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ON SPECIAL GENERATED GOLDBACH TYPE SEQUENCES OF EVENS WITH A COMMON PRIME

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ABSTRACT

The aim of this paper is to study some special sequences of positive even integers that can be expressed as a sum of two primes; one of the primes being common. These special sequences of positive integers are generated by an algorithm. The algorithm, when supplied with a 'suitable' even integer *n*, and a prime *p* such that $p < n$, generates the aforementioned sequence of even integers. The study of these generated sequences lead to very interesting conjectures and also a possible alternate method of tackling the Goldbach conjecture. A very interesting feature of the algorithm is that in terms of computational complexity, it is very efficient in expressing a 'suitable' even integer *n* as a sum of two primes.

Keywords: Special goldbach, type sequences, common prime.

INTRODUCTION AND DISCUSSION

There have been various results in Number Theory that have been motivated by the 'Goldbach Conjecture'. The conjecture states that every even integer greater than 4 is expressible as the sum of two odd primes (Richard, 1994). One of the important results motivated by the Goldbach conjecture is 'Every prime greater than 7 can be expressed as a sum of 3 odd primes', also called Goldbach's weak conjecture. This conjecture has been verified by Vinogradov for sufficiently large odd numbers. Another interesting result that has been proved is that the Generalized Riemann hypothesis implies Goldbach's weak conjecture (Deshouillers *et al*., 1997). It has also been proven that every large even integer is a sum of four squares of primes and 8330 powers of 2 (Jianya and Liu, 2000). Evidently, this area of Number Theory has become a very important area of research and is of interest to the Mathematical community in general. In this article, we conjecture that given a suitable positive integer *n* greater than 4 and a fixed prime *p*, such that $p < n$, a sequence of even integers generated by *n* can be found satisfying certain conditions and these conditions lead to various interesting results. We will first state the conjecture and follow it up with an algorithm and an example.

Conjecture

Conjecture: Let *n* be a given positive even integer greater than 4 and let *p* be a prime number less than *n*. Then one of the following conditions is satisfied:

a) *n* generates a sequence of even integers $m_1, m_{2.,} ... m_n$ and a sequence of primes $q, q_1, p_1, p_{2.,} ... p_n$ such that $n = q + q_1, m_1 = q + p_{1,} m_2 = q + p_{2,} ... m_n = q + p_n$. In other words, the even integer *n* and the sequence of even integers generated by *n* are expressible as the sum of two primes with one prime *q* in common

b) *n* generates a sequence of even integers $m_1, m_{2.,} ... m_n$ and *n* is expressible as a sum of two primes i.e. $n = q + q_1$ and also every even integer in a subset of the generated sequence is expressible as a sum of two primes with the prime q in common. In other words, the generated sequence of positive even integers $m_1, m_{2.,} ... m_n$ is either expressible as a sum of two primes with the prime *q* in common or expressible as the sum of the prime *q* and another composite positive integer.

c) *n* generates a sequence of even integers $m_1, m_{2.,} ... m_n$ and *n* is not expressible as a sum of two primes but every integer in a subset of the generated sequence is expressible as a sum of two primes with one prime in common. Consequently, if such an *n* is encountered again in a separate algorithmic process, then that *n* will not be expressible as a sum of two primes.

d) Given a range of positive even integers $[a, n]$, varying the prime *p*; $p < n$, enables us to write every even integer in $[a, n]$ as a sum of two primes.

Algorithm

An algorithm for the above conjecture is given below: Let *n* be an even integer greater than 4 and let the prime $p = 3$.

a) Start with the even number (greater than 4), *n*.
b) Check if $n = 3 + prime$

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c) If not, find the numbers less than n and relatively prime to n i.e. find $\phi(n)$
d) Consider $m_1 = n - \varphi(n)$
e) Now check if $m_1 = 3 + prime$
f) If not repeat steps c) to step e) with m_1 replacing n.
g) Stop when $m_k = 3 + prime$, for some integer k.
h) Denote the prime in step g) by q.
i) Check if $m_{k-1} - q = prime, m_{k-2} - q = prime, m_{k-3} - q = prime ... m_1 - q = prime$

We will work out the steps in the algorithm for a given even integer greater than 4, say 156.

