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MASS FORCES' DEPENDENCE ON THE SPEED AND MATHEMATICAL MODELS OF SOME NATURAL PHENOMENA

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ABSTRACT

In the methods for solving the equations of hydrodynamics, it is assumed that the mass forces are external and do not depend on the fluid velocity. Here we consider the case when the mass forces depend on the velocity. Such forces are friction forces, inertial forces, centrifugal forces, and Coriolis forces. It is used the statement proved by the author that the centrifugal forces and the Coriolis forces are real forces. On this basis, mathematical models of such natural phenomena as sea currents, tsunamis, dust whirlwinds, waterfalls, funnels, and whirlpools are proposed.

Keywords: Equations of hydrodynamics, friction forces, inertial forces, centrifugal forces, Coriolis forces, numerical model, sea currents, tsunamis, dust whirlwinds, waterfalls, funnels and whirlpools.

INTRODUCTION

In the methods for solving the hydrodynamic equations, it is assumed that the mass forces are external and do not depend on the velocity of the fluid (Khmelnik, 2010). Here we consider the case when the mass forces are a function of the velocity. Such forces are friction forces, inertial forces, centrifugal forces, and Coriolis forces.

Further, only stationary problems for a viscous incompressible fluid are considered. In Chapter 5 by Khmelnik (2010), it was shown that for absolutely closed systems, the equations of hydrodynamics take the following form:

$$\mu \cdot \Delta v + \rho \cdot F = 0 \tag{1}$$

A method for resolving such equations was proposed but it is not suitable in the case under consideration due to the dependence of mass forces on the speed. Next, we consider some special cases for which the solution of equation (1) can be obtained in analytical form.

After resolving these equations, the pressure is calculated according to the following equation:

$$\nabla p = -\frac{\rho}{2}\nabla(W^2) \tag{2}$$

where

$$W^2 = \left(v_x^2 + v_y^2 + v_z^2\right) \tag{3}$$

$$\nabla(W^2) = \begin{bmatrix} v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \\ v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \\ v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \end{bmatrix} \tag{4}$$

In equation (2), on the left there is the pressure gradient and on the right there is the density gradient of the kinetic energy of the flow. The volume density gradient integral determines the energy in the liquid volume as follows:

$$W_V = \frac{\rho}{2}\int_V \nabla(W^2) dV \tag{5}$$

This integral assumes that there is a volume center (the beginning of integration) where the energy density is equal to zero. The same integral determines the total pressure in the volume (if the pressure at the center is known) which has no physical meaning. Thus, the pressure at the center is an undefined constant of integration and cannot be determined by this method.

When this flow appeared, it displaced the environment, which was at a pressure p_0. It can be, for instance, the air at the atmospheric pressure p_0. Therefore, we can take the value of p_0 as the pressure at the zero point of integration. Then the pressure at any point of the flow can be found as follows:

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$$p = p_0 + \frac{\rho}{2} W \tag{6}$$

Fluid jet

The author has already considered the mathematical model of the fluid jet based on the equations of gravitomagnetism (Khmelnik, 2016). Another model based on the use of Coriolis forces is proposed below. It is possible that these models can be combined but no such attempt is made here and no comparison is made between the old and the new models.

It was pointed out in (Khmelnik, 2016) that experiments make it possible to establish only that a vortex is formed when the flow velocity exceeds a certain threshold value (Fernandez-Feria and Sanmiguel-Rojas, 2000). Another phenomenon observed when water flows out of a pipe is that the water compressed at the outlet of the pipe expands relatively quickly again (Prandtl and Tietjens, 1934). However, below we will consider a jet that retains its shape over a long distance.

Such a jet is created in machines for "hydroabrasive cutting that represents the processing of materials by cutting, where a jet of water or a suspension of abrasive material in water is used as a cutting tool instead of a cutter, emitted at high speed and under high pressure" (https://ru.wikipedia.org/wiki/Гидроабразивная_резка, in Russian) as shown in Figure 1. This method is applicable to cutting highly hard and brittle materials with high precision, safety, and efficiency.

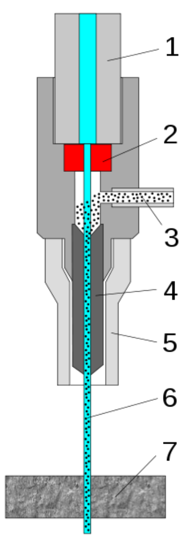

Fig. 1. The scheme of the installation of waterjet cutting (1 - high pressure water supply, 2 - nozzle, 3 - abrasive supply, 4 - mixer, 5 - casing, 6 - jet hit, 7 - cut material.)

Pressurized water is passed through the hole in the cutting head, which is a diamond, ruby or sapphire with a hole smaller than the point of a pin. As the water passes, its pressure and velocity increase **radically** to more than 6.2 × 10^8 [N/m^2] and 1000 [m/s] (water jet cutting, https://www.iqsdirectory.com/articles/water-jet-cutting.html). It is interesting to note that abrasive particles are added to water **after** it has acquired a high velocity under high pressure (Fig. 1).

It follows from these remarks that the power of the jet increases gradually and exceeds the power of the motor at the inlet of the jet. For some reason, the developers of this high-class method do not pay attention to this. Everything seems obvious: "*in nature, such a process, which occurs naturally, is called water erosion*" (https://ru.wikipedia.org/wiki/Гидроабразивная_резка, in Russian). Further, as a result of the simulation, it will be shown that the work of the jet is the work of the Coriolis forces. The opinion about the fictitiousness of these forces is very tenacious. However, a mathematically substantiated proof of the physical existence of these forces (Khmelnik, 2020, 2022) and examples of constructions (Khmelnik, 2023) have recently appeared, the existence of which is inexplicable in the absence of physical Coriolis forces.

The jet is an absolutely closed system and, consequently, is described by equation (1). In our case, the Coriolis forces are unknown, because depend on speeds. Therefore, it is necessary to perform the calculation in some additional assumption. We will assume that the jet boundaries (its radius) are known and the longitudinal velocity at the jet boundary is known.

It is convenient to consider the jet in the system of cylindrical coordinates $\{r, \varphi, z\}$. It observes the speeds of mass particles $\{v_r, v_\varphi, v_z\}$ directed along the radius, along the circumference, and along the central axis, respectively. Speed v_φ corresponds to the following angular speed:

$$\omega = v_\varphi / r \tag{7}$$

directed along the radius and along the central axis, respectively. There is still gravity. In the presence of the angular velocity ω, the centrifugal and Coriolis forces arise as follows:

$$F_r = 2\rho\omega v_z + \rho\omega^2 r \tag{8}$$

$$F_z = 2\rho\omega v_r \tag{9}$$

directed along the radius and along the central axis, respectively. There is the following force of gravity:

$$F_g = \rho g \tag{10}$$

In this case, equation (1) of hydrodynamics has the following form:

$$\mu \cdot \Delta v_r + 2\rho v_z \omega + \rho \omega^2 r = 0 \tag{11}$$
$$\mu \cdot \Delta v_\varphi + F_\varphi = 0 \tag{12}$$
$$\mu \cdot \Delta v_z + 2\rho\omega v_r - \rho g = 0 \tag{13}$$

where F_φ is an unknown function and

$$\Delta v = \begin{bmatrix} \Delta v_r \\ \Delta v_\varphi \\ \Delta v_z \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{r}+r\right)\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \phi^2} + \frac{\partial^2 v_r}{\partial z^2} \\ \left(\frac{1}{r}+r\right)\frac{\partial^2 v_\phi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{\partial^2 v_\phi}{\partial z^2} \\ \left(\frac{1}{r}+r\right)\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \phi^2} + \frac{\partial^2 v_z}{\partial z^2} \end{bmatrix} \tag{14}$$

