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INTEGRATED OPTIMAL SOLUTION FOR VARIABLE DETERIORATING INVENTORY SYSTEM OF VENDOR – BUYER WHEN DEMAND IS QUADRATIC

Ajay S Gor[1], *Nita H Shah[2] and Chetan Jhaveri[3]
[1]Pramukh Swami Science and H D Patel Arts College, Kadi
[2]Department of Mathematics, Gujarat University, Ahmedabad
[3]S L Institute of Business Administration, Ahmedabad, Gujarat, India

ABSTRACT

An integrated optimal policy for the vendor and the buyer is studied when units in inventory are subject to deterioration at different rates and demand is quadratic. It is shown numerically that the integrated approach reduces the total joint cost significantly when compared with the independent decision of both the players. To encourage the buyer to place order of larger size, a permissible trade credit is offered by the vendor to the buyer to settle the account. A negotiation factor is incorporated to share the benefits of cost savings.

Keywords: Integrated optimal strategy, deterioration, permissible delay period, quadratic demand.

INTRODUCTION

The inventory models under the assumption of linearly trended demand and exponentially time – varying demand are extensively analyzed by the researchers. When demand is taken as linear, it is assumed that the demand changes uniformly over the time which is not observed in the market, in general. On the other hand, an exponentially time – varying demand means exponential rate of change of demand which is unrealistic for the newly launched products in the market (Silver and Meal, 1969; Silver, 1979; Xu and Wang, 1991; Chung and Ting, 1993, 1994; Bose *et al*., 1995; Hariga, 1995; Giri and Chaudhuri, 1997; Lin *et al*., 2000; Mehta and Shah, 2003, 2004). In order to have alternative demand pattern quadratic demand is considered. This type of demand is partially constant, partially varies linearly with time and partially varies exponentially with time. This type of demand is observed for a substitutable products, seasonal goods, fashion apparels etc.

Most of the models so far derived where buyer is the sole decision maker for "when to order and how much to order?" The optimal solution of the buyer may not be agreeable for the vendor. So in competitive global market, an integrated policy should be thought when decision is to be made which is favorable to both the parties. The vendor – buyer integration was first studied by Clark and Scarf (1960). Banerjee (1986) developed a joint lot-size inventory model when production at vendor's end is finite. Goyal (1988) extended Banergee's model by relaxing the assumption of the lot – for –lot production.

Deterioration is defined as the decay, spoilage, evaporation and loss of utility of a product from the original one. Fruit and vegetables, cosmetics and medicines, electronic items, blood components, radioactive chemicals, agriculture produce are some of the examples of deteriorating commodities. For articles on deteriorating inventory one can refer to Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001). Yang and Wee (2005) derived a win – win strategy for an integrated system of vendor-buyer when units in inventory are subject to constant rate of deterioration and deterministic constant demand. Shah *et al*. (2008) extended above model by incorporating salvage value to the deteriorated units.

In this article, an integrated vendor – buyer inventory system is studied when units in inventory deteriorate at different rate and demand is quadratic. A negotiation factor is incorporated to share the cost savings. A permissible delay in payment is offered to the buyer by the vendor to make a joint strategy beneficial. A numerical example is illustrated to support the proposed model. Sensitivity analysis is carried out to visualize the changes in cost savings.

Assumptions and Notations

The mathematical model is developed under following assumptions and notations:

Assumptions

1. An inventory system of single – vendor and single – buyer is considered.
2. The demand is quadratic in time t, i.e. $R(t) = a(1 + bt + ct^2)$, where $a > 0$ is constant demand, $0 < b, c < 1$ are rate of linear and exponential demand and t is the time.
3. Shortages are not allowed and lead – time is zero.
4. The deterioration rates of items in vendor's and buyer's inventory are different and proportional to on

*Corresponding author email: nitahshah@gmail.com

hand stock in inventory. There is no repair or replacement of deteriorated units during a cycle time.

5. The permissible delay in payment is offered by the vendor to attract the buyer to cooperate in the integrated decision.