Example
The Algorithm for n=156 and prime p=3
a) $156 \neq 3 + prime$.
b) So find $\phi(156) = 48$.
c) $m_1 = 156 - \phi(156) = 156 - 48 = 108.$
d) Now check if $m_1 = 3 + prime$. But, $108 = 3 + 105$ (105 is not a prime)
e) If not repeat steps b) to step d) with m_1 replacing n.
f) Now $m_1 = 108.$
g) Find $\phi(108) = 36$.
h) $m_2 = 108 - \phi(108) = 108 - 36 = 72.$
i) Now check if $m_2 = 3 + prime$. But, $72 = 3 + 69$ (69 is not a prime)
j) So repeat steps b) to step d) with m_2 replacing m_1.
k) Now $m_2 = 72.$
l) So find $\phi(72) = 24$.
m) $m_3 = 72 - \phi(72) = 72 - 24 = 48.$
n) Now check if $m_3 = 3 + prime$. But, $48 = 3 + 45$ (45 is not a prime)
o) If not repeat steps b) to step d) with m_3 replacing m_2.
p) Now $m_3 = 48.$
q) So find $\phi(48) = 16$.
r) $m_4 = 48 - \phi(48) = 48 - 16 = 32.$
s) Now check if $m_4 = 3 + prime$. $32 = 3 + 29$ (29 is a prime)
t) So $q = 29.$
u) Check if $m_{k-1} - q = prime, m_{k-2} - q = prime, ..., m_1 - q = prime$. Note that $48 - q = 48 - 29 = 19(prime)$. So $48 = 19 + 29.$ $72 - q = 72 - 29 = 43(prime)$. So $72 = 29 + 43.$
v) Next, $108 - q = 108 - 29 = 79(prime)$. So $108 = 29 + 79.$ Finally, $156 - q = 156 - 29 = 127$. So $156 = 29 + 127.$

Note that this has not only accomplished that $156 = 29 + 127.$, but also that $72 = 29 + 43.$, $48 = 29 + 19.$ and $32 = 29 + 3.$ The common prime is 29.

The number of steps required to get the common prime 29 is 4.

Tests for the range 4-100
At this stage, we will introduce some terminology. An even integer for which the algorithm works is termed *easy even*, otherwise it is termed *difficult even*. The algorithm generates a sequence of even integers and we call these even integers *intermediate evens*. An intermediate even for an *easy even* is called a *valid intermediate even* if is expressible as a sum of two primes, *one prime being common*. Else it is called an *invalid intermediate even*.

Next we run the algorithm for even integers between 6 and 100 (both inclusive)

The results of the algorithm are given in table 1.

As observed in the above table, in the range 6-100, the algorithm works for 38 even integers and does not work for the remaining 10 even integers. So there are 38 easy evens in the range 6-100 and 10 difficult evens. The difficult evens are 28, 38, 52, 58, 68, 78, 80, 94, 96, 98. Also we observe that for all the easy evens in the range 6-100, the intermediate evens are all valid. The first even integer that is an easy even and also has an invalid intermediate even is 114. The intermediate evens for 114 are 24,36,54,78 and the common prime is 13. Observe that $114 = 13 + 101, 54 = 13 + 41, 36 = 13 + 23, 24 = 13 + 11$ But $78 = 13 + 65$ and 65 is not a prime. So 54, 36, 24 are valid intermediate evens and 78 is invalid intermediate even. From table 1, observe that 78 is a difficult even. So in this situation, the invalid intermediate even is also a difficult even. This seems to be true in general and is a very interesting observation. In other words, if a difficult even is encountered in the algorithmic process, it turns out to be an invalid intermediate even for other even integers. This is noted in part c) of the conjecture.

It would be interesting to observe the changes if the prime p in the algorithm is changed from 3 to 5. Changing the prime 3 to the prime 5 in the algorithm results in the following observations.

Table 1.