The Coriolis forces bring momentum and energy to the system. We can write the following equation of the law of conservation of momentum for such a movement:

$$(F_z + F_r + \rho g)dt = -\rho\left(dv_r + dv_z + \frac{1}{2} r \cdot d\omega\right) \tag{15}$$

where on the left there is the momentum added by the acting forces during the time *dt* and on the right there is the change in the momentum of the ring over the same time. Obviously,

$$dt = \frac{dz}{v_z} \tag{16}$$

Using equations (8), (9), (15), and (16), we get:

$$(2\omega v_r + 2\omega v_z + g)dz = -v_z\left(dv_r + dv_z + \frac{1}{2} dv_\varphi\right) \tag{17}$$

or

$$(2\omega v_r + 2\omega v_z + \omega^2 r + g) = -v_z\left(\frac{dv_r}{dz} + \frac{dv_z}{dz} + \frac{1}{2}\frac{dv_\varphi}{dz}\right) \tag{18}$$

Thus, the presence of the angular velocity ω, i.e. the rotation of the vortex follows from the law of conservation of momentum.

We will look for a solution to the set of equations (11), (12), and (13) in the following form:

$$v_r = b_{r0}(r) + b_r(r) \cdot \exp(\alpha\varphi + \chi z) \tag{19}$$
$$v_\varphi = b_\varphi(r) \tag{20}$$
$$v_z = b_{z0}(r) + b_z(r) \cdot \exp(\alpha\varphi + \chi z) \tag{21}$$

where α and χ are some constants and $b(r)$ are unknown functions of the argument r. Then equations (11), (12), and (13) after reduction by common factors will take the following form:

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_r}{\partial r^2} + \frac{\alpha^2}{r^2} b_r + \chi^2 b_r + \mu_{\text{inv}}\rho(2b_z\omega + \omega^2 r) = 0 \tag{22}$$

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_\varphi}{\partial r^2} + \frac{\alpha^2}{r^2} b_\varphi + \chi^2 b_\varphi + F_\varphi(r) = 0 \tag{23}$$

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_z}{\partial r^2} + \frac{\alpha^2}{r^2} b_z + \chi^2 b_z + 2\mu_{\text{inv}}\rho b_r \omega = 0 \tag{24}$$

$$2\rho b_{r0}\frac{b_\varphi}{r} - \rho g = 0 \tag{25}$$

$$2b_{z0}\frac{b_\varphi}{r} + \omega^2 r = 0 \tag{26}$$

where

$$\mu_{\text{inv}} = \frac{1}{\mu} \tag{27}$$

Using equations (8) and (25), we find:

$$b_{r0} = g/(2\omega) \tag{28}$$
$$b_{z0} = -0.5\omega r \tag{29}$$

From equation (23) follows that

$$\frac{dv_\varphi}{dz} = 0 \tag{30}$$

With equations (30) and (18), we find:

$$(2\omega v_r + 2\omega v_z) = -v_z\left(\frac{dv_r}{dz} + \frac{dv_z}{dz}\right) - g \tag{31}$$

Utilizing expressions (31), (19), and (21), we can obtain that

$$2\omega(b_r + b_z) = -b_z\chi(b_r + b_z) - g \tag{32}$$

or

$$\omega = -0.5(b_z\chi + g/(b_r + b_z)) \tag{33}$$

Exploiting expressions (33), (20), and (7), we find that

$$b_\varphi = -\frac{r}{2}(b_z\chi + g/(b_r + b_z)) \tag{34}$$

The following two expressions follow from equations (22), (24), (7), and (30):

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_z}{\partial r^2} + \frac{\alpha^2}{r^2} b_z + \chi^2 b_z + 2\mu_{\text{inv}}\rho b_r b_\varphi = 0 \tag{35}$$

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_r}{\partial r^2}+\frac{\alpha^2}{r^2}b_r+\chi^2 b_r+2\mu_{\text{inv}}\rho b_z b_\varphi=0 \quad (36)$$

One of the solutions to this set of equations can be:

$$b_r=b_z \quad (37)$$

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_z}{\partial r^2}+\frac{\alpha^2}{r^2}b_z+\chi^2 b_z+2\mu_{\text{inv}}\rho b_z b_\varphi=0 \quad (38)$$

So, the solution is reduced to resolving equations (37), (38), and (30), from which we find:

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_z}{\partial r^2}+\frac{\alpha^2}{r^2}b_z+\chi^2 b_z+\mu_{\text{inv}}\rho r b_z^2(\chi+g/2)=0 \quad (39)$$

$$b_\varphi=-\frac{r}{2}\left(b_z\chi+g/(2b_z)\right) \quad (40)$$

Let's also determine the energy flux density along the jet:

$$S_z=F_z v_z \quad (41)$$

or

$$S_z=2\rho\omega v_z^2=2\rho\omega b_z^2\exp^2(\chi z) \quad (42)$$

It follows that at $\chi > 0$ the energy flux increases as the jet moves, which is a consequence of the work of the Coriolis forces that constantly add energy to the jet. The power carried by the entire jet at $\exp^2(\chi z) \approx 1$ is equal to

$$S=\int_0^R 2\rho\omega b_z^2\cdot 2\pi r\cdot dr=4\rho\pi\int_0^R \omega b_z^2 r\cdot dr \quad (43)$$

Finally, we find the pressure inside the jet by using formulae (5) and (6) as follows:

$$p=-\frac{\rho}{2}\left(2b_z^2+b_\varphi^2\right) \quad (44)$$

This pressure solidifies the jet and holds the liquid within the jet.

Example 1.
Consider the conditions of one process from (https://ru.wikipedia.org/wiki/Гидроабразивная_резка, in Russian). "Water, compressed by one of the main components of the system, a high-pressure pump (4000 bar), passes through a water nozzle, which forms a jet with a diameter of 0.35 [mm], which enters the mixing chamber. In the mixing chamber, water is mixed with an abrasive (granite sand) and then it passes through a second, hard-alloy or diamond nozzle with an inner diameter of 1 [mm]. From this nozzle, a jet of water with an abrasive comes out at a speed of by about 1000 [m/s] and hits the surface of the material being cut".

Figure 2 shows the results of the calculation in the SI system for R = 0.001, $v_z = v_r$ = 1000. In Figure 2, the designations bz2, bz1, bz, omega, FzC, and Ff respectively stand for $(\partial^2 b_z)/(\partial r^2)$, $(\partial^2 b_z)/(\partial r^2)$, $b_z = b_r$, ω, $F_z = F_r$, and F_φ as functions of the radius r. The total power carried by the entire jet is $S = 1.6\cdot\times 10^7$ [W]. Finally, we find the pressure inside the jet by using expression (6): $p = 10^9$ [N/m^2].

