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IMPACT OF INFLATION AND CREDIT POLICIES ON A PRODUCTION LOT SIZE MODEL

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ABSTRACT

The main purpose of this paper is to study the impact of inflation and trade credit on a production lot size model, using a discounted cash flow (DCF) approach over an infinite planning horizon. A DCF approach permits a proper recognition of the financial implication of the opportunity cost and out-of-pocket costs in inventory analysis. It also permits an explicit recognition of the exact timing of the cash flows associated with an inventory system. Optimal solution for the model is derived and the effects of inflation and trade credit on the optimal replenishment policy are studied with the help of numerical example.

Keywords: EPQ, inventory, inflation, trade credit, discounted cash flow.

INTRODUCTION

In the conventional inventory models it is assumed that buyer's capitals are unrestricting and must be paid for the items as soon as items were received. But in practice, it is observed that supplier offers different credit policies to the buyers. Generally, there are two types of credit policies that are prevalent in the market. One such policy is where supplier allows a certain fixed period to settle the account. During this period the supplier is charging no interest, but beyond this period interest is charged. Another credit policy is *"d/D_1 Net D"*, which implies that a *d*% discount on sale price is granted if payments are made within D_1 days and the full sale price is due within *D* days from the date of invoice if the discount is not taken ($D_1<D$). In many situations, the supplier adopts this policy to promote his commodities, or to stimulate his demand.

In the past, a lot of work has been done for studying the inventory system behavior under the various trade-credit policies offered by the suppliers. Haley and Higgins (1973) developed an inventory model to determine economic order quantity under conditions of permissible delay in payments. Goyal (1985) presented the similar model with no penalty cost due to late payment. Chung (1989) then developed an alternative approach to the problem. Chand and Ward (1987) analyzed Goyal's problem under assumption of the classical economic order quantity model, obtaining different results. Teng (2002) amended Goyal (1985) model by considering the difference between unit price and unit cost and mathematically proved that it makes economic sense for a buyer to order less quantity and take benefits of the permissible delay more frequently. Shah (1993) and Aggarwal and Jaggi (1995) extended the Goyal's model to the case of deterioration. Jamal *et al*. (2000) further generalized the model to allow shortages. Hwang and Shinn (1997) in their paper jointly optimize retailer's optimal price and lot size under permissible delay in payments. Dye (2002) formulated the model with stock dependent demand for deteriorating items with partial backlogging under permissible delay in payment. They assumed initial stock dependent demand function. Chang, Hung and Dye (2004) extended this work by considering instantaneous stock-dependent demand. Chung (2000) presented the discounted cash flows (DCF) approach for the analysis of the optimal inventory policy in the presence of trade credit. Jaggi and Aggarwal (1994) extended his work for deteriorating items in the presence of trade credit using the DCF approach. Besides this there were several other interesting and relevant papers related to delay of payments such as Chu *et al.* (1998), Chung (2000), Sarker *et al.* (2000a,b), Shinn (1997), Huang (2003), Khouja and Mehrez (1996) and their references.

But all the above models assumed infinite replenishment rate, which is not true in real practice. So, we relax this assumption to finite replenishment rate. That is the well-known production lot size model. Huang (2007) in his paper determined optimal retailer's replenishment decisions in the EPQ model under two levels of trade credit policy. Recently Jaggi *et al.* (2004) formulated an inventory model for finite replenishment rate under permissible delay in payments. And Jaggi *et al*. (2006) extended the model for deteriorating items.

From a financial standpoint, an inventory represents a capital investment and must compete with other assets for a firm's limited capital funds. The effect of inflation is not usually considered when an inventory system is analyzed because most people think that inflation would not influence the inventory policy to any significant degree. Due to high inflation, the financial situation has changed

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in many developing countries. Besides this, inflation also influences demand of certain products. As inflation rate increases the value of money goes down which erodes the future worth of savings and forces one for more current spending. These spending may be on clothes, accessories, peripherals or daily household items that give rise to demand of these items. As a result, while determining the optimal inventory policy, the effect of inflation cannot be ignored. Many authors have developed different inventory models incorporating the concept of inflation under different assumptions. The fundamental result in the development of EOQ model with inflation is that of Buzacott (1975) who discussed EOQ model with inflation subject to different types of pricing policies.