Even Integer	Easy Even	Intermediate Evens	Valid Intermediate Even	Invalid Intermediate Even	Common Prime
6	Yes	None	N/A	N/A	3
8	Yes	None	N/A	N/A	5
10	Yes	None	N/A	N/A	7
12	Yes	8	8	N/A	5
14	Yes	None	N/A	N/A	11
16	Yes	None	N/A	N/A	13
18	Yes	8, 12	8, 12	N/A	5
20	Yes	None	N/A	N/A	17
22	Yes	None	N/A	N/A	19
24	Yes	16	16	N/A	13
26	Yes	None	N/A	N/A	23
28	No				
30	Yes	22	22	N/A	19
32	Yes	None	N/A	N/A	29
34	Yes	None	N/A	N/A	31
36	Yes	24,16	24,16	N/A	13
38	No				
40	Yes	None	N/A	N/A	37
42	Yes	22,30	22,30	N/A	19
44	Yes	None	N/A	N/A	41
46	Yes	None	N/A	N/A	43
48	Yes	32	32	N/A	29
50	Yes	None	N/A	N/A	47
52	No				
54	Yes	16,24,36	16,24,36	N/A	13
56	Yes	None	N/A	N/A	53
58	No				
60	Yes	44	44	N/A	19
62	Yes	None	N/A	N/A	59
64	Yes	None	N/A	N/A	61
66	Yes	46	46	N/A	43
68	No				
70	Yes	None	N/A	N/A	67
72	Yes	32,48	32,48	N/A	29
74	Yes	None	N/A	N/A	71
76	Yes	None	N/A	N/A	73
78	No				
80	No				
82	Yes	None	N/A	N/A	59
84	Yes	44,60	44,60	N/A	41
86	Yes	None	N/A	N/A	83
88	Yes	32,48	32,38	N/A	29
90	Yes	46,66	46,66	N/A	43
92	Yes	None	N/A	N/A	89
94	No				
96	No				
98	No				
100	Yes	None	N/A	N/A	97

The prime 3 is replaced by 5 in step b) and the algorithm is run for even integers between 6 and 100 (Table 2).

In the range 10-100, there are 36 easy evens and 10 difficult evens. The difficult evens are 32, 40, 44, 50, 56, 62, 82, 92, 98, and 100 and comparing with the results in table 1, we observe that all of these except 98 were easy evens when the algorithm was run with the prime 3. Thus running the algorithm with the primes p=3 and p=5, we were able to write every even integer in the range 6-100 except 98 as a sum of two odd primes. Replacing *p* by primes less than 100 but other than 3 and 5, enables us to write every even integer in the range 6 to 100 as a sum of two primes.

Table 2.

Even Integer	Easy Even	Intermediate Evens	Valid Intermediate Even	Invalid Intermediate Even	Common Prime
10	Yes	None	N/A	N/A	5
12	Yes	None	N/A	N/A	7
14	Yes	8	8	N/A	3
16	Yes	None	N/A	N/A	11
18	Yes	None	N/A	N/A	13
20	Yes	12	12	N/A	7
22	Yes	None	N/A	N/A	17
24	Yes	None	N/A	N/A	19
26	Yes	8, 14	8, 14	N/A	3
28	Yes	None	N/A	N/A	23
30	Yes	22	22	N/A	17
32	No				
34	Yes	None	N/A	N/A	29
36	Yes	None	N/A	N/A	31
38	Yes	12, 20	N/A	N/A	7
40	No				
42	Yes	None	N/A	N/A	37
44	No				
46	Yes	None	N/A	N/A	41
48	Yes	None	N/A	N/A	43
50	No				
52	Yes	None	N/A	N/A	47
54	Yes	36	36	N/A	23
56	No				
58	Yes	None	N/A	N/A	53
60	Yes	24, 44	24	44	19
62	No				
64	Yes	None	N/A	N/A	59
66	Yes	None	N/A	N/A	61
68	Yes	36	36	N/A	31
70	Yes	46	46	N/A	29
72	Yes	67	67	N/A	67
74	Yes	12, 20, 38	12, 20, 38	N/A	7
76	Yes	None	N/A	N/A	71
78	Yes	None	N/A	N/A	73
80	Yes	48	48	N/A	37
82	No				
84	Yes	79	79	N/A	79
86	Yes	24, 44	24	44	19
88	Yes	None	N/A	N/A	83
90	Yes	66	66	N/A	29
92	No				
94	Yes	None	N/A	N/A	89
96	Yes	64	64	N/A	59
98	No				
100	No				

Tests in an arbitrary range

After having verified the conjecture for the range 6-100, we will next test our algorithm for an arbitrary range of numbers 12000000-12000050 with prime p=3 and compile the results (Table 3).