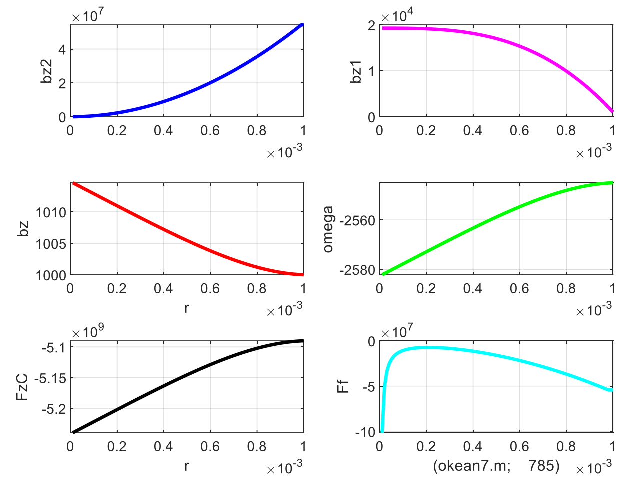

Fig. 2. The calculated parameters for R = 0.001, $v_z = v_r$ = 1000.

Example 2.
Example 1 was calculated for water with ρ = 1000 and μ = 0.0009. For water with an abrasive, we will assume that the density increased to ρ = 5000, and the coefficient of internal friction did not remain equal to μ = 0.0009. In this case we find that $S = 8.5\cdot\times 10^7$ [W], $p = 5\cdot\times 10^9$ [N/m^2].

Example 3.
For comparison, let's treat an air jet for which there are ρ = 1.3 and $\mu = 1.5\cdot\times 10^{-5}$. In this case, we find:
$S = 2.1\cdot\times 10^4$ [W], $p = 1.3\cdot\times 10^6$ [N/m^2].

Example 4.
Finally, let's consider an air jet with an abrasive. For this jet we can assume that ρ = 5000 and $\mu = 1.5\cdot\times 10^{-5}$. In this case we find that $S = 3.6\cdot\times 10^9$ [W] and $p = 5\cdot\times 10^9$ [N/m^2]. Thus, it is shown that the jet acquires energy as it propagates, and this energy is delivered to it by the Coriolis forces.

Sea currents and tsunamis
Sea currents, water and sand tsunamis can be represented as a limited or closed strip moving at a constant speed and at the same time maintaining the shape of the section along its entire length. These phenomena are striking in their grandiosity, organization, and demonstration of the

existence in its volume of an inexhaustible source of colossal energy. What is the internal structure of such a band and how is its engine arranged? Knowing the answers to these questions, one can further speculate about the cause of such phenomena. But first of all, the reader needs to find answers to the first questions, which, in principle, the reader can try to do without knowing the conditions for their occurrence.

First of all, there is a desire to identify such phenomena with an electromagnetic wave because it is also infinite, preserving its shape, moving and carrying energy. But in a vacuum, such a wave does not waste energy on its propagation path and therefore, does not need an internal source of energy. There are always internal losses in the fluid flow and therefore such a source is necessary. We will immediately begin with the assertion that the driving forces in the phenomena under consideration are the Coriolis forces. The opinion about the fictitiousness of these forces is very tenacious. However, a mathematically substantiated proof of the physical existence of these forces (Khmelnik, 2020, 2022) and examples of constructions (Khmelnik, 2023) have recently appeared, the existence of which is inexplicable in the absence of physical Coriolis forces.

The author has already considered mathematical models of flows and tsunamis based on the equations of gravitomagnetism (Khmelnik, 2017). Another model based on the use of Coriolis forces is proposed below. It is possible that these models can be combined but no such attempt is made here and no comparison is made between the old and the new models.

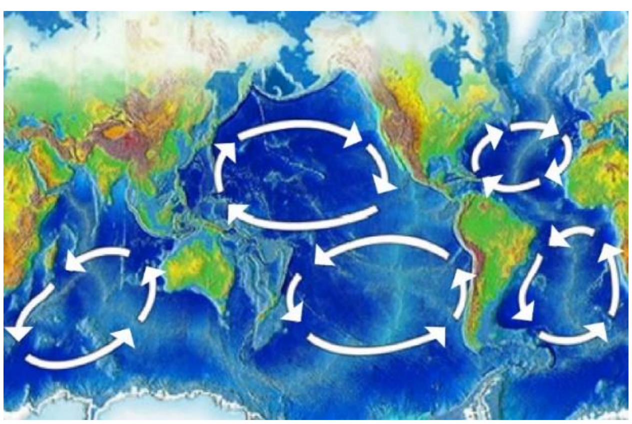

Fig. 3. Five major oceanic cycles.

The accepted ideas about the causes of ocean currents do not agree well with the existence of closed flow trajectories and the stability of the configuration and shape of the section. Typically, there are gradient currents, wind-driven currents, and tidal currents. These factors can cause the emergence of currents but they cannot support the existence (for centuries) of a closed trajectory of the current (since oppositely directed sections of this trajectory must be subjected to oppositely directed influences). However, flows, as a rule, are closed (as can be seen in Figure 3 that was borrowed from (https://ru.wikipedia.org/wiki/Общая_циркуляция_океана, in Russian)). Also, to explain these phenomena, they usually point to differences in the composition and properties of the waters of the jet and the surrounding waters. It is more natural (in our opinion) to assume that these differences are a consequence of the isolation of the jet, and not the cause of this isolation.

To explain the reasons for the existence of a tsunami, they usually point to the initial shock from an earthquake, and to predict the behavior of a tsunami (which is extremely important for practice), statistics from past tsunamis and seabed maps are used. Dozens of institutes and hundreds of scientists are engaged in this, see in (Kulikov *et al*., 2016). However, here we are interested in the <u>general patterns of tsunami motion</u>. Let's look at the pictures in Figure 4 that was borrowed from (Khmelnik, 2017). It seems unconvincing idea of the initial push as the reason for the long movement of this colossus. It seems that inside this "device" there is its own engine, and the resistance of the environment is only a catalyst, a force that presses on the gas pedal. So, there must be internal forces that ensure the movement, stability of the configuration and cross-sectional shape of these flows.

The jet is an absolutely closed system and consequently, is described by equation (1). In our case, the Coriolis forces are unknown, because depend on speeds. Therefore, it is necessary to perform the calculation in some additional assumption. We will assume that the jet boundaries (its radius) are known and the longitudinal speed at the jet boundary is known.

We will consider the mathematical model of the jet in the system of rectangular coordinates $\{x, y, z\}$. We will also assume that an ideal tsunami has the form of a flat wall with the x-, y-, z-axes directed horizontally along the wall, vertically, and along the thickness, respectively. In equation (1), the Lagrangian Δv in the rectangular coordinates is determined by the following formula:

$$\Delta v = \begin{bmatrix} \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \\ \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \\ \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \end{bmatrix} \quad (45)$$

The mass Coriolis forces created by the rotation of the Earth with the angular velocity ω_o can be define as

$$F_x = 2\rho\omega_{\mathrm{o}} v_z \quad (46)$$
$$F_z = 2\rho\omega_{\mathrm{o}} v_x \quad (47)$$

Fig. 4. The possible natural phenomena with colossal energies.

