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TIME DEPENDENT PROBLEM OF DIFFRACTION OF A SPHERICAL ACOUSTIC WAVE FROM AN ABSORBING HALF PLANE

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ABSTRACT

The diffraction of a spherical acoustic wave from an absorbing half plane due to an arbitrary time dependent source distribution in the presence of a moving fluid is considered. The convolution integral appearing in the process of calculating inverse temporal transform has been evaluated asymptotically to present the diffracted field. This procedure is applicable to any type of time dependence provided the duration of incoming signal is small as compared to the emitted signal.

Keywords: Time dependent problem, spherical acoustic wave, absorbing half plane.

INTRODUCTION

The transient wave phenomenon is an important field of discussion in the wave motion theory which gives a more complete picture of the wave processes. The present ability to produce short electromagnetic pulses, which are being used as diagnostic tools for the study of implosion and other wave material applications, requires the development of new time dependent techniques. Stimulated by various applications that require the explicit treatment of time dependent effects, many scientists worked on time dependent problems, for instance, Friedlander (1958), Rienstra (1981), Jones (1986), Lakhtakia *et al.* (1987), Lakhtakia *et al.* (1989), Sun *et al.* (1991), Asghar *et al.* (1991), Asghar *et al.* (1998).

The objective of this paper is to discuss the general time dependent problem of diffraction of a spherical acoustic wave from an absorbing half plane in a moving fluid, which is perhaps a first attempt for this type of boundary. The temporal Fourier transform has been applied to obtain the transfer function in the transformed plane. Once the transfer function is available, an inverse temporal transform is used to obtain the results in time domain. The convolution technique is employed to calculate the inverse temporal transform and the resulting convolution integral has been evaluated approximately. This method of calculations can be used for various types of incoming signals whose amplitude characteristic is negligible in certain parts of the frequency axis (low pass filters). These filters are important in applications because the results obtained for these filters can be used to analyze more general filters, see Papoulis (1962). Finally, we employ this procedure to calculate the diffracted field due to an impulsive point source and triangular pulse from an absorbing half plane.

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Formulation of the Problem

We consider the scattering of an acoustic wave from a semi-infinite absorbing plane occupying the position $y=0, x\le 0.$ The plane is assumed to be of negligible thickness and satisfies the absorbing boundary condition $p - u_n z_a = 0$ on both sides of its surface, Morse and Ingard (1961). Here, p is the surface pressure and u_n the normal derivative of the perturbation velocity, z_a the acoustic impedance of the surface and n a normal pointing from the fluid into the surface. The whole system is assumed to be placed in a fluid moving with subsonic velocity U parallel to the $x-$axis. The perturbation velocity $\mathbf{u}$ of the irrotational sound wave can be written as $\mathbf{u}=\nabla\psi.$ The resulting pressure in the sound field is given by

$$p = -\rho_0\left(\frac{\partial}{\partial t}+\mathrm{U}\frac{\partial}{\partial x}\right)\psi,$$

where ρ_0 is the density of the undisturbed stream. We consider a point source to be located at the position (x_0, y_0, z_0) and the time dependence is introduced through the function $f(t)$. Thus, we have to solve the following time dependent boundary value problem

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\left(\frac{1}{c}\frac{\partial}{\partial t}+M\frac{\partial}{\partial x}\right)^2\right]\psi = \delta(x-x_0)\delta(y-y_0)\delta(z-z_0)f(t), \quad (1)$$

$$\left\{\frac{\partial}{\partial y} \mp \beta M \frac{\partial}{\partial x} \mp \frac{\beta}{c}\frac{\partial}{\partial t}\right\}\psi(x,0^{\pm},z;t)=0,$$

$$(x<0), \tag{2}$$

$$\frac{\partial}{\partial y}\psi(x,0^{+},z;t)=\frac{\partial}{\partial y}\psi(x,0^{-},z;t),$$

$$\psi(x,0^{+},z;t)=\psi(x,0^{-},z;t),\ (x>0), \tag{3}$$

where $\beta=\rho_0 c/z_a$, c is the velocity of sound and $M=U/c$ is the Mach number. For subsonic flow, $|M|<1$ and $\operatorname{Re}\beta>0$.