In this range, the algorithm works for 3 numbers, 12000000, 12000020 and 12000032. The algorithm does not work for the remaining 23 numbers. We observe that the number of easy evens have declined and the number of difficult evens have increased.

With the prime ‘5’, the algorithm works for 4 numbers, 12000000, 12000002, 12000022 and 12000034 as seen in table 4.

We conjecture that if we run the algorithm with all primes p less than 12000000, we should be able to express each of the even integer less than 12000000 as a sum of two

Table 3.

Even Integer	Easy Even	Intermediate Evens	Valid Intermediate Even	Invalid Intermediate Even	Common Prime
12000000	Yes	None	N/A	N/A	3
12000020	Yes	None	N/A	N/A	3
12000032	Yes	8800000	N/A	N/A	8799997

Table 4.

Even Integer	Easy Even	Intermediate Evens	Valid Intermediate Even	Invalid Intermediate Even	Common Prime
12000000	Yes	8800000; 5600000; 3680000; 2272000; 1376000; 838400; 505600; 305920; 184064; 92160; 67584	5600000; 1376000; 67584	8800000; 3680000; 2272000; 838400; 505600; 305920; 184064; 92160.	67579
12000002	Yes	6857186; 3918398; 242238; 1139558; 672542; 362150; 217310; 133310; 79990; 49750; 29950; 17990; 11846; 5924; 2964; 2100; 1620; 1188; 828	362150; 17990; 5924; 2964; 2100; 1620; 828	6857186; 3918398; 2242238; 1139558; 672542; 217310; 133310; 79990; 49750; 29950; 11846; 1188;	3
12000022	Yes	8800000	N/A	N/A	8799997
12000034	Yes	8800000	N/A	N/A	8799997

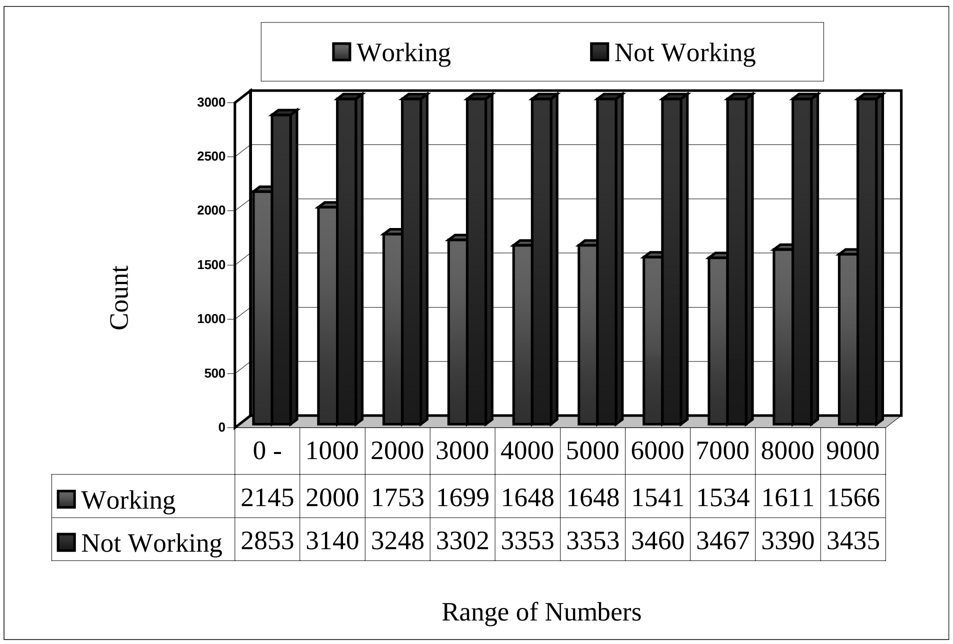

Fig. 1. Algorithmic Behaviour for even numbers from 4-100000.

primes. As seen above, for a fixed *p* in the algorithm, the number of easy evens appears to be decreasing as *n* gets larger. This phenomenon is observed for the range of even integers 4-100000 with the prime *p*=3 in the algorithm. Next, we explore this phenomenon.