For middle latitudes there is $\omega_{\mathrm{o}} \approx 10^{-5}$. We define the mass force of gravity as

$$F_y = -g\rho \quad (48)$$

Let's rewrite equation (1), taking into account formulas (45)-(48):

$$\mu \cdot \Delta v + \rho \begin{bmatrix} 2\omega_{\mathrm{o}} v_z \\ -g \\ 2\omega_{\mathrm{o}} v_x \end{bmatrix} = 0 \quad (49)$$

In this case, our problem takes the form of the following set of three equations with three unknowns v_x, v_y, v_z:

$$\mu \cdot \Delta v_x + 2\rho v_z \omega_{\mathrm{o}} = 0 \quad (50)$$
$$\mu \cdot \Delta v_y - \rho g = 0 \quad (51)$$
$$\mu \cdot \Delta v_z + 2\rho v_x \omega_{\mathrm{o}} = 0 \quad (52)$$

Next, we will look for a solution in the following form:

$$v_x = b_x \mathrm{ex} \quad (53)$$
$$v_y = b_y \quad (54)$$
$$v_z = b_z \mathrm{ex} \quad (55)$$
$$\mathrm{ex} = \exp(\beta x + \alpha y + \chi z) \quad (56)$$

where b_x, b_y, b_z, α, β, χ are some constants. Substituting formulas (53), (54), and (55) into (45), after differentiation we get:

$$\begin{bmatrix} \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix} = \begin{bmatrix} \beta^2 b_x \mathrm{ex} + \alpha^2 b_x \mathrm{ex} + b_x \chi^2 \mathrm{ex} \\ b_y \\ \beta^2 b_z \mathrm{ex} + \alpha^2 b_z \mathrm{ex} + b_z \chi^2 \mathrm{ex} \end{bmatrix} \quad (57)$$

or

$$\begin{bmatrix} \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix} = \begin{bmatrix} b_x \mathrm{ex} \\ b_y \\ b_z \mathrm{ex} \end{bmatrix} \quad (58)$$

Substituting formulae (53), (54), and (58) into equations (50), (51), and (52), we get:

$$b_x \mathrm{ex} + 2q b_z \omega_{\mathrm{o}} \mathrm{ex} = 0 \quad (59)$$
$$b_y - qg = 0 \quad (60)$$
$$b_z \mathrm{ex} + 2q b_x \omega_{\mathrm{o}} \mathrm{si} \quad = 0 \quad (61)$$

where

$$q = \frac{\rho}{\mu} \quad (62)$$

One of the solutions to the set of equations (59) and (61) can be:

$$b_x = b_z \tag{63}$$
$$b_x = 2q\omega_o \tag{64}$$

Utilizing equation (60), we also get:

$$b_y = qg \tag{65}$$

With formulas (46), (47), (53), (55), (63)-(65), we get:

$$v_x = v_z = 2q\omega_o \mathrm{ex} \tag{66}$$
$$F_x = F_z = (2\rho\omega_o)^2 \mathrm{ex} \tag{67}$$

Let us also determine the following density of energy fluxes circulating in a rectangular jet along the coordinate axes:

$$S_x = F_x\, v_x, \;\; S_z = F_z\, v_z \tag{68}$$

Using formulae (66), (67), and (68) we get:

$$S_x = S_z = (2\rho\omega_o)^3 \mathrm{ex}^2 \tag{69}$$

Thus, we have determined the analytical description of a jet with a rectangular cross-section. Specific values of all parameters can be determined with known statistics of measurements of such jets (ocean currents and tsunamis). The author does not have such information and would be glad to cooperate in any form.

Note that for $\chi > 0$, the value of the exponent increases with increasing z. Using this statement and equality (69), it follows that <u>the energy flux increases as the jet moves</u>. This is a consequence of the work of the Coriolis forces, which constantly add energy to the jet. That is why sea currents do not fade, and tsunamis accelerate and expand.

Finally, we find the pressure inside the jet by using formula (6):

$$p = -\frac{\rho}{2}\left(v_z^2 + b_y^2 + v_z^2\right)\mathrm{ex}^2 \tag{70}$$

This pressure turns the jet into an almost solid body and keeps the liquid in the jet volume.

Dust whirl and spinning top

A dust whirlwind is widely known, which is an almost vertical column of dust (Figs. 5 and 6). Such a vortex has a vertical axis of rotation, a height of several tens of meters, a diameter of several meters, a speed inside the vortex of by about 10 [m/s] and a lifetime of several tens of seconds (dust whirlwind, https://ru.wikipedia.org/wiki/Пыльный_вихрь, in Russian). There are also phenomena similar to it such as air, ash, water “whirlwinds”, see in Figure 7 borrowed from (underwater tornado in Aruba, https://www.youtube.com/watch?v=TenvpcI2HUE). This vortex has a diameter of by about 0.1 [m]. The causes of the occurrence of sand vortices are considered to be various atmospheric phenomena (wind, heating of the atmosphere). However, the very existence of a sand whirl - the preservation of form and movement is difficult to explain by the same reasons. In addition, such vortices also exist and move on Mars, where there is no atmosphere (dust whirlwind, https://ru.wikipedia.org/wiki/Пыльный_вихрь, in Russian). Therefore, when explaining such vortices, the main questions are about the source of energy and the causes of stability in such an unusual form.

Fig. 5. The dust whirlwind in a rural area.

Fig. 6. The dust whirlwind on a town street.

Khmelnik (2017) has already considered the mathematical model dust vortex. In contrast to this, the proposed model below, where the Coriolis forces are explicitly used [the reality of which is substantiated by Khmelnik (2020, 2022)], as internal forces that ensure the stability of the vertical jet. It is possible that both of these models can be combined, but no such attempt is made here and no comparison is made between the old and new models.

Next, the question is considered: "How does a dusty whirlwind is arranged?" And the question "How does it arise?" remains unanswered.

Fig. 7. The underwater tornado in Aruba (water "whirlwind").