Aggarwal and Jaggi (1987) determined economic order quantity with inflation under all unit discounts of deteriorating item. Sarkar and Haixu (1994) studied effect of inflation and the time value of money on order quantity and allowable shortage. Aggarwal *et al.* (1997) investigated the economic ordering policies in the presence of trade credit with inflation for non-deteriorating items. Further Liao *et al.* (2000) proposes an inventory model for deteriorating items under inflation under a situation in which the supplier provides the purchaser a permissible delay of payments. Recently Jaggi and Goel (2005) investigated the economic ordering policies of deteriorating items with trade credit under inflationary conditions.

Further, it is interesting to note that the primary benefit of taking trade credit is that one can have savings in purchase cost and opportunity cost, and the presence of inflation makes it more realistic. In particular, when the unit purchase cost is high, replenishment rate is finite and inflation is present, the saving due to trade credit appears to be more significant than without trade credit, as trade credit helps in increasing the sales and also paying later indirectly reduces the cost. Now, when there is inflation, an organization might increase the selling price of the item. However, it is much more likely that the selling price will be held so that it becomes more competitive. We make the later assumption in our modeling with trade credit policy of later type i.e. *"d/D_1 Net D"* using discounted cash flow (DCF) approach. The discounted cash flow (DCF) approach permits a proper recognition of the financial implication of the opportunity cost and out of pocket cost in the inventory analysis. Therefore, in this paper we develop a production lot size model by incorporating some of the realistic phenomenon viz., inflation, trade credit, and discounted cash flow approach. Moreover, we investigate under what condition it is advantageous for the organization to take discount or to for go the discount.

1. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used to develop the mathematical model:

Assumptions:

1. The demand rate is known, constant and continuous during the planning horizon under consideration.
2. Replenishment rate is known and constant.
3. Shortages are not allowed.
4. Lead-time is negligible.
5. Planning horizon is infinite.
6. The terms of credit policy are *"d/D_1 Net D"*, which implies that a *d%* discount on sale price is granted if payments are made within D_1 days and the full sale price is due within D days from the date of invoice, if the discount is not taken (where $D_1 < D$).

Notations:

λ : demand rate per unit time
R : replenishment rate per unit time
Q : order quantity
T : inventory cycle length
$A(t)$: ordering cost per order at time t
$q(t)$: instantaneous inventory level at any time t
$C(t)$: unit purchase cost of item bought at time t
I : out of pocket inventory carrying charge per unit per unit time
r : opportunity cost (discount rate)
k : constant inflation rate

2. MODEL FORMULATION

Let $q(t)$ be the inventory level at any time t. Since it is a continuous system and the replenishment rate is finite, the time period $0 \le t \le t_1$ is inventory-building time as production is on, and demand is also occurring simultaneously. Thus the differential equation describing the instantaneous state of inventory level $q(t)$ over $(0, t_1)$ is given by

$$\frac{dq(t)}{dt} = R - \lambda \qquad 0 \le t \le t_1 \qquad (1)$$

and, the time period $t_1 \le t \le T$ is inventory-downtime as there is no production, and only demand is occurring. Thus the differential equation describing the instantaneous state of inventory level $q(t)$ over (t_1, T) is given by