Notations

A_b	Buyer's ordering cost per order
A_v	Vendor's ordering cost per order
C_b	Buyer's purchase cost per unit
C_v	Vendor's purchase cost per unit
I_b	Inventory carrying charge fraction per unit per time unit for buyer
I_v	Inventory carrying charge fraction per unit per time unit for vendor
θ_b	Deterioration of items in buyer's inventory system
θ_v	Deterioration of items in vendor's inventory system; $0< \theta_v < \theta_b <1$
$I_b(t)$	Buyer's inventory level at instant of time t
$I_v(t)$	Vendor-buyer combined inventory level at instant of time t
$R(t)$	$= a(1 + bt + ct^2)$ demand rate at the time t, where $a > 0$ is constant demand and $0 < b, c < 1$ are rate of change of demand and t is the time.
n	Number of time of orders kept by buyer during cycle time.
K_b	Buyer's total cost per time unit
K_v	Vendor's total cost per time unit
K	Integrated total cost for both vendor and buyer per time unit.
T	Vendor's cycle time (a decision variable)
T_b	(= T/n), buyer's cycle time (a decision variable)
M	Permissible delay period offered by the vendor to the buyer (a decision variable)
r	Continuous interest rate

Mathematical Model

Figure 1 represents time-varying inventory status for the vendor and the buyer.

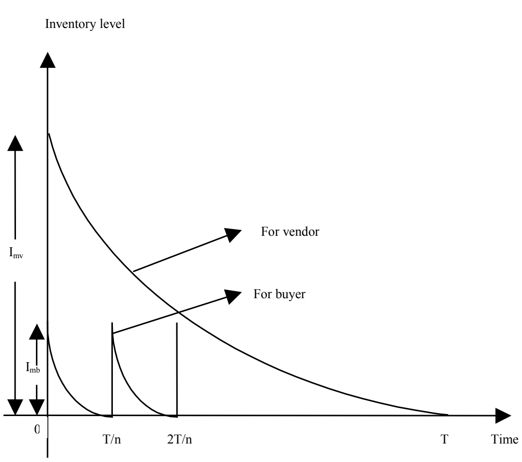

Fig. 1. Time – Inventory status of Vendor and Buyer.

The inventory depletes due to quadratic demand and deterioration rate for both vendor and buyer. The rate of change of inventory for the vendor and the buyer is given by the differential equations:

$$\frac{dI_b(t)}{dt} + \theta_b I_b(t) = -a(1 + bt + ct^2)\quad 0 \le t \le \frac{T}{n} \qquad (3.1)$$

and

$$\frac{dI_v(t)}{dt} + \theta_v I_v(t) = -a(1 + bt + ct^2)\quad 0 \le t \le T \qquad (3.2)$$

with the boundary conditions $I_b(\frac{T}{n})=0$ and $I_v(T) = 0$, $I_b(0) = I_{mb}$ and $I_v(0) = I_{mv}$.

The solutions of the differential equation are

$$I_b(t) = \frac{a}{\theta_b}[e^{\theta_b(\frac{T}{n}-t)} - 1] + \frac{ab}{\theta_b^2}[1 - e^{\theta_b(\frac{T}{n}-t)}] + \frac{2ac}{\theta_b^3}[e^{\theta_b(\frac{T}{n}-t)} - 1] + \frac{aT}{n^2\theta_b^2}[n\theta_b b - 2cn + cT\theta_b]\, e^{\theta_b(\frac{T}{n}-t)} + \frac{a}{\theta_b^2}[2ct - \theta_b bt - \theta_b ct^2] \qquad (3.3)$$

and

$$I_v(t) = \frac{a}{\theta_v}[e^{\theta_v(T-t)} - 1] + \frac{ab}{\theta_v^2}[1 - e^{\theta_v(T-t)}] + \frac{2ac}{\theta_v^3}[e^{\theta_v(T-t)} - 1] + \frac{aT}{\theta_v^2}[\theta_v b - 2c + cT\theta_v]\, e^{\theta_v(T-t)} + \frac{a}{\theta_v^2}[2ct - \theta_v bt - \theta_v ct^2] \qquad (3.4)$$