Figure 8 shows the top in its simplest form. In this case, a top with mass m has the form of a cylinder with radius R and height h, inclined to the horizontal plane at the angle α. The spinning top has

- initial rotation of the top around its own vertical axis with the angular velocity ω_1,
- rotation of a top inclined at the angle α to the horizontal plane around its own axis with the angular velocity ω_2,
- precession around the circumference of a top inclined at the angle α to the plane, with the angular velocity ω_3,
- linear precession velocity v; this is the speed of movement of p. B on the radius AB, rotating with the angular velocity ω_3 (Fig. 8):

$$v = \omega_3 h\cos(\alpha) \tag{71}$$

It was shown by Khmelnik (2023) that

$$\omega_3 = \frac{\pi R^2}{q + \frac{\pi R^2}{4}}\omega_1 \tag{72}$$

$$\omega_2 = \frac{(b-a)}{(a+b)}\omega_1 \tag{73}$$

where

$$q = h^2 - \alpha\left(\frac{2}{\pi}h^2 - R^2\right) \tag{74}$$

$$a = 4\pi R^2 \tag{75}$$

$$b = \pi^2 R^4 / q \tag{76}$$

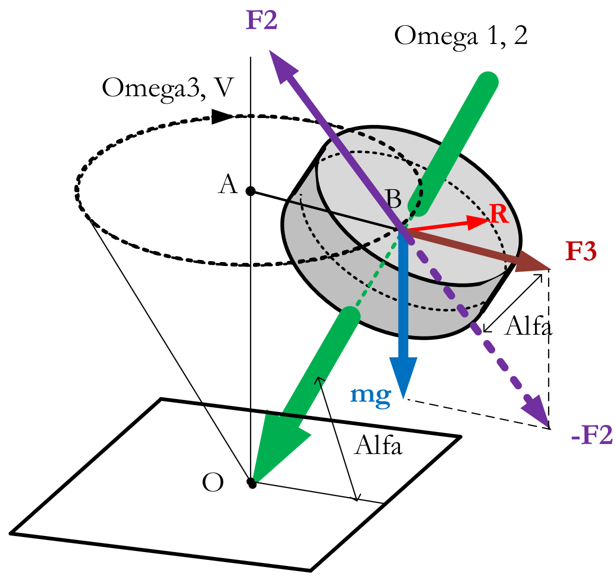

Fig. 8. The simplest form of the spinning top.

In Khmelnik (2023), a mathematical model of a top was constructed, in which the following designations for the acting forces and mechanical moments were adopted (Figure 8):

- Coriolis force acting on point B at radius AB,

$$F_2 = -2m\omega_2 \times v \tag{77}$$

- centrifugal force directed horizontally AB from the center,

$$F_3 = m\omega_3^2 h\cos(\alpha) \tag{78}$$

- force of gravity directed vertically down,

$$F_4 = -mg \tag{79}$$

- total force as a vector sum,

$$F_s = F_2 + F_3 + F_4 \tag{80}$$

- horizontal and vertical projections of this force, F_{sx} and F_{sy}, respectively;
- overturning moment of rotation of the top around the pivot point

$$M = hF_{sx}\sin(\alpha) - hF_{sy}\cos(\alpha) \tag{81}$$

- pressure force on the pivot point, directed along the rod,

$$T = -hF_{sx}\cos(\alpha) - hF_{sy}\sin(\alpha) \tag{82}$$

The mathematical model of the top constructed in this way makes it possible to answer the question of why the top does not fall. This question is not new. It was asked in 1890 by Prof. John Perry in (Perry, J. Spinning Tops. The "Operatives' Lecture" of the British Association Meeting at Leeds, 6th September, 1890, page 93). He wrote "*... in a spinning top, obviously, only with rotation does life and stability appear, or, in other words, only then do forces act that oppose the earth's gravity, which tends to overturn the spinning top. Where do these forces come from and how are they explained?*" The questioner intuitively feels that the initial push cannot give the energy that is needed for a long and vigorous rotation. The answer obtained in Khmelnik (2023) is that real (not fictitious) Coriolis and centrifugal forces act in the top.

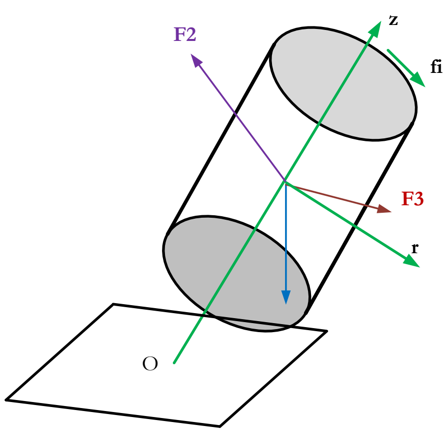

Fig. 9. The axes and forces for the cylinder.

The forces listed above, obviously, act on each point representing the elementary mass of the top. Let us consider such body forces in the cylindrical coordinate system $\{r, \varphi, z\}$, where the z-axis coincides with the axis of the cylinder, i.e. inclined at the angle α to the horizontal plane (Figure 9). Let's denote such body forces as F_r, F_φ, F_z. To calculate these forces, we note first that instead of the forces F_2, F_3, F_4, it is necessary to use, respectively, the forces

$$f_2 = \frac{\rho}{m}\frac{r}{R}F_2,\ f_3 = \frac{\rho}{m}\frac{r}{R}F_3, f_4 = \frac{\rho}{m}F_4 \tag{83}$$

Here the coefficient (ρ/m) appears because all the forces related to the entire mass of the top must refer to the mass unit ρ, the specific mass of the material. The coefficient (r/R) appears because the forces F_2 and F_3 calculated relative to the radius of the top must be calculated relative to the radius r of the current point. Denote the vector sum:

$$f_s = f_2 + f_3 + f_4 \tag{84}$$

We also denote the projections of this force on the z-axis and on the r-axis as f_{sz} and f_{sr}, respectively. Then we get:

$$F_r = f_{sr} \tag{85}$$
$$F_\varphi = 0 \tag{86}$$
$$F_z = f_{sz} \tag{87}$$

Suppose now that the <u>material of the top is a liquid but it nevertheless retains its cylindrical shape</u>. In such a liquid top, the laws of hydrodynamics must be observed, i.e. the mass forces F_r, F_φ, F_z that have arisen in it must cause internal flows representing mass fluid currents with velocities v_r, v_φ, v_z. It can be assumed that these internal flows do not change the distribution of masses inside the top and therefore do not change the mass forces and velocities of the top as a whole. However, the body forces F_r, F_φ, F_z will have to do work to overcome internal friction, i.e. the lifetime of such a top will be limited.

Obviously, hydrodynamics should be used to describe a liquid top. Such a top is an absolutely closed system and therefore, is described by equation (1).

We will consider the mathematical model of a liquid top in the system of cylindrical coordinates $\{r, \varphi, z\}$. Let us assume that there are velocities in it directed along these axes v_r, v_φ, v_z and which are functions of these coordinates.