$$\frac{dq(t)}{dt} = -\lambda \qquad t_1 \le t \le T \qquad (2)$$

The solution of equation (1) using condition, $q(t) = 0$ at $t = 0$ is

$$q(t) = (R - \lambda)\mathrm{T} \qquad 0 \le t \le t_1 \qquad (3)$$

and the solution of equation (2) using condition, $q(t) = 0$ at $t = T$ is

$$q(t) = \lambda(T - t) \qquad t_1 \le t \le T \qquad (4)$$

Equating the equations (3) and (4) at $t = t_1$, we have

$$t_1 = \frac{\lambda}{R}T \qquad (5)$$

The optimal order quantity can be calculated using the following condition

$$Q = \lambda T = Rt_1 \tag{6}$$

Now at the beginning of each cycle there will be cash outflow of ordering cost and purchase cost. Assuming continuous compounding of inflation, the ordering cost and unit cost of the item at any time t are

$$A(t) = A_0 e^{kt}$$
$$C(t) = C_0 e^{kt} \tag{7}$$

where $A_0 = A(0)$ and $C_0 = C(0)$ are ordering cost and cost of item at time zero.

Since the purchases at time t, amounts to $QC(t)$, and the credit policy available to the organization is *"d/D_1 Net D"*. Now we are to investigate whether taking discount is advantageous for the organization or to make the payment in full at D, assuming the organization makes periodic purchases of Q units. Under these circumstances, using DCF approach we discuss the following two cases.

Moreover, the inventory carrying cost is proportional to the value of the inventory, the out-of-pocket (physical storage) inventory carrying cost per unit time at time t is $IC(t)q(t)$. The present worth of this out-of-pocket carrying cost is obtained by continuous discounting $IC(t)q(t)$ at the cost of capital r. Further, since the purchases at time t, amounts to $QC(t)$, and the credit policy available to the organization is *"d/D_1 Net D"*. Under these circumstances, this paper discusses below two cases to investigate whether the organization should take or forgo the discount using the DCF approach.

Case1. When discount is taken

This case presents the situation when purchases are made at the beginning of the cycle and payments are made on the D_1^{th} day after taking the discount. Now since the planning horizon is infinite and periodic purchases are made of Q units each time.

Let $t_0, t_1, t_2, \ldots, t_{i-1}, t_i, \ldots$ be the replenishment points and $t_i - t_{i-1} = T$, $t_i = T * i$

The various components of present worth of total variable cost for the i^{th} cycle are
ordering cost, purchase cost, inventory carrying cost.
Now, using the DCF approach, various costs are calculated as follows:

1. Present worth of ordering cost for the i^{th} cycle, $C1$ is

$$C1 = A(t_{i-1})e^{-r\,t_{i-1}} = A_0 e^{(k-r)t_{i-1}} \tag{8}$$

Since in this case purchases are made at the beginning of the cycle and payments are made on the D_1^{th} day after taking the discount. Therefore, effective purchase price for Q units in the i^{th} cycle at time t_{i-1} is $Q(1-d)C(t_{i-1})e^{-kD_1}$.

2. Hence, the present worth of purchase cost for the i^{th} cycle, $C2$ is

$$\begin{aligned} C2 &= Q(1-d)C(t_{i-1})e^{-kD_1}e^{-r\,t_{i-1}} \\ &= Q(1-d)C_0 e^{-kD_1}e^{(k-r)\,t_{i-1}} \end{aligned} \tag{9}$$

Now, for calculating the present worth of the out of pocket inventory carrying cost for the i^{th} cycle, it is assumed that duration of each cycle is same viz. T.

Moreover, the inventory carrying cost is proportional to the value of the inventory, the out-of-pocket (physical storage) inventory carrying cost per unit time at time t is $IC(t)q(t)$. The present worth of this out-of-pocket carrying cost is obtained by continuous discounting $IC(t)q(t)$ at the cost of capital r.

