2, S'=0 \| (\vec{\sigma}_i) \| 1/2, 1/2, S=1 \rangle$

= $(-1)^{\frac{1}{2}+\frac{1}{2}+S+1}\left((2S+1)(2S'+1)\right)^{1/2}\begin{Bmatrix} 1/2 & S' & 1/2 \\ S & 1/2 & 1 \end{Bmatrix}\langle 1/2 \| \vec{\sigma}_i \| 1/2 \rangle$

= $(-1)^{\frac{1}{2}+\frac{1}{2}+1+1}\sqrt{3}\begin{Bmatrix} 1/2 & 0 & 1/2 \\ 1 & 1/2 & 1 \end{Bmatrix}\langle 1/2 \| \vec{\sigma}_i \| 1/2 \rangle$

= $-\sqrt{3} \times \frac{1}{\sqrt{6}} \times \sqrt{6} \quad = -\sqrt{3}$

Using Eq. (7.1.8) of Ref [R.Edmonds, 1974],

$\langle 1/2, 1/2, S'=0 \| (\vec{\sigma}_j) \| 1/2, 1/2, S=1 \rangle$

= $(-1)^{\frac{1}{2}+\frac{1}{2}+S'+1}\left((2S+1)(2S'+1)\right)^{1/2}\begin{Bmatrix} 1/2 & S' & 1/2 \\ S & 1/2 & 1 \end{Bmatrix}\langle 1/2 \| \vec{\sigma}_j \| 1/2 \rangle$

= $(-1)^{\frac{1}{2}+\frac{1}{2}+0+1}\sqrt{3}\begin{Bmatrix} 1/2 & 0 & 1/2 \\ 1 & 1/2 & 1 \end{Bmatrix}\langle 1/2 \| \vec{\sigma}_j \| 1/2 \rangle$

= $\sqrt{3}$

Therefore, $\langle S \| (\vec{\sigma}_i - \vec{\sigma}_j)/2 \| S \rangle \ = \ -\sqrt{3}$

Hence, for l=2, $S=1, S'=0$ and $J = L$=2, Eq. (A.3) gives, D = $\sqrt{6} f(\vec{r})$

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