Equation (1), taking into account expressions (85), (86), and (87), takes the following form:

$$\mu \cdot \Delta v_r + f_{sr} = 0 \tag{88}$$
$$\mu \cdot \Delta v_\varphi = 0 \tag{89}$$
$$\mu \cdot \Delta v_z + f_{sz} = 0 \tag{90}$$

where the Lagrangian Δv in the cylindrical coordinates is given by the following formula:

$$\Delta v = \begin{bmatrix} \Delta v_r \\ \Delta v_\varphi \\ \Delta v_z \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{r}+r\right)\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \phi^2} + \frac{\partial^2 v_r}{\partial z^2} \\ \left(\frac{1}{r}+r\right)\frac{\partial^2 v_\phi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{\partial^2 v_\phi}{\partial z^2} \\ \left(\frac{1}{r}+r\right)\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \phi^2} + \frac{\partial^2 v_z}{\partial z^2} \end{bmatrix} \tag{91}$$

With formulae (89) and (91), it follows that

$$\frac{\partial^2 v_\phi}{\partial r^2} = 0, \qquad \frac{\partial v_\phi}{\partial r} = C, \qquad v_\phi = Cr \tag{92}$$

where C is some constant. A priori we can write down that

$$v_\phi = \omega r \tag{93}$$

Therefore, formula (89) corresponds to formula (93). In this case, the set of equations (88), (89), and (90) degenerates into the following set of two equations:

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \phi^2}+\frac{\partial^2 v_r}{\partial z^2}+f_{sr}/\mu=0 \tag{94}$$

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \phi^2}+\frac{\partial^2 v_z}{\partial z^2}+f_{sz}/\mu=0 \tag{95}$$

The solution to this set of equations is:

$$v_r = b_r(r)\cdot \exp(\alpha\varphi+\chi z) \tag{96}$$
$$v_z = b_z(r)\cdot \exp(\alpha\varphi+\chi z) \tag{97}$$

where α and χ are some constants and $b(r)$ are unknown functions of the argument r. Then equations (94) and (95) after reduction by common factors will take the following forms:

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_r}{\partial r^2}+\frac{\alpha^2}{r^2}b_r+\chi^2 b_r+f_{sr}/\mu=0 \tag{98}$$

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_z}{\partial r^2}+\frac{\alpha^2}{r^2}b_z+\chi^2 b_z+f_{sz}/\mu=0 \tag{99}$$

The above-defined functions $f_{sr}(r)$ and $f_{sz}(r)$ can be resolved by numerical differentiation. The pressure inside the cylinder can be found by using formula (6) as follows:

$$p=-\frac{\rho}{2}\left(v_r^2+v_\varphi^2+v_z^2\right) \tag{100}$$

This pressure turns the liquid cylinder of the top into a solid body and keeps the liquid in the volume of the cylinder. Therefore, the liquid cylinder behaves like a solid top. The extension of this statement to the "dusty cylinder" requires more rigorous proofs. But for the time being we will be satisfied with a reference to the experiments set by nature: water and dust vortices are identical (Figures 5, 6, and 7). Another analogy follows from the second section: a jet of water behaves like a hard drill, when an abrasive is added to the water, this drill becomes even harder.

Waterfalls

The amazing ability of trout to climb up the waterfall is known. This issue, in particular, was actively dealt with by Viktor Schauberger, but his ideas did not take an acceptable form in science. Recently, videos of amazing experiments have appeared on the Internet, which, nevertheless, remove a touch of mystery in the abilities of trout. Figure 10 shows frames from the video film (https://drive.google.com/file/d/1Y2yUVJu4y5AxilCJP-lHBfhJ37I_5HQJ/view?usp=sharing), where a rigid spiral rises upstream or along a jet of water. Obviously, it is necessary to look for an explanation exclusively in the properties of the jet, assuming that the stream and the waterfall can break up into a set of jets.

Fig. 10. The video experiments with the jets.

Our task is to obtain such a mathematical model that would allow us to substantiate such amazing phenomena and experiments. Let's first consider the mathematical model of the jet adopted in the second section.

In the system of cylindrical coordinates $\{r, \varphi, z\}$, the jet hydrodynamic equations have the form of equations (11)-(14) and the momentum conservation law in the form of equation (18) is satisfied. These equations have a solution in the form of equations (19)-(21), (7), (28), (29). Let's rewrite these functions accordingly:

$$v_r = g/(2\omega)\ + b_r(r)\cdot\exp(\alpha\varphi+\chi z) \tag{101}$$
$$v_\varphi = b_\varphi(r) \tag{102}$$
$$v_z = -0.5\omega r + b_z(r)\cdot\exp(\alpha\varphi+\chi z) \tag{103}$$
$$\omega = v_\varphi/r = b_\varphi(r)/r \tag{104}$$

The unknown functions $\omega(r)$, $b_r(r)$, $b_z(r)$, and $b_\varphi(r)$ are the consequence of causes (34)-(36). Let's rewrite these equations:

$$\omega = -0.5(b_z\chi + g/(b_r+b_z)) \tag{105}$$
$$b_\varphi = \omega r \tag{106}$$

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_z}{\partial r^2}+\frac{\alpha^2}{r^2}b_z+\chi^2 b_z+2\mu_{\text{inv}}\rho b_r\omega r=0 \tag{107}$$

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_r}{\partial r^2}+\frac{\alpha^2}{r^2}b_r+\chi^2 b_r+2\mu_{\text{inv}}\rho b_z\omega r=0 \tag{108}$$

From the experiments considered in the introduction, it can be assumed that in the jet the central region rotates in one direction, and the outer region rotates in the other, and therefore, there is a certain radius r_0, for which there is the following equality:

$$\omega(r_0)=0 \tag{109}$$

With formulae (105) and (109), we find:

$$\left(b_z(r_0)\chi+g/\left(b_r(r_0)+b_z(r_0)\right)\right)=0 \tag{110}$$

or

$$b_r(r_0)=-g/\left(\chi b_z(r_0)\right)-b_z(r_0) \tag{111}$$

It follows from equations (107), (108), and (109) that for $r = r_0$ the following equation is executed:

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b}{\partial r^2}+\frac{\alpha^2}{r^2}b+\chi^2 b=0 \tag{112}$$

where $b = b_z$ or $b = b_r$. We will look for a solution in the following form:

$$b_z=A_z\cos(\beta r),\, b_r=A_r\cos(\beta r) \tag{113}$$

where A_z, A_r, β are some coefficients. Then equation (113) takes the following form:

$$-r^3+r^2\left(\frac{\chi}{\beta}\right)^2-r+\left(\frac{\alpha}{\beta}\right)^2=0 \tag{114}$$

This equation is easily resolved numerically and its solution is $r = r_0$. If the value of the constant A_z is defined, then the value of the constant A_r is determined by formula (111):

$$A_r\cos(\beta r_0)=-g/\left(\chi A_z\cos(\beta r_0)\right)-A_z\cos(\beta r_0) \tag{115}$$

or

$$A_r=-g/\left(\chi A_z\cos^2(\beta r_0)\right)-A_z \tag{116}$$

Example 1.
Let $\beta = 20$, $\alpha = 1.4$, $\chi = 2.5$. In the computer software MATLAB, polynomial (114) takes the following form:
p = [–1 0.0156 –1 0.005]; r_0 = roots(p) = 0.05;

This solution is the only real solution to this polynomial. Let A_z = – 4.5. Using formula (116) we find that A_r = – 5.34. With the parameters found, the functions $b_z(r)$, $b_r(r)$, $\omega(r)$ are determined, which are shown in Figure 11.