Now, the total inventory in each cycle will be same, which is given by

$$= \int_0^T q(T)dt = \int_0^{t_1} q(t)dt + \int_{t_1}^T q(t)dt$$
$$= \int_0^{t_1} (R-\lambda)t\,dt + \int_{t_1}^T \lambda(T-t)dt$$

The inventory carrying cost for the i^{th} cycle will be

$$= IC(t_{i-1})(1-d)e^{-kD_1}\int_o^T q(t)\,dt$$

3. And the present worth of inventory carrying cost for the i^{th} cycle, $C3$ is

$$\begin{aligned} C3 &= IC_0(1-d)e^{-kD_1}e^{(k-r)t_{i-1}}\int_o^T q(t)\,e^{-r\,t}dt \\ &= IC_0(1-d)e^{-kD_1}e^{(k-r)t_{i-1}} \end{aligned} \tag{10}$$
$$\left\{\int_0^{t_1}(R-\lambda)te^{-rt}dt + \int_{t_1}^T \lambda(T-t)e^{-rt}dt\right\}$$
$$= IC_0(1-d)e^{-kD_1}e^{(k-r)t_{i-1}}$$
$$\frac{1}{r^2}\left\{R\left(1-e^{-r\frac{\lambda}{R}T}\right) - \lambda(1-e^{-rT})\right\}$$

The present worth of future cash flows in the i^{th} cycle is the sum of $C1$, $C2$ and $C3$ i.e.,

$$PW1_i(T) = [\;A_0 + Q(1-d)C_0 e^{-kD_1} +$$

$$IC_0(1-d)e^{-kD_1}\frac{1}{r^2}\left\{R\left(1-e^{-r\frac{\lambda}{R}T}\right)-\lambda(1-e^{-rT})\right\}$$

$$\lambda(1-e^{-rT})\ \Big]e^{-K(i-1)T}$$

where $K = r - k$ $(r > k)$ (11)

Now, the present worth of all future cash flows is calculated as

$$PW1_\infty(T)=\sum_{i=1}^{\infty}PW1_i(T) \tag{12}$$

Using (11), we get

$$PW1_\infty(T)=\frac{A_0+Q(1-d)C_0e^{-kD_1}+IC_0(1-d)e^{-kD_1}\frac{1}{r^2}\left\{R\left(1-e^{-r\frac{\lambda}{R}T}\right)-\lambda(1-e^{-rT})\right\}}{\left(1-e^{-KT}\right)} \tag{13}$$

Since $PW1_\infty(T)$ is a pseudo–convex function (*Appendix A*), therefore, the local minimum will be global minimum. Hence, the necessary condition for finding the global optimal value of T is

$$\frac{dPW1_\infty(T)}{dT}=0 \tag{14}$$

which gives

$$(e^{KT}-1)M_1\left\{\lambda+\frac{I}{r^2}\left(r\lambda e^{-r\frac{\lambda}{R}T}-r\lambda e^{-rT}\right)\right\}$$

$$=K\left\{A_0+M_1\lambda T+\frac{IM_1}{r^2}\left(R\left(1-e^{-r\frac{\lambda}{R}T}\right)-\lambda(1-e^{-rT})\right)\right\}$$

where $M_1 = C_0(1-d)e^{-kD_1}$ (15)

The optimal value of replenishment cycle T (say T_1^*) can be obtained from equation (15) using Solver (Add-In Tool of MS Excel). The optimal values of future cash flows and order quantity (say $PW1_\infty(T_1^*)$ and Q_1^*) can be obtained by substituting the value of T_1^* in the equation (13) & (6) respectively.

Case2. When discount is not taken

In this case, the purchases are made at the beginning of the cycle and payments are made on the D^{th} day taking the benefit of full credit period.

Now, using the DCF approach, various costs are calculated as follows (*Following the same approach as in Case1*):

1. Present worth of ordering cost for the i^{th} cycle, *D1* is

$$\begin{aligned} D1 &= A(t_{i-1})e^{-r\,t_{i-1}} \\ &= A_0e^{(k-r)t_{i-1}} \end{aligned} \tag{16}$$

Since the payments are made after availing full credit period i.e. D days. Therefore, the effective purchase price for Q units in the i^{th} cycle at time t_{i-1} is $=QC(t_{i-1})e^{-kD}$.