Solution of the Problem

We define the temporal Fourier transform pair as

$$\begin{cases}\phi(x,y,z;w)=\int_{-\infty}^{\infty}\psi(x,y,z;t)e^{iwt}dt\\ \psi(x,y,z;t)=\frac{\operatorname{Re}}{\pi}\int_0^{\infty}\phi(x,y,z;w)e^{-iwt}dt.\end{cases} \tag{4}$$

Now, taking temporal transform of Eqs.(1)-(3), we obtain

$$\left\{\left(1-M^2\right)\frac{\partial^2}{\partial x^2}+2ikM\frac{\partial}{\partial x}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\right\}\phi$$
$$=\delta(x-x_0)\delta(y-y_0)\delta(x-x_0)F(w), \tag{5}$$

$$\left\{\frac{\partial}{\partial y}\mp\beta M\frac{\partial}{\partial x}\pm ik\beta\right\}\phi(x,0^{\pm},z;w)=0, \qquad (x<0), \tag{6}$$

$$\begin{cases}\frac{\partial}{\partial y}\phi(x,0^{+},z;w)=\frac{\partial}{\partial y}\phi(x,0^{-},z;w),\\ \phi(x,0^{+},z;w)=\phi(x,0^{-},z;w),\end{cases} \qquad (x>0), \tag{7}$$

where $F(w)$ is temporal transform of $f(t)$ and its value is negligible outside an interval $(-\Omega,\Omega)$ (Papoulis, 1962) and $k=w/c$. We observe that the mathematical problem in the transformed $w-$plane is the same as in case of steady state (harmonic time dependent) problem, Asghar *et al.* (1991) except a multiplicative factor $F(w)$ in the right hand side of the wave equation. Thus, we can employ the results of Asghar *et al.* (1991) directly to write down the diffracted field as

$$f(x,y,z;w)=\frac{AF(w)}{\sqrt{w}}$$
$$\times e^{-iw(MR_1\cos q_1-R_{12})/c_1+ip/4}, \tag{8}$$

where

$$A=\sqrt{\frac{c_1}{\left(1-M^2\right)}}\left\{\frac{BR_{12}}{R+R_0}BM\cos\theta_0-2\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\right\}$$
$$\times\frac{1}{4\pi\sqrt{2\pi R_0RR_{12}}\left(\cos\theta+\cos\theta_0\right)L_+\left(K\xi_2\cos\theta\right)L_-\left(-K\xi_2\cos\theta_0\right)}, \tag{9}$$

$X-X_0=R_1\cos\theta_1,\ c_1=c\sqrt{\left(1-M^2\right)},\ k=K\sqrt{\left(1-M^2\right)},$

$L(\nu)=1+\frac{B\left(K-M\nu\right)}{\sqrt{K^2\gamma^2-\nu^2}}=L_-(\nu)L_+(\nu)$, see Asghar et. al. (1991).

Now, taking the inverse temporal transform of Eq.(8), we get

$$\psi(x,y,z;t)=A\frac{\operatorname{Re}}{\pi}.$$
$$\int_0^{\infty}F(w)e^{-iw(MR_1\cos\theta_1-R_{12})/c_1+i\pi/4}\frac{1}{\sqrt{w}}e^{-iwt}dw. \tag{10}$$

It is important to note that $L_+(Kx_2\cos q)$ and $L_-(-Kx_2\cos q)$ are independent of K. The integral appearing in Eq.(10) may be solved using the convolution theorem. Thus,

$$\psi(x,y,z;t)=\int_0^{\infty}f(t)g(t-\tau)d\tau, \tag{11}$$

where

$$g(t)=A\frac{\operatorname{Re}}{\pi}\int_0^{\infty}e^{-iw(MR_1\cos\theta_1-R_{12})/c_1+i\pi/4}$$
$$\frac{1}{\sqrt{w}}e^{-iwt}dw=A\frac{\operatorname{Re}}{\pi}\int_0^{\infty}G(w)e^{-iwt}dw$$
$$=\frac{H\left(t-\left(R_{12}-MR_1\cos\theta_1\right)/c_1\right)}{\sqrt{\pi\left(t-\left(R_{12}-MR_1\cos\theta_1\right)/c_1\right)}}, \tag{12}$$

where $H(.)$ is the Heaviside function. Now, we give a brief description of the approximate method for evaluating the convolution integral (11) between two functions $f(t)$ and $g(t)$. For that, Eq.(8) can be rewritten as