Using $I_b(0) = I_{mb}$ and $I_v(0) = I_{mv}$, the purchase quantities for the buyer and the vendor are

$$I_{mb} = \frac{a}{\theta_b}[e^{\frac{\theta_b T}{n}} - 1] + \frac{ab}{\theta_b^2}[1 - e^{\frac{\theta_b T}{n}}] + \frac{2ac}{\theta_b^3}[e^{\frac{\theta_b T}{n}} - 1] + \frac{aT}{n^2\theta_b^2}[n\theta_b b - 2cn + cT\theta_b]\, e^{\frac{\theta_b T}{n}}$$

and

$$I_{mv} = \frac{a}{\theta_v}[e^{\theta_v T} - 1] + \frac{ab}{\theta_v^2}[1 - e^{\theta_v T}] + \frac{2ac}{\theta_v^3}[e^{\theta_v T} - 1] + \frac{aT}{\theta_v^2}[\theta_v b - 2c + cT\theta_v]\, e^{\theta_v T}$$

During the cycle time [0, T] , the buyer's holding cost is

$$IHC_b = n\, C_b\, I_b \int_0^{\frac{T}{n}} I_b(t)\, dt \;,$$

Ordering cost is $OC_b = n\, A_b$ and number units deteriorated is ($I_{mb} - \frac{T}{n} R\left(\frac{T}{n}\right)$).

Hence cost due to deterioration of units is

$CD_b = n\, C_b\, (I_{mb} - \frac{T}{n} R\left(\frac{T}{n}\right)$).

Hence, the buyer's total cost, K_b per time unit is

$$K_b = \frac{1}{T}[IHC_b + CD_b + OC_b] \quad (3.5)$$

The vendor's inventory in the joint two-echelon inventory model is the difference between the vendor-buyer combined inventory and the buyer's inventory. Therefore, vendor's holding cost is

$$IHC_v = C_v\, I_v \left[\int_0^{T} I_v(t)\, dt - n \int_0^{\frac{T}{n}} I_b(t)\, dt \right]$$

The units deteriorated at vendor's inventory system is (I_{mv} - n* I_{mb}). Hence, cost due to deterioration of units is CD_v = $C_v(I_{mv} - n\, I_{mb})$.

and vendor's ordering cost is $OC_v = A_v$.

The vendor's total cost K_v per time unit is

$$K_v = \frac{1}{T}[IHC_v + CD_v + OC_v] \quad (3.6)$$

The joint total cost; K is is the sum of K_b and K_v. Since $T_b = \frac{T}{n}$, K is the function of discrete variable n and continuous variable T.

Computation Procedure

There are two cases:

Case 1: when vendor and buyer make decision independently.

For given value of n, K_b can be minimized by solving $\frac{\partial K_b}{\partial T_b} = 0$ for T_b.

This solution (n, T_b) minimizes K_v provided

$$K_v(n\text{-}1) \geq K_v(n) \leq K_v(n+1) \quad (4.1)$$

gets satisfied. Then the total cost without integration; K_{NJ} is given by

$$K_{NJ} = \min_n[\min_n K_b + K_v] \quad (4.2)$$

Case 2 : when vendor and buyer make decision jointly.

The optimum values of T and n must satisfy the following condition simultaneously:

$$\frac{\partial K}{\partial T} = 0 \text{ and } K(n\text{-}1) \geq K(n) \leq K(n+1) \quad (4.3)$$

The total integrated cost is $K_J = \min_{T,n}[K_b + K_v]$ (4.4)

Clearly, $K_J \leq K_{NJ}$. Hence, total cost savings, Sav_J is defined as $Sav_J = K_{NJ} - K_J$. Let the buyer's cost saving, Sav_b be defined as $Sav_b = \alpha\, Sav_J$,where α is the negotiation factor and $0 \leq \alpha \leq 1$.When negotiation factor equal to 1, all saving goes to buyer; when it is equal to zero, all saving goes to the vendor. When negotiation factor is 0.5, total savings is equally distributed between the vendor and the buyer. The present value of unit cost after a time interval M permissible credit period is $e^{-r\,m}$ where r is discounting rate. Solving the following equation

$$R(T)\, C_b\, (1 - e^{-r\,m}) = Sav_b \quad (4.5)$$

The buyer's permissible delay in payment can be computed as

$$M = \frac{1}{r} \ln\left[\frac{C_b R(T)}{C_b R(T) - Sav_b}\right] \quad (4.6)$$