2. Hence, the present worth of purchase cost for the i^{th} cycle, *D2* is

$$\begin{aligned} D2 &= QC(t_{i-1})e^{-kD}e^{-r\,t_{i-1}} \\ &= QC_0e^{-kD}e^{(k-r)\,t_{i-1}} \end{aligned} \tag{17}$$

3. And, the present worth of inventory carrying cost for the i^{th} cycle, *D3* is

$$D3 = IC_0e^{-kD}e^{(k-r)t_{i-1}}\left\{\int_0^{t_1}(R-\lambda)te^{-rt}\,dt+\int_{t_1}^{T}\lambda(T-t)e^{-rt}\,dt\right\}$$

$$=IC_0e^{-kD}e^{(k-r)t_{i-1}}\frac{1}{r^2}\left\{R(1-e^{-r\frac{\lambda}{R}T})-\lambda(1-e^{-rT})\right\} \tag{18}$$

The present worth of future cash flows in the i^{th} cycle is sum of *D1*, *D2* and *D3* i.e.,

$$PW2_i(T)=\Bigg[A_0+QC_0e^{-kD}+IC_0e^{-kD}\frac{1}{r^2} \tag{19}$$

$$\left\{R\left(1-e^{-r\frac{\lambda}{R}T}\right)-\lambda\left(1-e^{-rT}\right)\right\}\ \Big]e^{-K(i-1)T}$$

and the present worth of all future cash flows is

$$PW2_\infty(T)=\frac{A_0+QC_0e^{-kD}+IC_0e^{-kD}\frac{1}{r^2}\left\{R\left(1-e^{-r\frac{\lambda}{R}T}\right)-\lambda\left(1-e^{-rT}\right)\right\}}{\left(1-e^{-KT}\right)} \tag{20}$$

Again, it is easy to show that $PW2_\infty(T)$ is a pseudo–convex function. Therefore, the necessary condition for finding the global optimal value of T is

$$\frac{dPW2_\infty(T)}{dT}=0 \tag{21}$$

which gives

$$(e^{KT}-1)M_2\left\{\lambda+\frac{I}{r^2}\left(r\lambda e^{-r\frac{\lambda}{R}T}-r\lambda e^{-rT}\right)\right\}=$$

$$=K\left\{A_0+M_2\lambda T+\frac{IM_2}{r^2}\left(R\left(1-e^{-r\frac{\lambda}{R}T}\right)-\lambda(1-e^{-rT})\right)\right\}$$

where $M_2 = C_0e^{-kD}$ (22)

The optimal value of T (say T_2^*) can be obtained from equation (22) using Solver (Add-In Tool of MS Excel). The optimal values of future cash flows and order quantity (say $PW2_\infty(T_2^*)$ and Q_2^*) can be obtained by

Table 1

k	Optimal Results	d=1.5%			d =2%			d =2.5%			NET90
		D1=10	D1=20	D1=30	D1=10	D1=20	D1=30	D1=10	D1=20	D1=30	D=90
0.08	T(days)	68.78	68.85	68.93	68.95	69.03	69.1	69.13	69.2	69.28	68.86
	Q	188.43	188.64	188.84	188.91	189.12	189.32	189.39	189.6	189.81	188.66
	PW($)	1138447	1135971	1133501	1132707	1130244	1127786	1126967	1124516	1122070	1135713
0.09	T(days)	74.19	74.3	74.38	74.39	74.48	74.57	74.58	74.67	74.76	74.37
	Q	203.28	203.53	203.78	203.8	204.05	204.3	204.32	204.57	204.82	203.75
	PW($)	1326532	1323285	1320047	1319840	1316610	1313388	1313148	1309934	1306728	1320465
0.10	T(days)	81.14	81.25	81.36	81.34	81.45	81.56	81.55	81.66	81.77	81.41
	Q	222.29	222.59	222.9	222.85	223.16	223.46	223.42	223.73	224.04	223.05
	PW($)	1589719	1585394	1581082	1581695	1577393	1573101	1573671	1569391	1565121	1579000
0.11	T(days)	90.46	90.6	90.73	90.69	90.83	90.96	90.92	91.06	91.2	90.87
	Q	247.84	248.21	248.58	248.47	248.84	249.22	249.1	249.48	249.85	248.95
	PW($)	1984285	1978344	1972422	1974264	1968353	1962461	1964243	1958362	1952499	1966605