Easy evens vs. Difficult evens in the long run

Even for this small range of numbers, the easy evens (blue bars-evens for which the algorithm works) decreases and the difficult evens (red bars-evens for which the algorithm fails) increases as *n* gets larger (Fig. 1). This along with the Prime Number Theorem might tempt one to speculate that the Goldbach conjecture might be false. But then if we were to vary the prime *p* in the algorithm and run the algorithm for all primes less than any given *n*, one might be tempted to speculate that the Goldbach conjecture is true. Further study could be done to determine the relationship between the even integers in the generated sequence to shed some light on these observations.

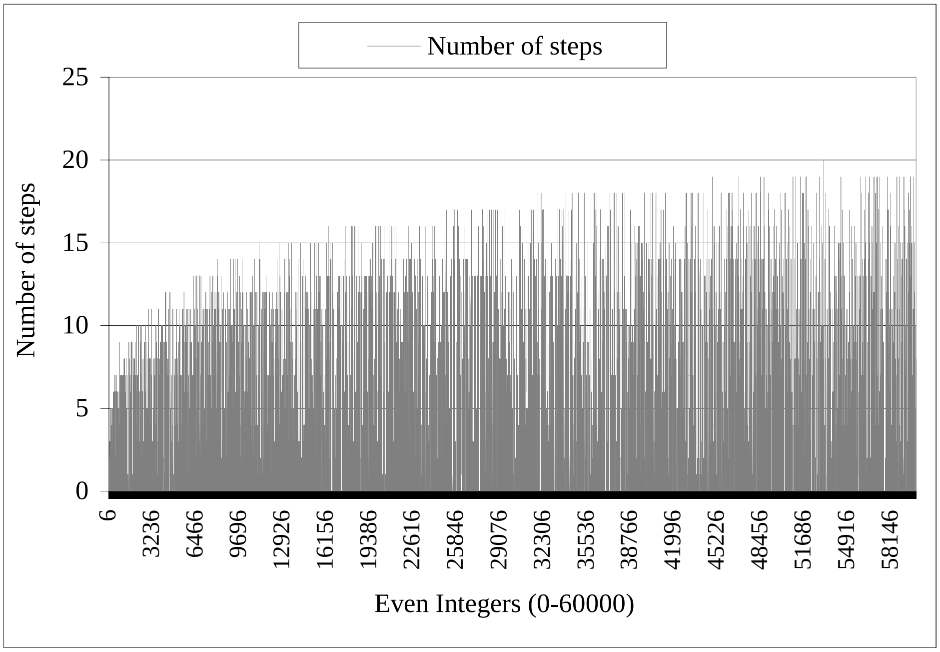

Fig. 2. Efficiency of Algorithm.

Efficiency of the Algorithm

The algorithm was tested for its efficiency for even integers in the range 0-60000 with the prime p =3 in the algorithm. We will denote the number of steps required in the algorithm to express an easy even as a sum of two primes by $\chi(\mathrm{n})$. Fig. 2.

The algorithm is efficient in writing the given easy even as a sum of two primes and the maximum number of steps encountered for the range 6-60000 is 20. The graph of $\chi(\mathrm{n})$. seems to display the behavior of a logarithmic function. Also note that by the Prime number theorem, the number of primes less than or equal to a real number x is denoted by $\pi(\mathrm{x})$ and is approximated by $\frac{x}{\ln(x)}$. One might again be tempted to explore the relation between $\chi(\mathrm{n})$ and $\pi(\mathrm{n})$.

CONCLUSION

The program written for the algorithm was run on a personal computer with usual resources and an interested reader with access to better computing facilities can run the program by varying the prime p and make more interesting observations. The conjecture that any even that is a '*difficult even'* turns out to be an '*invalid intermediate even'* when it occurs again in a separate algorithmic process is interesting and this lends structure and validity to the algorithm. This has been verified to be true for all the *invalid intermediate evens* tried by the author. The algorithm also presents a different way of looking at the Goldbach conjecture. An interested reader can adopt the algorithm and the various observations and with more work, both numerically and mathematically, come up with interesting concrete results.

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