Numerical Example and Sensitivity Analysis

To illustrate proposed model, consider following parameter values in proper units:

[a b c A_b A_v C_b C_v I_b I_v θ_b θ_v r] = [40000 0.03 0.04 600 3000 25 15 0.11 0.10 0.20 0.10 0.03]

In table 1, the solution is exhibited for independent and joint decision. The buyer's cost and cycle time increase when both players agree for joint decision. The vendor benefits \$12332 and buyer looses \$10601. This compels buyer not to agree for joint decision. To encourage and attract the buyer to cooperate, vendor offers the buyer a permissible delay in payment of 0.02867 yrs with equal sharing of the benefits. This reduces integrated total cost defined as $PJCR = \frac{K_{NJ} - K_J}{K_J}$ by 4.2 %.

Table 1. The optimal solution with and without considering joint strategy

	Case 1	Case 2
n	3	1
T_b	0.065865	0.186356
T	0.197594	0.186356
K_b	18274.40	28875.20
K_v	22673.70	10341.40
K	40948.10	39216.60
PJCR	-	4.2285
M	-	0.02867

The sensitivity analysis for buyer's deterioration rate; θ_b, demand; a, exponential demand rate; c; and inventory carrying charge fraction of buyer; I_b is exhibited in table 2 to 5. From table 2, it is observed that increase in deterioration of units in buyer's inventory system decreases total cost savings and permissible delay in payments. The increase in fixed demand; a results in significant decrease cost savings and permissible trade credit is exhibited in table 3. In table 4;c; the exponential rate of change in demand is studied. It is observed that there is significant increase in total cost savings and allowable delay period. The total integrated cost is very sensitive to changes in c. The changes in I_b is studied in table 5. The smaller buyer's inventory carrying charge fraction is required for total cost savings and larger allowable delay period.

CONCLUSION

In this study, a mathematical model is developed for an optimal ordering policy in the joint strategy of vendor-buyer inventory system when demand is quadratic. The deterioration rate of units in vendor-buyer inventory system is considered to be different. It is established that integrated policy lowers the total cost of an inventory system, even though the buyer's cost increases significantly. To encourage the buyer for cooperation, a promotional incentive in terms of trade credit is offered by vendor to the buyer to settle the account.

Table 2. Sensitivity Analysis of deterioration rate

θ_b	0.10	0.15	0.20	0.25	0.30
K_{NJ}	38697	39676.80	40948.10	42339	43773.70
K_J	35135.54	37232.30	39216.60	41105.17	42911.39
PJCR	9.2034	6.1610	4.2285	2.9141	1.9699
M	0.0589	0.0404	0.02867	.0204	0.014

Table 3. Sensitivity Analysis of demand rate

a	24000	32000	40000	48000	56000
K_{NJ}	31784.7	36656.20	40948.10	44829.10	48398.80
K_J	30246.25	35015.21	39216.60	43014.68	46508.20
PJCR	4.84043	4.4767	4.2285	4.0471	3.9062
M	0.04235	0.03391	0.02867	0.02505	.02238

Table 4. Sensitivity Analysis of demand rate

c	0.04	0.06	0.08	0.10
K_{NJ}	40948.10	40990.10	41034	41081.30
K_J	39216.60	38750.58	38258.80	37735.50
PJCR	4.2285	5.4635	6.7631	8.1443
M	0.02867	0.03705	0.04587	0.05524

Table 5. Sensitivity Analysis of inventory carrying charge fraction of buyer

I_b	0.066	0.088	0.11	0.132
K_{NJ}	38873	39917.10	40948.10	41964.50
K_J	34772.25	37068.43	39216.60	41245.22
PJCR	10.549	7.1364	4.2285	1.7140
M	0.06786	0.04715	0.02867	0.01191

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