substituting the value of T_2^* in equations (20) & (6) respectively.

4. COMPARSION OF TWO CASES

In this section we investigate that under what condition it is advantageous for the organization to take discount or to for go the discount. For this we define, η, the benefit as a percentage of the present worth of all future cash flows when discount is taken, as

$$\eta = \frac{PW2_{\infty}(T_2) - PW1_{\infty}(T_1)}{PW1_{\infty}(T_1)} = \frac{PW2_{\infty}(T_2)}{PW1_{\infty}(T_1)} - 1 \quad (23)$$

Now assuming there is long-term relationship between buyer and supplier, the ordering cost can be neglected i.e. $A_0 = 0$. With this assumption, equation (23) reduces to

$$\eta = \frac{e^{-k(D-D_1)}}{1-d} - 1 \quad (24)$$

$$\frac{d\eta}{dk} = -(D - D_1)\frac{e^{-k(D-D_1)}}{1-d} < 0 \quad (25)$$

From the above equation it is clear that percentage benefit decreases with k (inflation rate). Also, if $k(D - D_1)$ is large enough, i.e., inflation rate is high, then $e^{-k(D-D_1)}$ could be smaller than ($1-d$), so that η is negative and it would be certainly better for the purchasing firm to forgo the discount offered by the supplier. We will illustrate the formulation with the help of numerical examples. The purpose is to see which case would be better for the organization i.e. to take discount or to forgo the discount.

5. NUMERICAL EXAMPLE AND OBSERVATIONS

Let λ = 1000 units/year, R = 2000 units/year, C_0 =\$80, r = 0.15, I = 0.12 and A_0 =\$100. The variation in the optimal solution for different values of k (0.08, 0.09, 0.10, 0.11), D_1 (10, 20, 30 days) and d (1.5%, 2.0%, 2.5%) is shown in the *Table1*. In this table, various credit policies i.e. 1.5/10, 1.5/20, 1.5/30, 2/10, 2/20, 2/30, 2.5/10, 2.5/20 and 2.5/30 have been compared with Net /90.

The following inferences can be made from the results obtained:

1. The ordering policies in the shaded portion of *Table1* are more costly than the ordering policy under (Net/90), so it is beneficial for the buyer to forgo discount and go for Net/90 credit policy while ordering policies in the un-shaded portion are less costly than the ordering policy under (Net/90), hence here taking discount is advantageous for the buyer.
2. As inflation rate (k) increases there is marginal increase in cycle length and order quantity (T and Q), but there is considerable increase in the present worth of all future cash flows ($PW_{\infty}(T)$).
3. It can be seen that for larger value of $k(D-D_1)$ it is advantageous for the firm to forgo discount, whereas for smaller value of $k(D-D_1)$ taking discount is more beneficial than availing full credit period (for some values of d and D_1). The same result has been proved in the analysis.
4. For higher values of discount (i.e. $d \geq 0.02$ & $D_1 \geq 20$), it is always better to take discount.

CONCLUSIONS

In this paper a production lot size model has been developed by incorporating some of the realistic phenomenon viz., inflation, trade credit, and discounted cash flow approach. As inflation suggests one to procure more, that means more investment in inventory, which is highly correlated with the return on investment. Hence, it is important to consider the effects of inflation and time value of money in formulating inventory replenishment policy. Further, the credit policy in payment has become a very powerful tool to attract new customers and a good incentive policy for the buyers. In keeping with this reality, these factors are incorporated into the present model. The model is very useful in retail business. It can be used for electronic components, domestic goods and other products that are likely to have these characteristics.