$$\phi(x,y,z;w) = F(w)G(w).$$

The p'th moment of the function $f(t)$ denoted by m_p, is defined by

$$m_p = \int_{-\infty}^{\infty} t^p f(t)dt.$$

Using the moment theorem (Papoulis, 1962),

$$F(w) = \sum_{p=0}^{\infty} \frac{m_p}{p!}\left(-iw\right)^p.$$

Thus, the field $f(x,y,z;w)$ takes the form

$$f(x,y,z;w) = G(w)\sum_{p=0}^{\infty} \frac{m_p}{p!}(-iw)^p. \qquad (13)$$

Since $(iw)^p G(w)$ is the Fourier transform of the p'th derivative of $g(t)$, therefore the inverse temporal transform of Eq. (13) term wise yields

$$\psi(x,y,z;t) = m_0 g(t) - \frac{m_1}{1!}\frac{dg}{dt} + \dots + (-1)^p \frac{m_p}{p!}\frac{d^p g}{dt^p}. \qquad (14)$$

Thus, $\psi(x,y,z;t)$ appears as a series in terms of derivatives of $g(t)$ and the moments of $f(t)$. For the validity of Eq. (14), it is assumed that Eq.(13) converges for every w. Even, if this happens, the convergence is slow and a large number of terms is required for a satisfactory evaluation of $f(t)$. However, if we assume that the temporal transform $G(w)$ is negligible outside an interval $(-\Omega,\Omega)$ and in this interval $F(w)$ is sufficiently smooth, so that for $|w| < W,$ a small number of terms in Eq.(13) suffices to approximate adequately $F(w)$, that is,

$$F(w) \approx m_0 + \dots + \frac{m_k}{k!}(-iw)^k, \qquad |w| < \Omega.$$

Thus, Eq.(14) takes the form

$$\psi(x,y,z;t) \approx m_0 g(t) - \frac{m_1}{1!}\frac{dg}{dt} + \dots + (-1)^p \frac{m_k}{k!}\frac{d^k g}{dt^k}. \qquad (15)$$

The above assumption roughly stated in terms of time domain says that the duration of $f(t)$ should be small as compared to the duration of $g(t)$.

Examples

(1) Consider the impulsive point source $f(t) = d(t)$, where $d(t)$ is the Dirac delta function. In this case

$$m_0 = 1, \; m_1 = m_2 = \dots = m_k = 0,$$

and the field $\psi(x,y,z;t)$ given by Eq. (15) takes the form

$$\psi(x,y,z;t) = \frac{H\left(t - \left(R_{12} - MR_1\cos\theta_1\right)/c_1\right)}{\sqrt{\pi\left(t - \left(R_{12} - MR_1\cos\theta_1\right)/c_1\right)}}. \qquad (16)$$

Here, we observe that the diffracted field given by Eq. (16) corresponds to the field presented in Asghar *et al*. (1996), which proves the validity of this method.

(2) As a second example, we consider the triangular pulse

$$f(t) = \begin{cases} 1 - \dfrac{|t|}{T}, & \text{if } |t| < T, \\ 0, & \text{if } |t| > T, \end{cases}$$

whose temporal transform is

$$F(w) = \frac{4\sin^2(Tw/2)}{Tw^2}.$$

The first three moments of $f(t)$ are given by

$$m_0 = T, \quad m_1 = 0, \quad m_2 = T^3/6.$$

Substituting these values in Eq. (15), we obtain the field

$$\psi(x,y,z;t) \approx \frac{AT}{\sqrt{\pi\left(t - R_{13}\right)}}\left\{1 + \frac{T^2}{16\left(t - R_{13}\right)^2}\right\},$$

where A is given by Eq. (9) and

$$R_{13} = \left(R_{12} - MR_1\cos\theta_1\right)/c_1.$$

Concluding Remarks

In this paper, we have successfully employed the method of moments to present the asymptotic diffracted field from an absorbing half plane due to an arbitrary time dependent source with the assumption that the duration of incoming signal is small as compared to the emitted signal. As far as the validity of the method is concerned, we have provided a useful check in Example (1). It is interesting to note that the field due to a rigid half plane can be obtained by taking the absorption parameter b to be zero and the still fluid results by taking Mach number M equal to zero in Eq. (15) respectively.

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