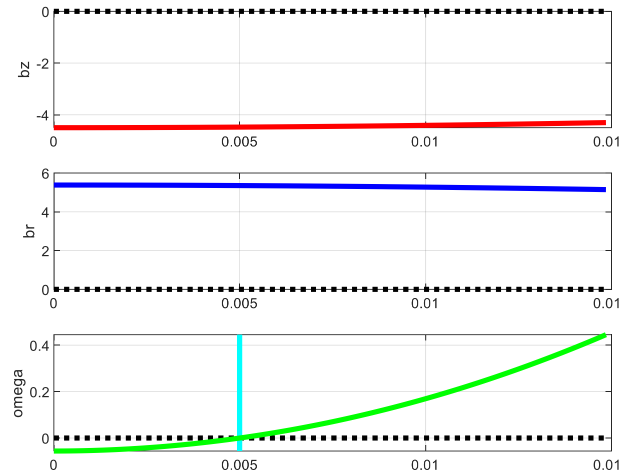

Fig. 11. The parameters ω, b_r, and b_z.

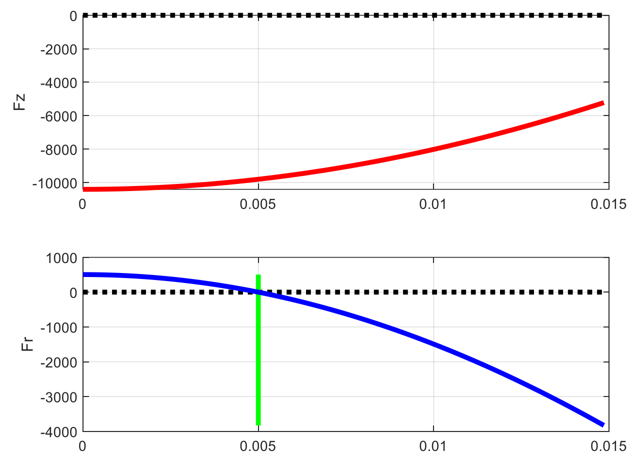

Fig. 12. The forces F_r and F_z.

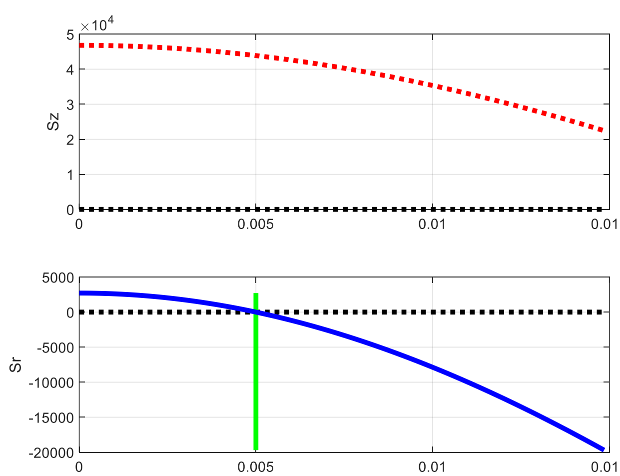

Fig. 13. The energy fluxes S_r and S_z.

It can be seen that at a constant direction of the longitudinal and radial velocities, the direction of rotation of the jet changes at a certain radius to the opposite.

The second section defines the mass forces such as the gravity forces, the centrifugal forces, and the Coriolis forces, see in formulas (8) and (9):

$$F_r = 2\rho\omega v_z + \rho\omega^2 r \tag{117}$$
$$F_z = 2\rho\omega v - \rho g \tag{118}$$

Figure 12 shows the graphs of the forces F_r and F_z. Let us also define the energy fluxes flowing along the radius and along the jet:

$$S_r = F_r v_r \tag{119}$$
$$S_z = F_z v_z \tag{120}$$

These functions are shown in Figure 13. It is seen that

• radial energy flows are directed oppositely and do NOT exit the volume of the jet, thanks to which its integrity is preserved,

• the longitudinal energy flow in the central part of the jet is directed upwards; it is these streams that carry the trout up the falls. It is important to note that the water flow in the central part of the jet is unidirectional with the flow in the outer part of the jet!

These facts are possible only if there is a rotation of the jet around the vertical axis, i.e. $\alpha \neq 0$. The obtained functions allow us to calculate the power P consumed by the jet and the flow rate Z of water in the jet:

$$S_z = 2\pi\left(\int_{r_o}^{r} S_z r dr - \int_0^{r_o} S_z r dr\right) \tag{121}$$
$$Z = 2\pi\left(\int_{r_o}^{r} b_z r dr - \int_0^{r_o} b_z r dr\right) \tag{122}$$

Remark to both the second section and the previous section

In these sections under consideration, in essence, the same mathematical problem is solved. However, the previous section uses the fact that polynomial (114) can have a solution at $r = r_0$ for $0 < r_0 < R$. The second section does **not** address the impact of such a decision, i.e. by default, it is assumed that such a solution exists for $r_0 > R$.

Funnels and whirlpools

First, we will consider a funnel with water. The issue of establishing the water cycle in a funnel and its connection with the direction of the Coriolis force, created by the rotation of the Earth, is very often discussed. The period of this rotation T depends, as is known, on the period of daily rotation of the Earth T_s and the latitude of the location of the funnel:

$$T = T_s/\sin(\alpha) \tag{123}$$

For $\alpha = \pi/4$ we have, for instance,

$$T = 24 \cdot \frac{3600}{\sin\left(\frac{\pi}{4}\right)} \approx 10^5 \tag{124}$$

The angular frequency of such rotation is $\omega_0 \approx 10^{-5}$. Such a speed of rotation of water in the funnel could not be noticeable. Therefore, the actual speed of rotation is not related to the rotation of the Earth. Therefore, in the mathematical description of the funnel, it can be considered as a structure that is motionless relative to the Earth.

The jet arising in the funnel will be considered by analogy with the jet considered in the second section. For the convenience of the reader, we rewrite the main formulas from the second section. In the jet, mass particle speeds v_r, v_φ, v_z are observed, directed along the radius, along the circumference, and along the central axis, respectively. Speed v_φ corresponds to the angular speed as follows:

$$\omega = v_\varphi/r \tag{125}$$

In a whirlpool, the rotation of water is a precession that occurs when the water moves towards the center of the whirlpool, which follows from the law of conservation of momentum and is proved in the same way as in the second section. In the presence of the angular velocity ω, the centrifugal, Coriolis, and gravity forces, respectively, arise as follows:

$$F_r = 2\rho\omega v_z + \rho\omega^2 r \tag{126}$$
$$F_z = 2\rho\omega v_r \tag{127}$$
$$F_g = \rho g \tag{128}$$

Verin (2016) performed a comprehensive analysis of a funnel, a waterfall, and other natural phenomena. However, he did not analyze the influence of the Coriolis forces and the reasons for the appearance of rotation. Therefore, the radial and vertical velocities were not considered. The following discussion refers only to the calculation of these velocities.