We present an analytic formulation of the inventory problem and discuss about the pseudo convexity of the cost function. We also compare the two cases i.e., when discount is taken and when discount is not taken. Further, we investigate under what condition it is advantageous for the organization to take discount or to for go the discount. It has been proved theoretically, that for larger value of $k(D-D_1)$ it is advantageous for the firm to forgo discount, where as for higher values of d_1 it is always advantageous for the firm to take discount. Finally a numerical example is solved and sensitivity of the solution to changes in the values of different parameters has been discussed. The results show that the total cost is sensitive with changes in inflation (k), discount rate (d_1), and the credit period (D_1).

APPENDIX

In this section, it has been shown that $PW1_\infty(T)$ is a pseudo-convex function on its appropriate domain. For this, following results from *Bazaraa and Shetty* [4] are used.

Definition: Let S be a non-empty convex set in E^n. The function $f: S \to E^1$ is said to be convex on S if $f(\lambda x_1 + (1-\lambda)x_2) \le \lambda f(x_1) + (1-\lambda) f(x_2)$ $\forall$ x_1 *and* x_2 $\in$ S and for each $\lambda \in (0, 1)$.

Result: Let $p: U \to E^1$ and $q: U \to E^1$, where E^1 is 1-dimentional real Euclidean space & U is a non-empty convex set in E^n (n-dimensional real Euclidean space). Consider the function $f: U \to E^1$ defined by

$$f(x) = \frac{p(x)}{q(x)}$$

Then $f(x)$ is pseudo convex if:

- p is convex and differentiable on U and $p(x) \ge 0$, $\forall x \in U$
- q is concave and differentiable on U and $q(x) > 0$, $\forall x \in U$

From equation (13), we have

$$PW1_\infty(T) = \frac{A_0 + Q(1-d)C_0 e^{-kD_1} + IC_0(1-d)e^{-kD_1} \frac{1}{r^2}\left\{R\left(1-e^{-r\frac{\lambda}{R}T}\right) - \lambda(1-e^{-rT})\right\}}{\left(1-e^{-KT}\right)} \quad \text{(A1)}$$

Here let

$$p(T) = A_0 + \lambda T(1-d)C_0 e^{-kD_1} + \quad \text{(A2)}$$
$$IC_0(1-d)e^{-kD_1} \frac{1}{r^2}\left\{R\left(1-e^{-r\frac{\lambda}{R}T}\right) - \lambda(1-e^{-rT})\right\}$$

and $q(T) = 1 - e^{-KT}$ (A3)

We see, $p(T) \ge 0$ for all $T > 0$ and

$$\frac{dp(T)}{dT} = \lambda(1-d)C_0 e^{-kD_1} +$$
$$IC_0(1-d)e^{-kD_1} \frac{1}{r^2}\left\{r\lambda e^{-r\frac{\lambda}{R}T} - r\lambda e^{-rT}\right\}$$

$$\frac{d^2p(T)}{dT^2} = IC_0(1-d)e^{-kD_1}\left\{\lambda e^{-rT} - \frac{\lambda^2}{R^2} e^{-r\frac{\lambda}{R}T}\right\}$$

> 0 for $T > 0$.

Hence, $p(T)$ is convex for $T > 0$.

Now from (A3), we see $q(T) > 0$ for all $T > 0$ and

$$\frac{dq(T)}{dT} = Ke^{-KT}$$

$$\frac{d^2q(T)}{dT^2} = -K^2 e^{-KT} < 0 \quad \text{for all } T > 0$$

Hence, $q(T)$ is concave for $T > 0$.

Hence, using the above results $PW1_\infty(T)$ is pseudo convex.

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