Hydrodynamics equation (1), taking into account the Coriolis forces, can have the following form:

$$\mu \cdot \Delta v_r + 2\rho v_z \omega + \rho\omega^2 r = 0 \tag{129}$$
$$\mu \cdot \Delta v_\varphi + F_\varphi = 0 \tag{130}$$
$$\mu \cdot \Delta v_z + 2\rho\omega v_r - \rho g = 0 \tag{131}$$

where F_φ is an unknown function and

$$\Delta v = \begin{bmatrix} \Delta v_r \\ \Delta v_\varphi \\ \Delta v_z \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{r}+r\right)\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \phi^2} + \frac{\partial^2 v_r}{\partial z^2} \\ \left(\frac{1}{r}+r\right)\frac{\partial^2 v_\phi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{\partial^2 v_\phi}{\partial z^2} \\ \left(\frac{1}{r}+r\right)\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \phi^2} + \frac{\partial^2 v_z}{\partial z^2} \end{bmatrix} \quad (132)$$

The Coriolis forces bring momentum and energy to the system. The momentum conservation law equation takes the following form:

$$(2\omega v_r + 2\omega v_z + \omega^2 r + g) = -v_z\left(\frac{dv_r}{dz} + \frac{dv_z}{dz} + \frac{1}{2}\frac{dv_\varphi}{dz}\right) \quad (133)$$

Thus, the presence of the angular velocity ω, i.e. the rotation of water in a funnel follows from the law of conservation of momentum.

We will look for a solution to the set of equations (129), (130), and (131) in the following form:

$$v_r = b_{r0}(r) + b_r(r)\cdot\exp(\alpha\varphi + \chi z) \quad (134)$$
$$v_\varphi = b_\varphi(r) \quad (135)$$
$$v_z = b_{z0}(r) + b_z(r)\cdot\exp(\alpha\varphi + \chi z) \quad (136)$$

where α and χ are some constants and $b(r)$ are unknown functions of the argument r. Then equations (129), (130), (131), and (133) will take the following form:

$$b_{r0} = g/(2\omega) \quad (137)$$
$$b_{z0} = -0.5\omega r \quad (138)$$
$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_z}{\partial r^2} + \frac{\alpha^2}{r^2}b_z + \chi^2 b_z + 2\mu_{\text{inv}}\rho b_r b_\varphi = 0 \quad (139)$$
$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_r}{\partial r^2} + \frac{\alpha^2}{r^2}b_r + \chi^2 b_r + 2\mu_{\text{inv}}\rho b_z b_\varphi = 0 \quad (140)$$
$$b_\varphi = -0.5r(b_z\chi + g/(b_r + b_z)) \quad (141)$$

Looking at photographs of waterfalls, one can notice that there is such a radius r_0, at which there are no radial currents in the area of smaller radius $r \le r_0$, i.e. $b_r = 0$. From here and from (139) it follows that r_0 is the root of the following equation:

$$\left(\frac{1}{r}+r\right)\frac{\partial^2 b_z}{\partial r^2} + \frac{\alpha^2}{r^2}b_z + \chi^2 b_z = 0 \quad (142)$$

Let's designate the function representing the solution of this equation as b_{z9}, and the value of this function at $r = r_0$ as bz_{90}. Thus, for $r = r_0$ we have:

$$b_r(r_0) = 0, b_z(r_0) = b_{z90} \quad (143)$$

The left side of equation (140) coincides with equation (142) and therefore, this part, as a function, also vanishes at $r = r_0$ and the value of this part is $b_z(r_0) = 0$, i.e.

$$b_r(r_0) = b_z(r_0) = 0 \quad (144)$$

Utilizing formulas (127) and (141), we find that

$$\omega(r_0) = \frac{b_\varphi(r_0)}{r_0} = 0 \quad (145)$$

i.e. the inner surface of the whirlpool opening does not rotate.

Thus, the whirlpool as the set of equations (139), (140), and (141) can be calculated in exactly the same way as the outer part of the jet in the fifth section. However, this calculation applies only to the surface of the whirlpool.

Verin (2016) showed that the shape of an ideal funnel is determined by an equation of the form:

$$h = H\left(1 - \left(\frac{R}{r}\right)^2\right) \quad (146)$$

where H is the undisturbed water level, h is the height of the funnel at the level of radius r, and R is the inner radius of the funnel (Figure 14). In this case, the linear speed of rotation b_φ does not depend on the depth. This means that it is enough to solve the equations for the surface with the coordinates (h, r), and then use the found functions b_r, b_z, and b_φ for any value of h.

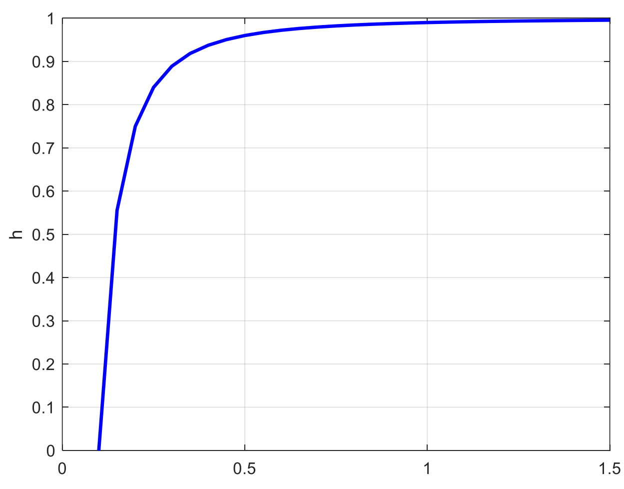

Fig. 14. The shape of the envelope of an ideal water funnel.

Verin (2016) writes the following:

What happens below the surface of a water funnel? In order to answer this question, we do not even need to make any additional calculations (this is one of the advantages of the ideal "portrait" of the phenomenon!). Indeed, knowing the shape of the funnel envelope, we can determine the imaginary surfaces of equal pressures under water. The fact is that these surfaces of equal pressures completely repeat the shape of the funnel surface itself, since the centrifugal force has a horizontal direction and is perpendicular to gravity, as a result of which the vertical pressure change is determined only by the magnitude of the water column (Figure 15). This figure shows the envelope lines of surfaces of equal pressures differing by the same value of ΔP. In fact, this is the same curve drawn several times with the same vertical offset.

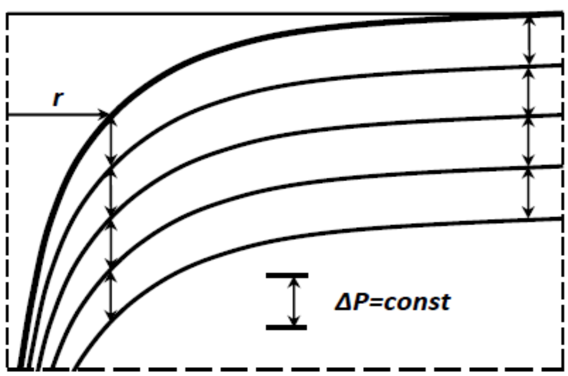

Fig. 15. The surfaces of equal pressure that follow the shape of a funnel.

When comparing the mathematical models of a waterfall and a whirlpool, the question remains unanswered: why does the waterfall jet have a central part with a water flow and an oppositely directed energy flow, while the central part is empty in a whirlpool?

CONCLUSION

Some natural phenomena related to the problems of hydrodynamics but not resolved by hydrodynamics methods are considered. The reason for this is that in these problems the main role is played by the Coriolis forces, and hydrodynamics does not operate with these forces. In turn, the reasons for this are, first, the existing idea of the fictitiousness of the Coriolis forces and second, the lack of methods for solving problems with velocity-dependent mass forces. In the article, these restrictions are removed and therefore, the corresponding mathematical problems are formulated and resolved.

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